MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Reversibilität und Dualität

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Zentrales Thema der Tagung war die Rolle von Zeitumkehr und Dualität in der Theorie der Markoffschen Prozesse und in einigen ihrer Anwendungen. Auf der theoretischen Seite wurde über verschiedene neuere Entwicklungen vorgetragen, insbesondere über Exkursionstheorie, terminale Verteilungen, Potentialtheorie Markoffscher Prozesse mit mehrdimensionalem Parameter, Reversibilität ohne Dualität.

Bei den Anwendungen ging es vor allem um Zeitumkehr und unendlich dimensionale Diffusionsprozesse in der Populationsgenetik, um den Zusammenhang von Reversibilität und Gibbs-Verteilungen bei der Zeitentwicklung von unendlichen Teilchensystemen und um stochastische Variationsprinzipien. Neben den 28 Vorträgen gab es informelle workshops über Prinzipien der Zeitumkehr, über Randwertprobleme in Verbindung mit der Feynman-Kac-Formel und über die Anwendung reversibler Diffusionsprozesse bei der Modellierung von Mesonen.

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Vortragsauszüge

REVERSIBILITY WITHOUT DUALITY

J. Azéma, Paris

One studies relationships between additive functionals, measures, and h-transforms when there don't exist duality hypothesis. Also, the recurrent case is discussed.

THE ASYMPTOTIC BEHAVIOR OF THE EMPIRICAL PROCESS OF BROWNIAN MOTION

E. Bolthausen, Berlin

Empirical measures of recurrent continuous time Markov processes seem to have a similar or even better asymptotic behavior than the empirical measures of independent random variables. This is made precise in the case of Brownian motion on the d-dimensional Torus T^d. Let ξ_t, $t \geq 0$, be this process and if A is a Borel subset of T^d, let $\mu_t(A) = \frac{1}{t}\int_0^t 1_A(\xi_s)ds$.

In dimension d=1 μ_t has continuous density with respect to Lebesque measure ℓ_t, the local time. Then, as $t \to \infty$, $t(\ell_t - 1)$ converges weakly in the space of continuous functions to the process $2(B(x) - \int_{T^1} B(y)dy)$, $x \in T^1$, where B is the ordinary Brownian bridge. Furthermore a.s. $(\sqrt{n/\log\log n})(\ell_t - 1)$ is relatively compact as $t \to \infty$ and has a suitable set of limit points.

In dimensions d>1 similar statements can be proved in suitable Sobolev spaces of generalized functions.

PROBLEMS OF REVERSING

Kai Lai Chung, Stanford

A brief review of reversing in Markov chains and in Brownian motion is given. A general Hunt process having a finite lifetime ζ can be reversed from ζ to yield a natural reverse which is left continuous and has the moderate Markov property (Chung and Walsh). This implies an analytic duality which is not quite the same as classical duality (as discussed by Getoor). When the latter duality is assumed the reversal leads to Nagasawa's formula.

Question 1: what does the natural reverse add to the structure of the process? Question 2: can we use true reversing to solve various problems clearly or unclearly involving reversing? For instance, can the last exit decomposition for the equilibrium potential be obtained in this way, as suggested by the physicist's notion of reversibility? A most challenging example is the Kellogg-Evans theorem in potential theory which became Hunt's Hypothesis (H). All known probability proofs of this result use some form of reversing. Why? and what is the natural condition for its validity? By contrast, Hunt's Hypothesis (B) has a reverse formulation due to Azéma. Finally two elementary problems of reversing in Brownian motion are mentioned.

LEVY SYSTEMS FOR CHUNG PROCESSES

E. Çinlar, Evanston

The theory of Chung processes does not fit in the extensive theory developed for "standard" or "right" processes because of the intricate discontinuity properties of the sample paths of the former. Although it is possible to transform a Chung process into a nearly Hunt process, the required Ray-Doob-Meyer-Knight compactification enlarges the state space, so that one loses the charm of the original discrete state space. Our objective is to illustrate another path for treating Chung processes without changing the state space. This involves using Lévy's ideas combined with the powerful tools developed for regenerative systems by Maisonneuve and Meyer (to cite only the principals). One is able to obtain a Lévy system fairly quickly, and using it, one can give various last exit-first entrance decompositions with greater ease than has been possible before.

TOWARD A POTENTIAL THEORY FOR SEVERAL MARKOV PROCESSES

E.B. Dynkin, Ithaca

If $X_t = (X^1_{t_1},\dots,X^n_{t_n})$ is a family of time-reversible Markov processes, then a class of measures M on the product of the state spaces with the finite energy integral can be used as a starting point for a potential theory. We call B a null set if $\mu(B) = 0$ for all $\mu \in M$. We construct a Dirichlet space as a completion of the set of potentials $f_n(x) = \int g(x,y)\mu(dy)$ $(g(x,y) = g'(x',y')$ $g''(x'',y'')$ where g' is the Green's function for X).

An element h of the Dirichlet space H is called regular if $E_\mu h(X_t) \to \mu(h)$ as $t \to o$ for all $\mu \in M$.

A class of harmonic functions associated with the family X_t is introduced and the Dirichlet problem for this class is solved for a certain class of domains. The main tool are additive functionals of the family X which correspond to all $\mu \in M$.

TRANSITION FUNCTIONS AND MARKOV-PROCESS MEASURES

M.P. Ershov, Essen

The two questions were discussed:

1) Given an initial distribution and a Kolmogorov-Chapman transition function, does there always exist the corresponding Markov-process measure?

2) If not always, under what possibly weak conditions there is such a measure?

These questions were considered in a joint paper by A. Wakolbinger and M. Ershov.

The answer to question 1 is "no" (A. Wakolbinger constructed a negative example). A positive result was formulated in terms of the existence of a "weak" disintegration for three-dimensional distributions (determined by the initial distribution and transition function) with respect to the two "outer" time-points.

STOPPED DISTRIBUTIONS FOR MARKOV PROCESSES IN DUALITY

N. Falkner, Columbus

Let X, $\hat{X}$ be standard Markov processes in duality. Assume that semipolar sets are polar and that the resolvents are strong Feller. Let μ be a measure such that μU is sigma finite and let ν be another measure. If $\mu U \geq \nu U$ and if there exists a set C such that for every polar set Z, $\nu(Z) = \mu(Z \cap C)$ then there exists an $(\mathcal{F}_t)$-stopping time T such that $\mu P_T = \nu$ (and conversely). Here $(\mathcal{F}_t)$ is the usual completion of $(\mathcal{F}_t^o)$, where $\mathcal{F}_t^o = \sigma(X_s : 0 \leq s \leq t)$.
At the time of my talk, I had assumed that $\hat{X}$ was continuous on $[0,\hat{\zeta})$ and without holding points. I thank Joe Glover for helpful discussions which led

to the elimination of these unnecessary hypotheses. After further discussions with Joe and Michael Sharp, I am quite optimistic that the strong Feller hypothesis can also be dropped.

DUALITY WITH RESPECT TO SPACE REVERSAL

H. Föllmer, Zürich

Let (X_t) be a Markov process with semigroup (P_t) on some state space E, and let C be a convex class of functions on E which admits an integral representation in terms of probability measures on some space $\hat{E}$, i.e. $C = \{\int K(\cdot,y)\mu(dy) | \mu \text{ p.m. on } \hat{E}\}$. If C is invariant under (P_t), then we can introduce a dual process $(\hat{X}_t)$ with semigroup $(\hat{P}_t)$ on $\hat{E}$ via the relation $P_t K(\cdot,y)(x) = \int K(x,z)\hat{P}_t(y,dz)$.

We consider in particular the case $K(x,y) = I_{\{x \geq y\}}$ with some partial ordering on E and give applications to some infinite particle systems with an ordered state space at each site.

STATIONARY MEASURES OF REVERSIBLE PROCESSES IN DIMENSIONS ONE AND TWO

J. Fritz, Budapest

We consider interacting diffusion processes in $\mathbb{R}^{\mathbb{Z}^d}$ of type

$$dx_k = C_k(x)dt + \sigma_k(x)dW_k, \quad k \in \mathbb{Z}^d,$$

where W_k is a family of independent Wiener processes. If $U_V(X)$ is an interaction and $H_k = \sum_{V \ni k} U_V(X)$ then such processes are formally reversible with respect to Gibbs measures with interaction U_V if

$$C_k = \frac{1}{2} \exp(H_k(X)) \frac{\partial}{\partial X_k}(\sigma_k^2(x)\exp(-H_k(x))).$$

Using a reversing and coupling technique involving free energy we show that under some natural regularity conditions every stationary measure is a Gibbs state, at least if $d \leq 2$. Earlier results by Holley-Stroock and by Fritz are improved in the sense that the restriction $\sigma_k(x) = \sigma(x_k)$ is removed.

DUALITY, REVERSIBILITY, AND EXCURSIONS

R.K. Getoor, San Diego

The basic assumptions and notations for a pair of standard processes X and $\hat{X}$ in duality relative to a σ-finite measure $\xi(dx) = dx$ on a Lusinian state space E were recalled. Some extensions of these were then noted. Especially important being the notion of a dual density p(t,x,y). The relationsship between this concept of dual density for X and $\hat{X}$ and the ordinary duality of space-time processes over X and $\hat{X}$is explained. Of particular importance is the fact that if X and $\hat{X}$ have dual density, then so do (X,T) and ($\hat{X}$,$\hat{T}$) where T and $\hat{T}$ are dual terminal times-(X,T) being X killed at T.

The main results have to do with excursions from a closed optional homogeneous set M which under the present assumptions has the form

$$M = <t>0: (X_{t-}, X_t) \in \Gamma\}$$

where $\Gamma \in \mathcal{E} \times \mathcal{E}$. Then the dual of M, $\hat{M}$, is defined by

$$\hat{M} = \{t>o: (X_{t-}, X_t) \in \hat{\Gamma}\}$$

where $\hat{\Gamma} = \{(x,y): (y,x) \in \Gamma\}$. Let R be the debut of M and $\hat{R}$ the debut of $\hat{M}$. Then R and $\hat{R}$ are dual terminal times and we let q(t,x,y) be the dual density for (X,R) and ($\hat{X}$,$\hat{R}$). If $(*P^x, B)$, resp. $(*\hat{P}^x, \hat{B})$, are the Maisonneuve exit systems associated with M, resp. $\hat{M}$, then one defines kernels

$$Q_t^*(dx) = *P^x[X_t \in dx;\ t<R] = q^*(t,x,y)dy,$$
$$\hat{Q}_t^*(dx) = *\hat{P}^x[X_t \in dt;\ t<R] = \hat{q}^*(t,x,y)dy.$$

Finally let $\nu = \nu_B$, $\hat{\nu} = \hat{\nu}_{\hat{B}}$ be the bi-measures associated with B and $\hat{B}$. The following formulas are of interest. If $x \in E(R)$ = regular pts for R, then

$$P^*[R \in dr, X_{R-} \in dy, X_R \in dz] = \hat{q}^*(r,y,x)dr\ \hat{\nu}(dz,dy), \quad \text{and for all } x$$
$$*P^x[R \in dr, X_{R-} \in dy; X_R \in dz] = \eta\ (r,x,y)dr\ \hat{\nu}(dz,dy) \quad \text{where}$$
$$\eta(r,x,y) = \int q^*(s,x,z)\hat{q}^*(r-s,y,z)dz$$

independent of s, 0<s<r .

The main result is the construction of measures $P^{x,\ell,y}$ on s governing the law of the excursion conditioned to start at x, and at y, and have length ℓ. If $\hat{P}^{x,\ell,y}$ denotes the dual object then $r_\ell P^{x,\ell,y} = \hat{P}^{y,\ell,x}$, where r_ℓ is the reversal from ℓ operator. Applications of these measures to various particular excursions were given. More generally it is possible to write the law governing the excursion straddling an arbitrary stopping time in terms of appropriately conditioned versions of the $P^{x,\ell,y}$.

MARKOV PROCESSES WITH IDENTICAL LAST EXIT DISTRIBUTIONS

J. Glover, Rochester

Let X and Y be two transient locally Hunt Markov processes. If X and Y enjoy the same last exit distributions from bounded open sets, then Y is equivalent to a time-change of X by the inverse of a strictly increasing continuous additive functional. This result can also be interpreted (with natural auxiliary hypotheses) as a statement in potential theory involving equilibrium measures.

BIRTH TIMES, DEATH TIMES AND DUALITY

M. Jacobsen, Kopenhagen

For Markov chain paths with finite lifetime, there is an obvious definition of the dual $\hat{\tau}$ for any given random time τ. Based on concrete examples in this setup, a general definition of $\hat{\tau}$ is proposed for τ optional or co-optional. With this definition $\hat{\tau}$ is cooptional, optional, coterminal, terminal for τ optional, cooptional, terminal, coterminal respectively.

One reason for introducing duals of random times, is that they may help understand better the striking duality in appearance between the results characterising regular birth times and regular death times for a given Markov process.

TIME REVERSAL IN INFINITE PARTICLE SYSTEMS

H. Künsch, Zürich

A discrete time Markov chain on an infinite product space is considered. It is supposed that the transition kerne $P(x,.)$ is a product measure for fixed x and that points far away interact weakly. Then we prove that an equilibrium μ is Gibbsian iff the reversed process $\hat{P}(z,.)$ is Gibbsian for all z. In the reversible case $\hat{P} = P$, the local specifications of μ can be directly expressed by P, but reversibility occurs only for very special P. In the non-reversible case, a complicated equation is given which determines in principle $\hat{P}$ and then also the local specifications of μ. Some examples where $\hat{P}(x,.)$ is again a product measure, but different from $P(x,.)$, are discussed, but this also happens only in few situations.

SOME ANALYTICAL RESULTS ON THE ORNSTEIN-UHLENBECK PROCESS

P.A. Meyer, Strasbourg
(joint work with D. Barrey)

For simplicity, consider the one-dimensional O-U semigroup P_t, defined as follows on $L^2(\mu)$ (μ is standard Gaussian measure on $\mathbb{R}$) = if

$$f = \sum a_K H_K \quad \text{(normalized Hermite polynomials)}$$

then $P_t f = \sum a_K e^{-Kt/2} H_K$

If $\int f d\mu = 0$, i.e. $a_o = o$, we may define the "Riesz potentials"

$$R^\varepsilon f = \sum \frac{a_K}{k^\varepsilon} H_k \quad \text{for } Re(\varepsilon) \geq 0$$

It isn't difficult to see that R^ε is bounded from L^p to L^p for $1<p<\infty$. On the other hand, the following results are known.

Gross logarithmic Sobolev inequality (J. Funct. Anal. 1975)

$$R^{1/2} \text{ maps } L^2 \text{ into } L^2 \log L$$

Feissner's extension (TAMS 1975)

$$R^{1/2} \text{ maps } L^2 \log^n L \text{ into } L^2 \log^{n+1} L \text{ for } n \in \mathbb{Z}$$

The general result can be shown to be

$$R^{\varepsilon + i\eta} \text{ maps } L^p \log^\alpha L \text{ into } L^p \log^{\alpha + p\varepsilon} L$$
$$\text{for } \varepsilon \geq 0,\ 1<p<\infty,\ \alpha \in \mathbb{R},\ \eta \in \mathbf{R}.$$

BALAYAGE FOR DUAL PROCESSES

J.B. Mitro, Cincinnati

For a pair of Markov processes in duality, the relationship between exit systems is investigated using an auxiliary process, defined on a random time interval, in which the original processes may be embedded. For a given closed homogeneous set M with associated exit system (*P,B) it is possible to describe a corresponding homogeneous set $\hat{M}$ (for the dual) and then obtain what might be called the "dual" exit system $(*\hat{P},\hat{B})$. However, in general B and $\hat{B}$ are not dual additive functionals in the classical sense (i.e., embed into a single random measure for the auxiliary process). We describe how B and $\hat{B}$ may be constructed from the random measure $1_{\underset{\sim}{M}^c}\, dt$ (where $\underset{\sim}{M}$ embeds both M and $\hat{M}$) via "adapted α-balayage" for the auxiliary process.

TIME REVERSAL IN POPULATION GENETICS

M. Nagasawa, Zürich

In Population Genetics (especially in the theory of evolution) it is important to compute (or "predict") the past history of a (mutant) gene (e.g. the age of an allele) given the present gene frequency, besides the prediction of the future (e.g. extinction or fixation of an allele). In 1975 Maruyama-Kimura presented a formula, which enables us to calculate the past history. Their idea was formulated in terms of the so called diffusion approximation, but their paper contains some ambiguous arguments. One trial to make their arguments clearer was reported based on a paper by Nagasawa-Maruyama, Adv. Appl. Prob.11(1979), 457-478, in which time reversal of diffusion processes is exploited.

MONKEYS LIVE EVERYWHERE

M. Nagasawa, Zürich

Based on a story of monkey populations which I reported two years ago, an application was explained: If we assume that excited monkeys are running in a meson, we can calculate the mass spectrum of mesons (especially with spin 0 and 1); π^0, π^+, η, ρ, ω,...,J/ψ, χ, ψ,...,γ,... . This is a part of joint work with K. Yasue (Geneva).

ITERATIONS OF MAPS OF THE UNIT INTERVAL

C.J. Preston, Bielefeld

Some recent results of Singer, Guckenheimer and Misiurewicz were presented concerning the iterates of continuous functions mapping the unit interval back into itself.

ENTRANCE AND EXIT BOUNDARY FOR ORNSTEIN UHLENBECK PROCESSES

U. Rösler, Göttingen

There exists a lot of literature on Martin boundary, but only a few examples of Markov processes with continuous time and space where the Martin boundary is known. This lecture is an attempt to give an example and to find some geometric meaning. Consider the process X_t as a solution of

$$dx_t = (a^o(t)+a^1(t)\,x_t)dt+b(t)dW_t$$

in $\mathbb{R}^n$. a^o is a measurable vector, a^1, b are measurable matrices. Besides existence assumption we assume nondegeneracy, i.e. det $b(t)\neq 0$ for all $t\in\mathbb{R}$. For these processes we can find all minimal harmonic (also time dependent) functions, the Martin exit and entracne boundary, and a description of the bounded harmonic functions. A geometric meaning is given for that part in the boundary, which corresponds to bounded harmonic functions, more precisely to the support of the measure μ on the Martin boundary, which represents the function identically 1 in the integral representation.

PROCESSUS DE DIFFUSION ASSOCIES AUX MESURES DE GIBBS SUR $\mathbb{R}^{\mathbb{Z}^d}$

G. Royer, Paris

We consider the following infinite system of stochastic differential equations:

$$X_t^u = \xi^u+B_t^u-\tfrac{1}{2}\int_0^t (\phi'(X_s^u) + \sum_{v\neq u}\phi'_{u,v}(X_s^u,X_s^v))ds$$

where $u\in\mathbb{Z}^d$, ξ is a function on $\mathbb{Z}^d$ belonging to a space E of functions of moderate growth, B^u is a family of independent standard brownian motion; the "interaction" ϕ, $\phi_{u,v}$ is a system of functions on $\mathbb{R}$ and $\mathbb{R}$ respectively, whose prototype is: ϕ a polynomial bounded from below, $\phi_{u,v}(x,y)=\frac{1}{2}(x-y)^2$, if u,v are nearest neighbours in $\mathbb{Z}^d$, $\phi_{u,v}=0$ otherwise. We show that the equation defines a diffusion process with values in E whose invariant and reversible measures are exactly the so-called Gibbs measure on E. When $\inf(\phi'')>0$ there is a unique invariant measure.

ONE-DIMENSIONAL DIFFUSIONS AND THEIR EXIT SPACES

P. Salminen, Abo

Consider a one-dimensional transient diffusion on an interval with a and b as left- and right hand end point, respectively. Both boundary points are assumed to be killing boundaries. The exit space (= the minimal part of the Martin compactification) of this process is the closed interval [a,b]. Denote the minimal functions with k_y, $y\in[a,b]$.
In this paper an explicit expression for the lifte-time distribution of a k_y-process is obtained. In the case both a and b are natural boundary points this distribution is closely connected with the last exit distributions

of the k_a- and k_b-processes. The behaviour of a k_y-process at the point of the convergence (i.e. at the point y) is explained via a limit procedure in an instantaneous jump diffusion (jump occurring at the point y).

Further, the time-reversal properties in the k_a- and k_b-processes are examined and a new, direct proof of D. Williams' time reversal theorem is given.

ANOTHER LOOK AT ENERGY FOR DUAL PROCESSES

M.J. Sharpe, La Jolla

Let u(x,y) be the potential kernel density for a pair of standard processes in duality. We examine various notions of energy related to the kernel u, aiming to explain the nature of the difficulties caused by lack of symmetry, unpredictable lifetimes, Martin boundary issues and the like. The work is joint with R.K. Getoor.

INFINITE DIMENSIONAL DIFFUSION PROCESSES OCCURRING IN POPULATION GENETICS

T. Shiga, Tokyo

The purpose of my talk is to describe the wandering phenomena for the continuous time Ohta-Kumura model.

Let $X_o = \{x = \{x_n\}_{n=\infty}^{\infty} : x_n \geq 0 \ \sum_{-\infty}^{\infty} x_n = 1 , \ \sum_{-\infty}^{\infty} n^{2k} x_n < \infty \ \forall k \geq 1\}$

Denote by $\{x(t),\ P_x\}_{x \in X_o}$ the diffusion process on X_o generated by

$$L = \tfrac{1}{2} \sum_n \sum_m x_n (\delta_{n,m} - x_m) D_n D_m + \gamma \sum_n (x_{n+1} + x_{n-1} - 2x_n) D_n$$

and define the empirical mean process $\bar{x}(t) = \sum_n n x_n(t)$ and the empirical centered random distribution process $\mu_t^\omega = \sum_n x_n(t) \delta_{\{n - \bar{x}(t)\}}$, i.e. $\langle \mu_t^\omega, f \rangle = \sum_n f(n - \bar{x}(t)) x_n(t)$.

Then we can show the following

I° $\bar{x}(t)$ behaves similar to Brownian motion as $t \to \infty$. In particular, $\bar{x}(t)$ is recurrent and the law of iterated logarithm holds.

II° $\exists \nu$: 1-dim symmetric prob. distribution. s.t.

$$\lim_{t\to\infty} \frac{1}{t} \int_0^t \mu_s^\omega \, ds \underset{(\text{weakly})}{\Longrightarrow} \nu \qquad P_x\text{-a.s.} \qquad (\forall x \in X_o)$$

Proof is essentially based on the duality methods.

COMPARABLE MARKOV-PROCESSES

M. Sieveking, Frankfurt

Consider Feller semigroups P_t, Q_t on a state space X.

Definition For $0\leq T\leq\infty$ P_t, Q_t are called F comparable ($P_t \underset{T}{\sim} Q_t$) if there exist constants γ, c s.th. $\frac{1}{c} P_{\frac{1}{\gamma}t} \leq Q_t \leq c P_{\gamma t}$ $(0\leq t\leq T)$. P_t, Q_t are called comparable ($P_t \sim Q_t$) if there is c>0 such that

$$\frac{1}{c}\int_0^\infty P_t dt \leq \int_0^\infty Q_t dt \leq c\int_0^\infty P_t dt$$

Theorem 1 (Arousson-Serrin ...) If $X = \mathbb{R}^n$, P_t, Q_t transient with elliptic operators in divergence form then $\forall T < \infty$ $P_t \underset{T}{\sim} Q_t$

Conjecture The same is true with $\mathbb{R}^n$ replaced by an open bounded set with smooth boundary.

Theorem 2 (Hueber, Sieveking) If $X \subset \mathbb{R}^n$ bounded open with smooth boundary, and P_t, Q_t are smooth with elliptic differential generator having Hölder continuous coefficients, then $P_t \sim Q_t$.

ELECTRIC NETWORK THEORY APPLIED TO MARKOV CHAINS

J.L. Snell, Murray Hill

Results of Peter Doyle are discussed. Consider a random walker on a graph who moves with equal probability along available edges. Griffeath conjectured that the probability, starting at 0, of reaching a set S before returning to 0 (escape to S) decreases if edges are removed. Doyle proved this by the observation that the probability of escape to S equals the current that flows from i to S in an associated electric network. Removing edges means replacing resistors by infinite resistors. This increases the effective resitance between i and S and decreases the current flow (escape probability). Here the existence of energy (reversibility) and the fact that current flows to minimize energy dissipation is used. What is the corresponding probability proof?

A walk on an infinite graph is recurrent or transient according as the resistance to infinity is infinite or finite. Shorting decreases resistance to infinity and cutting increases it. The dual techniques, introduced by Rayleigh, are used to show that random walks on " reasonable" graphs are recurrent in two dimensions and transient in three or more.

Non-compact simply connected 2-dimensional Riemann surfaces are parabolic or hyperbolic according as Brownian motion on the surface is recurrent or transient (Kaufmann). Equivalently, according as resistance to infinity of the surface, considered as a conductor, is infinite or finite. Shorting and cutting leads to dual criteria for the surface to be of parabolic or hyperbolic type.

CANONICAL EVOLUTIONS FOR SPIN SYSTEMS

W.G. Sullivan, Dublin

On the state space $\{-1,+1\}^N$ we consider a certain family of Markov transition functions P_t^β with generators $\{G^\beta(x,y): \beta \geq 0\}$ such that $G^\beta(x,y)=0$ unless x,y satisfy $\{j: x_j \neq y_j\} = \{i,i+1\}$ for some $i \in N$ and $x_i=y_{i+1}=-x_{i+1}=-y_i$ and the nonzero terms are chosen in such a way that the Markov chain μ_β with $\int x_i d_{\mu_\beta}=0$ and $\int x_{i-1}x_i d_{\mu_\beta}=(1-i^{-\beta})/(1+i^{-\beta})$ is a reversible invariant distribution for P_t^β. For these evolutions we find

$$\mu_{\beta_1} P_t^{\beta_2} \longrightarrow \mu_{\beta_2} \qquad \text{for } \beta_1 \leq 1,\ \beta_2 \geq 0;$$

$$\longrightarrow \tfrac{1}{2}(\delta_+ + \delta_-) \qquad \text{for } \beta_1 > 1,\ \beta_2 \leq 1$$

$$\not\longrightarrow \mu_{\beta_2} \qquad \text{for } \beta_1, \beta_2 > 1,\ \beta_1 \neq \beta_2.$$

DUALITY AND MARTINGALE PROBLEM

A. Wakolbinger, Zürich

We discuss the following question: Does duality of two operators $\mathcal{L}$, $\hat{\mathcal{L}}$ (defined on a subset $\mathfrak{D}$ of the bounded continuous functions of some polish space) imply duality of the Markov families (P_x), $(\hat{P}_x)$, which we assume to be solutions of well-posed martingale problems w.r. to $\mathcal{L}$ resp. $\hat{\mathcal{L}}$. For $\mathcal{L} = \hat{\mathcal{L}}$ (case of "reversibility") and a locally compact state space this has been investigated recently by Fukushima and Stroock with the help of Dirichlet forms. In case of infinite dimensional diffusions we can get a positive answer in case of $\mathcal{L} = \hat{\mathcal{L}}$, if the dynamic P_x is a weak limit of the "local dynamics with frozen initial conditions outside". Concerning the general question mentioned at the beginning we get an affirmative

answer at least under the following conditions:
$\mathfrak{D}$ is weakly dense in $L^1(\mu)$; P_μ is invariant;
$f(X_{L-t})$ is a P_μ-semimartingale with abs. continuous compensator (L constant).

$X_{\zeta-}$

J.B. Walsh, Vancouver

We introduce the model of electrons and holes moving in a semi-conductor as a physical model of dual processes, one process being the motion of the electrons, the other, of the holes.
We derive some of the standard results of duality theory, using two tools: h-path processes, and reversibility. We use the knowledge of $X^h_{\zeta-}$ to get some knowledge about the excessive function h, where X^h_t is the h-path transform of X. Among these results is the balayage formula for potentials. We give a heuristic explanation, in terms of the physical dual processes introduced above, of why the dual kernel enters into this formula.

STOCHASTIC CALCULUS OF VARIATIONS

K. Yasue, Geneva

The ordinary calculus of variations is extended to include certain continuous semimartingales. Let X, DX and D_*X be a continuous semimartingale, its mean forward derivative and mean backward derivative, respectively. Then a stationary point of a functional $J = E(\int_0^T L(X(t),DX(t),D_*X(t))dt)$ defined on a certain class of continuous semimartingales is given by the one that satisfies the Euler-Nelson equation $\frac{\partial L}{\partial X} - D\frac{\partial L}{\partial D_*X} - D_*\frac{\partial L}{\partial DX} = 0$ for almost every t. As an application, we have the following probabilistic representation of a solution to the Navier-Stokes equation, $\partial u/\partial t + u \operatorname{grad} u - \operatorname{div} \operatorname{grad} u + \operatorname{grad} p = 0$, $\operatorname{div} u = 0$, with vanishing initial condition $u(x,0)=0$:
$u(x,t) = E(\int_0^T \{\frac{1}{2}(DX(s))^2 - p(X(s),s)\}ds \mid X(t)=x)$, $T>t>0$. Here X is a diffusion process with uniform distribution which makes the function $J_{NS}(X) = E(\int_0^T \{\frac{1}{2}(DX(t))^2 - p(X(t),t)\}dt)$ stationary.

Berichterstatter: H.R. Künsch / A. Wakolbinger (Zürich)

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