MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 25/1981

Differentialgeometrie im Großen

7.6. bis 13.6.1981

Die Tagung fand unter der Leitung von Herrn S.S. Chern (Berkeley) und Herrn W. Klingenberg (Bonn) statt. Insgesamt haben 46 Mathematiker an der Tagung teilgenommen. 27 Wissenschaftler waren aus dem Ausland angereist, was das große Interesse an dieser Tagung besonders deutlich macht. Es wurden 26 Vorträge gehalten. Darunter waren drei Übersichtsvorträge in denen über neue Entwicklungen in verschiedenen Gebieten der Differentialgeometrie im Großen berichtet wurde: Verallgemeinerungen der Theorie der geschlossenen Geodätischen, Differential-Systeme, Einstein Mannigfaltigkeiten.

Weitere Hauptthemen in Diskussionen und Vorträgen waren: Die Theorie der Raumkurven und ihre Anwendungen in der Biologie, "Tightness" von Untermannigfaltigkeiten, komplexe Mannigfaltigkeiten, Mannigfaltigkeiten mit Rand, Laplace Operator Riemannscher Mannigfaltigkeiten.

Es folgt die Zusammenstellung der Vortragsauszüge.

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Vortragsauszüge

D. V. ANOSOV

On the Variational Theory of Closed Geodesics

The talk was devoted to extending the scope of the variational theory of closed extremals, which at the present time deals primarily with the Riemannian case. Well-known extensions include:

1) Manifolds with boundary.
2) Mechanical systems (in the classical sense).
3) Finsler case.

New Extensions (due to S.P. Novikov) include: A charged particle in a magnetic field. The Hamiltonian equations of motion can be written either in terms of "dynamic" impulses (as is common in physics) or in terms of the usual "kinematic" impulses (in this case one has to modify the basic symplectic form).

The latter version is preferable in some cases when there is no vector potential, e.g. for a particle on a sphere surrounding a magnetic monopole.

Variational treatment of such problems involves two difficulties arising from the functionals under consideration:

a) They need not be bounded from below.
b) They can be "multi-valued".

However, sometimes one can obtain some results on the existence of closed extremals.

A. BACK

Curvature, Invariant Theory and Equivariant Geometry.

The equivariant isometry type of a Riemannian G-manifold M is given by three basic invariants; the submersed metric on $\bar{M}_o/G$, the fiber metric on G/H along $\bar{M}_o$, and a $\bar{G} = N(H)/H$

connection in the principal bundle $\bar{M}_o \to \bar{M}_o/G$. Here M_o is the union of principal orbits, H is the principal isotropy subgroup and $\bar{M}_o = M_o \cap M^H$. Application of the Schur lemma describes the fiber metric in terms of $\bar{G}$ invariant functions on $\bar{M}_o$. In the absence of singular orbits, these basic invariants may be specified arbitrarily. Near singular orbits, the invariant theory of the slice representation decides the realizability question. The case of equivariant Kervaire spheres was discussed in detail, with nice formulas for the Ricci tensor and obstructions to equivariant metrics of positive curvature being among the results.

W. BALLMANN

On the Ergodicity of Geodesic Flows

Let M be a compact Riemannian manifold of non-positive curvature. Denote by φ_t the geodesic flow on the unit tangent bundle. φ_t preserves the Liouville measure. For v, w orthonormal let $J_{v,w}$ be the stable Jacobi field along the geodesic determined by v such that $J_{v,w}(0) = w$. Define

$$\mathcal{K}(v, w) = \lim_{T\to\infty} \sup \frac{1}{T}\int_0^T K(\varphi_t v, J_{v,w}(t))\, dt,$$

where $K(x,y)$ denotes the sectional curvature of the plane spanned by x and y.

Theorem (M. Brin, W. Ballmann) If there exists a unit vector v such that $\mathcal{K}(v, w) < 0$ for all unit vectors $w \perp v$, then φ_t is ergodic.

This generalizes a previous result of Pesin. Note that the assumption of the theorem is satisfied if there exists a unit vector v such that $K(v, w) < 0$ for all unit vectors $w \perp v$.

T. BANCHOFF

Degree and Class for Tight Surfaces

For a smooth immersion $f : M^2 \longrightarrow E^3$, the parallel class $c''(f)$ is the supremum of the number of critical points of non-degenerate height functions (= the sup of the number of tangent planes to $f(M)$ parallel to a fixed plane) and the class $c(f)$ is the sup of the number of tangent planes passing through a fixed line. One also defines $d(f)$, the degree of f to be the sup of the number of (transversal) intersections of $f(M)$ with a given line. The immersion is tight if $c''(f)$ is the minimum value, $4 - \chi(M^2)$.

The following results relating class and degree for tight surfaces were shown:

Theorem 1: If $\chi(M^2) \neq 2$, then $c(f) \geqslant d(f) - \chi(M^2)$ with equality if f is tight.

Theorem 2: For $\chi(M^2) \neq 2$ and any $m > 0$ there is a tight $f_m : M^2 \longrightarrow E^3$ for which $c(f_m) \geqslant 4 - \chi(M^2) + 2m$, and moreover such an f_m can be found in any neighborhood of any tight immersion.

Extensions to polyhedral embeddings and immersions, and to differentiable or polyhedral stable mappings were presented. For any orientable surface with $\chi(M^2) \neq 2$ there is a polyhedral tight embedding of degree 4 and a smooth tight embedding of degree 6. Orthogonal projections of the Veronese surface into E^3 give examples of stable mappings of class 3.

V. BANGERT

Riemannian manifolds with boundary and Blaschke manifolds.

A geodesic $c : [a,b] \longrightarrow M$ in a Riemannian manifold M with boundary ∂M is called a chord of M, if $a < b$, $c(a) \in \partial M$,

$c(b) \in \partial M$, while $c((a\ b)) \subseteq \mathring{M}$. Using Santalo's Formula one can derive the following estimate for the infimum ℓ of the lengths of chords of a compact M:

$$(*) \quad \ell \leq k_m \frac{\mathrm{vol}\ (M)}{\mathrm{vol}(\partial M)} \quad \text{where} \quad k_m = m \frac{\mathrm{vol}(S^m)}{\mathrm{vol}(S^{m-1})}, \quad m = \dim M - 1.$$

It was proved that equality holds in (*) if and only if M is isometric to a hemisphere. A closely related result is the following: Suppose all chords of M have equal length. Then M is isometric to a hemisphere.
In particular this last theorem indicates the relation to the Blaschke manifolds mentioned in the title.

J.-P. BOURGUIGNON

A Preview of A.Besse's book "Einstein manifolds"

The subject of the book is existence and classification of Einstein metrics on a given manifold M. Recently many new interesting examples have been obtained (via the solution of the Calabi conjecture or on manifolds of cohomogeneity one or on non compact Kähler manifolds), but there still remain very attractive open questions in the subject. After a brief introduction to the field, the talk discussed very recent contributions such as: a new presentation of G. Jensen's examples of "exotic" Einstein metrics on spheres as distances spheres in the quaternionic projective space; the possibility of extending the 4-dimensional Thorpe-Hitchin-obstruction and its generalization due to A. Polombo for Ricci-pinched metrics to higher dimensions; applications of the solution of the Calabi conjecture by S.T. Yau to constructing Kähler-Einstein metrics in particular on K3 surfaces; cohomogeneity one examples by L. Bérard Bergery of generalized Hirzebruch surfaces; results on the theory of moduli for Einstein metrics by N. Koiso; the discussion of the limiting case of a lower estimate for the first eigenvalue of the Dirac operator involving the scalar curvature due to T. Friedrich.

K.T. CHEN

Poincaré Dual of Chern classes and Indices of Hopf type

Let E be a holomorphic k-plane bundle over a compact complex manifold N. Let $s_1,...,s_r$ be holomorphic sections of E and let $\Delta = \Delta(s_1,...,s_r)$ be the degeneracy cycle. Set $k' = k - r + 1$. Let $\Delta_1,...,\Delta_l$ be the irreducible components of Δ of codimension k' in N, for each of which a multiplicity (index) m_i can be defined. If Δ is of codimension $\geqq k'$ and $\Delta(s_1,...,s_{r-1})$ of codimension $> k'$, then $\sum m_i \Delta_i$ is Poincaré dual to the k'-th Chern class of E.

This theorem can be proved in a C^∞ framework and improves known results by providing multiplicities. A generalization of the Bezout theorem follows as a corollary.

S. S. CHERN

Exterior Differential Systems

Professor Chern gave a preview of a book on exterior differential systems, he is writing with Griffith, Gardner and Bryant. His talk included the following topics:

1) normal forms of differential systems,
2) Cartan-Kähler theorem,
3) Cartan-Kuranishi theorem, proof by cohomology
4) Applications to isometric imbedding of Riemannian manifolds.

D. DE TURCK

The "Manifold" of Ricci Curvatures

Let M be a compact manifold without boundary (dim M $\geq$ 3), and g_o a Riemannian metric on M such that R_o = Ric (g_o) is everywhere nonsingular. The structure of the image of a neighborhood of g_o in the space of $C^{k+\alpha}$ (or H^s) metrics on M under the mapping Ric which sends metrics to their Ricci curvature tensors was examined. Also the related equation Ric (g) = T + fg, where T is an invertible tensor and f is a scalar-valued function of $x \in M$ (and also possible of trT, as in the Einstein equation) was considered .

J. GIRBAU

Deformations of transversally Holomorphic Foliations

The aim of the lecture was to explain some results obtained by Haefliger, Sundararaman and the speaker concerning the theory of deformations of transversally holomorphic foliations. For such foliations a theorem analogous to the classical theorem of Kuranishi for compact complex manifolds was stated. The relation between the deformations of transversally holomorphic foliations and the deformations of compact complex V-manifolds was discussed. It was shown how to compute the Kuranishi spaces of some interesting complex V-manifolds.

E. HEINTZE

Resolution of Singularities and the Invariant Spectrum

Hironaka's theorem on resolution of singularities can be used to prove the following.

Let M be a compact n-dim. Riemannian manifold, $\lambda_o < \lambda_1 < \lambda_2 < \ldots$ the eigenvalues of the Laplace operator with eigenspaces

E_{λ_i} and G a compact group of isometries.

<u>Theorem</u> (Brüning, Heintze): There exists an asymptotic expansion

$$\sum e^{-\lambda_i t} \dim E^G_{\lambda_i} \sim (4\pi t)^{-n/2} \sum_{\substack{\mu=0,\ldots,\infty \\ \nu=0,\ldots,\text{rank } G}} a_{\mu\nu} t^{\mu/2} (\log t)^{\nu}$$

for t going to zero.

The main difficulty is to find the asymptotic expansion of $\int_{G\times M} e^{-\frac{d^2(x,gx)}{t}} f(x,g)\,dg\,dx$ for any C^∞ function $f : G \times M \to \mathbb{R}$. A local version of the resolution theorem is used to simplify the crossings in the zero set of $d^2(x,gx)$ in $G \times M$.

W. KLINGENBERG

<u>Closed Curves on S^2</u>

A proposal for defining a selfintersection number $\vee(c)$ for a closed curve c on S^2 is made. If c^m denotes the m-fold covering of c one has the formula

$$\vee(c^m) = m^2 \vee(c) + m - 1.$$

Moreover, in the space ΛS^2 of all closed curves, the subset of curves $c_1 \cup \ldots \cup c_k$ formed by circles c_j which touch each other is suggested as to represent a deformation retract of ΛS^2 : $\vee(c_1 \cup \ldots \cup c_k) = k - 1$. For fixed k, the family of curves $\{c_1 \cup \ldots \cup c_k\}$ can serve as 'unstable manifold' of the space of k-fold covered great circles.

N. H. KUIPER

Conformally tight subsets of three space

A compact connected subset X of conformal space S^N is called conformally tight or taut in case $H_i(b \cap X, \mathbb{Z}_2) \longrightarrow H_i(X, \mathbb{Z}_2)$ is injective for all round balls $b \subset S^N$ and all i. (c^i)

(c^o) means that $b \cap X$ is connected

(c^1) means that any closed curve $\subset b \cap X$ bounds in $b \cap X$ if it bounds in X.

This is analogous to tightness (replace round balls b by halfspaces h in E^N), which for smooth submanifolds (roughly) means minimal absolute total curvature.

$SO(n) \subset R^{n^2}$ and many other classical models ($U(n)$, Grassmanian) are tight and taut. The most beautiful example is the Veronese surface, an embedded real projective plane in $S^4 \subset E^5 \subset R^9$. All taut X in $E^2 \subset E^2 \cup \infty = S^2$ are the following: point, round circle S^1, and the limit Swiss cheese obtained from S^2 by deleting the interiors of an everywhere dense set of disjoint round discs.

Theorem: All taut ANR sets X in $E^3 \subset E^3 \cup \infty = S^3$ are the following: point, S^1, S^2 and the conformal (Moebius) transforms of standard round tori, so called nonsingular Cyclide of Dupin surfaces.

J. LAFONTAINE

About conformally flat manifolds.

The following theorem was proved; let (M^{2n}, g) be a conformally flat riemannian metric of zero scalar curvature. Suppose M^{2n} is compact and $H^n(M^{2n}, \mathbb{R}) \neq 0$ (or that $H^n(\tilde{M}^{2n}, \mathbb{R}) \neq 0$ for some finite covering $\tilde{M}^{2n}$). Then the universal riemannian covering of (M^{2n}, g) is either $(\mathbb{R}^{2n}$, eucl.) or $(S^n, \text{can}) \times (H^n, \text{can})$.

S. MURAKAMI

Parallel submanifolds of complex space forms

Let $\bar{M}(c)$ be a space of constant holomorphic sectional curvature c with $c \neq 0$. Isometric immersions of a complete Riemannian manifold (M, g) into $\bar{M}(c)$ were considered. The following theorem was recently obtained by H. Naitoh.

Theorem 1: Assume that the immersion has parallel second fundamental form. Then, either one of the following occurs

(a) (M, g) has Kähler structure and the immersion is holomorphic

(b) (M, g) is immersed in totally real way, and is contained in a totally real totally geodesic submanifold

(c) (M, g) is immersed in totally real way, not totally geodesic, and is contained in a totally geodesic Kähler submanifold $M^r(c)$ with $r = \dim M$.

This theorem allows a classification of parallel submanifolds in $\bar{M}^r(c)$. The case (a) and (b) have been done by Nakagawa-Takagi and Ferus-Takeuchi respectively. For the case (c) with $c > 0$, one has

Theorem 2: Totally real submanifolds with parallel second fundamental form being locally symmetric if (M, g) has no euclidean factor, then (M, g) is a compact irreducible symmetric space of one of the following types:
$SU(n)/SO(n)$ $(n \geq 3)$, $SU(2n)/Sp(n)$ $(n \geq 2)$, $SU(n)$, E_6/F_4.
And the immersion is unique, up to rigidity.

A. M. NAVAIRA

Some remarks about the Riemannian Curvature Operator on a Riemannian almost-product manifold

The speaker has found 36 different classes of Riemannian almost-product structures on a given Riemannian manifold (M, g) according to the behaviour of ∇P, ∇ being the Levi-

Civita connection and P being the tensor field defining the structure. When $\nabla P = 0$, the two mutually orthogonal distributions given by P, are foliations with totally geodesic leaves, so the manifold is locally product.

Then, the curvature tensor field satisfies $R(M,N,L,O) = R(M,N,PL,PO)$ for any vector fields M,N,L and O.

In this talk equivalent definitions for several classes of almost-product manifolds were given. Several conditions on the curvature tensor field that are satisfied by the manifolds of some families with defining conditions weaker than $\nabla P = 0$ were analyzed.

M. OBATA

A certain differential Equation arising from the Scalar Curvature Function.

The following differential equation on a complete Riemannian manifold was considered:

$$(*) \quad \text{Hess } f + \left(\frac{R}{n-1} g - \text{Ric}\right) f = 0.$$

The complete conformally flat Riemannian manifolds admitting a non-trivial solution to $(*)$ are completely determined. Counter examples to the Fischer-Marsden conjecture are involved.

The scalar curvature R is considered as a map $\mathfrak{M}^s \longrightarrow \mathfrak{F}^{s-2}$ ($\mathfrak{M}^s$ is the space of the Riemannian metrics whose derivatives upto order s are L_2-integrable, $\mathfrak{F}^s$ is the space of functions with similar properties) when M is compact. Then dR_g is not surjective if and only if $(*)$ has a non-trivial solution.

$(*)$ is also obtained from the Einstein equation on a static-space-time with perfect fluid as a matter field. Conformally flat such space-times are also completely determined.

P. PICCINNI

Quaternionic differential forms and symplectic Pontrjagin classes

The "Dieudonné determinant with sign" of a quaternionic hermitian matrix was considered; it is defined by a polynomial in the entries of the matrix. It was shown that such a polynomial can be used to write the representative forms of the symplectic Pontrjagin classes of a quaternionic vector bundle, avoiding complex and real notations.

W. POHL

The Probability of Linking of Random Closed Space Curves

Given two closed curves of random shape, but fixed length, randomly placed inside a bounded domain in ordinary space, what is the probability that the two curves link? This problem arises in molecular biology, particularly in the study of the action of topological enzymes on DNA.

A formula can be given for the expected value of the square of the linking number. The basic result is the kinematic linking formula. Let C_o, C_1 be closed space curves of fixed shape, C_o fixed in position and C_1 moveable. Let dK_1 denote the kinematic measure of the positions of C_1, and λ the linking number. Then

$$\int \lambda^2 (C_o, C_1) dK_1 = \pi \int_0^d (A_o(r) A_1(r) + B_o(r) B_1(r)) dr,$$

where d denotes the smaller of the diameters of C_o and C_1, and A_i, B_i the associated functions of C_i. These associated functions, can be related to geometry and can be computed in various ways.

E. RUH

Almost flat manifolds

Definition. A compact riemannian manifold M is called ε-flat if $|K| \cdot d^2 \leq \varepsilon$, where K is the sectional curvature of M and d is its diameter.

The following result is a common generalization of Bieberbach's theorem on compact euclidean space forms and Gromov's theorem on almost flat manifolds.

THEOREM. Let M denote a compact riemannian manifold of dimension n. There exists a constant $\varepsilon = \varepsilon(n) > 0$ such that any ε-flat M is diffeomorphic to $\Gamma \backslash N$, where N is a simply connected nilpotent Lie group and Γ is a finite extension of a lattice $L \subset N$.

The main idea in the proof is to solve a certain partial differential equation. To prove sobvability, the operator in question is compared to the Laplace operator.

Chun-li Shen

The Gap Phenomena of Yangs-Mills Fields over the complete Manifold.

It is still an unsolved problem whether a sourceless $SU(2)$ gauge field over S^4 is a self-dual (or anti-self-dual) field. So it is very interesting to study the relation between sourceless fields and self-dual (or anti-self-dual) fields.

The following theorem was proved:

Theorem: Let M be a four-dimensional complete self-dual Riemannian manifold, R the scalar curvature of M, f is the strength of a Yang-Mills field over M with the gauge group $SU(N)$, $|f^-|$ the norm of the anti-self-dual part of f. If

(i) $|f^-| < \frac{R}{12}$,

(ii) there is a point $x_o \in M$ such that

$$\int_{B_k} |f^-|^2 \, dvol = o(k^2) ,$$

where B_k is a geodesic ball with the center x_o and radius k , (For example, this is true if the action of this field is finite.) then, this Yang-Mills field must be a self-dual field.

G. Stanilov

On the Geometry of Almost Hermitian Manifolds

One has a decomposition $R = R_1 + R_1^\perp + R_2^\perp + R_3^\perp$ for the curvature tensor R of any almost Hermitian manifold. Some geometrical interpretations of the components of R were given.

Theorem 1: The conditions

i) $H_R(p,\chi) = c(p)$, ii) $R_1 = \frac{c(p)}{4}(\pi_1 + \pi_2)$,

where H_R is the holomorphic curvature function, are equivalent. The mapping $R \to R_1$ preserves the holomorphic curvature, i.e. $H_R(p,\chi) = H_{R_1}(p,\chi)$.

Theorem 2: The conditions

i) $A_R(p,E^y) = c(p)$, ii) $R_1^\perp = \frac{c(p)}{16}(3\pi_1 - \pi_2)$,

where $A_R(p,E^y) = K_R(p,E^y) - K_R^*(p,E^y)$,

are equivalent. The mapping $R \to R_1^\perp$ preserves the function A_R .

Theorem 3: Let (M,g,I) be a 2n-dimensional connected QK_2-manifold with $n \geq 3$. If the relation

$$\lambda S(R) + \lambda^* S^*(R) = cg$$

where λ, λ^*-const. , $c \in FM$, holds, then c is also a global constant on M .

G. Tsagas

The Spectrum of the Laplace Operator

Let (M,g) be a compact Riemannian manifold of dimension n. Let $\Lambda^q(M)$ be the vector space of exterior q-forms on M, where $q = 0,1,...,n$. $Sp^q(M,g)$ denotes the spectrum of the Laplace operator on $\Lambda^q(M)$.

The following problem was considered. Does $Sp^q(M,g)$ determine the geometry of (M,g) ? The answer is in general negative. For special Riemannian manifolds the problem remains open. The following results were stated:

Theorem: Let (M,g), (M',g') be 2 compact, orientable Riemannian manifolds with $Sp^q(M,g) = Sp^q(M',g')$ (which implies that $\dim(M) = \dim(M') = n$). If n is given, then one can find at least one integer q, between o and n such that (M,g) has constant sectional curvature k, if and only if (M',g') has constant sectional curvature k' and $k = k'$.

Corollary: Let (S^n,g_o) be the standard Euclidean sphere. If $n \geq 6$, then $Sp^{[n/3]}(S^n,g_o)$ determines completely the geometry on (S^n,g_o). If $n \in [2,5]$ then $Sp^o(S^n,g_o)$ determines completely the geometry on (S^n,g_o).

J. H. White

Applications of the Theory of Space Curves to the Structure of DNA

Recent controversy has developed as to the exact model for the mathematical structure of DNA. This comes from the problem of replication; how the molecule uses its own structure as a template and takes substances form the cellular environment to reproduce itself. The lecture discussed the recent electrophoresis experiments of Bauer, Wang, Cozzarelli and Brown from a global geometric point of view and laid to rest many of these objections. It also discussed the recent crystallization work of many biochemists reaffirming the double helical structure.

F.-E. Wolter

Cut Loci

Recently distance geometry in bordered mfs. has been studied by various authors, see e.g. the article by S. Alexander in Lecture Notes 838.

Let M be a submf. of a complete Ri. mf. $(\overline{M},g)$ and boundary ∂M a closed top submf..

Defining the distance $d(\ ,\ )$ on M by $d(p,q) := \inf\{\text{length } c \,/\, c$ rectifiable path in M from p to $q\}$ we assume (M,d) to be a complete metric space whose topology agrees with the submf. topology of M.

For any closed subset A in M one defines the cut locus $Cu(A)$ as closure of all points $q \in M\setminus\partial M$ where a minimal join from A to q cannot be minim. prolonged beyond q. If now g is Hölder continuous then $Cu(A)$ is closure of all points $q \in (M\setminus\partial M)$ where at least 2 minimal joins from A to q end up with distinct tangents, and $M\setminus(\partial M \cup Cu(A))$ is the maximal open set in $(M\setminus\partial M)$ where $d^2(A,.)$ is C^1. If $\partial M = \emptyset$, let D be the set of points where $d^2(p,.)$ is not differentiable: if $D = \emptyset$ then M is diffeo. to $\mathbb{R}^n$; if D contains an isolated point M is homeo. to S^n. Let g be C^∞, ∂M not necessarily empty, then a min. join from any closed set A to a point $q \in M\setminus\partial M$ can be prolonged minim. beyond q iff there is a number K such that in a suitable chart $|c_q - c_{\bar{q}}| < K d(q,\bar{q})$ with $c_{\bar{q}}$ the tangent vector at $\bar{q}$ of any normalized min. join from A to $\bar{q}$, $|.|$ being the norm related to the chart.

W. Ziller

Existence of Closed Geodesics on Spheres

The existence of several short closed geodesics for a riemannian metric on S^n was discussed. Let $g(n) = 2n-S-1$ where $n = 2^k+S$, $0 < s < 2^k$.

Theorem (W. Ballmann, G. Thorbergsson, W. Ziller): If M is simply connected and the metric satisfies $1/4 < \delta \leq K \leq 1$, then

(i) there exist $g(n)$ closed geodesics without self-intersections and lengths in $[2\pi, 2\pi/\sqrt{\delta}] \subset [2\pi, 4\pi)$.

(ii) if all closed geodesics of length $< 4\pi$ are non-degenerate (an open and dense condition on the set of metrics) then there exist $\frac{n(n+1)}{2}$ such closed geodesics

(iii) if all closed geodesics with lengths in $[2\pi, 2\pi/\sqrt{\delta}]$ have the same length α, then all geodesics are closed of length α

(iv) there exist no closed geodesics with length in $(2\pi/\sqrt{\delta}, 4\pi)$. If there exists a closed geodesic of length 2π or $2\pi/\sqrt{\delta}$ or 4π, then M is isometric to a sphere with constant curvature.

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