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Mathematical Economics

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Organisers: G. Debreu (Berkeley)
W. Hildenbrand (Bonn)
D. Sondermann (Bonn)

This was the third meeting on Mathematical Economics in Oberwolfach. Unlike the first two meetings which mainly aimed to further the interaction between mathematicians and economists and therefore were organized around longer invited papers of survey character, the third conference was devoted to recent developments in mathematical economics and game theory. All plenary communications were strictly limited to fifteen minutes. After the plenary sessions smaller working groups were formed for more detailed presentations and further discussions. Although the "fifteen-minutes-rule" was met with some scepticism in the beginning, it was generally agreed at the end of the conference that it has proofed highly efficient since it provides a maximum of information and still leaves more time than usual for informal working and discussion groups. There was again a session on open problems, which are also recorded in this report.

Vortragsauszüge

Neighboring Information and Distributions of Agents' Characteristics under Uncertainty

By Beth Allen
University of Pennsylvania

For many economic problems concerning uncertainty, the specification of agents' information must be an inherent part of the economic model. In order to discuss distributions of agents' information (i.e., in "large" economies), just as we discuss distributions of preferences and endowments, one needs to define a topological structure for information.

Let $(\Omega, \mathfrak{A}, \mu)$ be an abstract probability triple, to be interpreted as the set of states of the world, (measurable) events, and the (objective of subjective) prior (σ - additive) probability occurrence of various events. Information is a sub - σ - field $\mathfrak{G}$ of $\mathfrak{A}$. Let $\mathfrak{A}^{**}$ denote the space of equivalence classes of sub - σ - fields of $\mathfrak{A}$, modulo the equivalence relation of having the same completion. Define the complete metric d on $\mathfrak{A}^{**}$ by $d(\mathfrak{G}, \mathfrak{H}) = \sup_{G \in \mathfrak{G}} \inf_{H \in \mathfrak{H}} \mu(G \Delta H) + \sup_{H \in \mathfrak{H}} \inf_{G \in \mathfrak{G}} \mu(G \Delta H)$

(This has been introduced in the probability theory literature to study the rate of martingale convergence.)

Suppose that there are l commodities and that economic agents have initial endowments $e \in \mathbb{R}^l_{++}$. Let $\mathcal{U} = \{u \in C^0(\mathbb{R}^l_+, \mathbb{R}):$ u is strictly monotone (increasing) and strictly concave$\}$. Agents have state - dependent (cardinal) utilities formally specified by $\mathcal{A}$ - measurable mappings $f: \Omega \to \mathcal{U}$ (endowed with the topology of uniform convergence on compact subsets of $\mathbb{R}^l_+$ and given the Borel σ - field). Assume that the support of $\mu \circ f^{-1}$ has a closed convex hull which is a compact subset of $\mathcal{U}$. Then, given information $\mathcal{G}$, traders choose demands $x(\cdot): \Omega \to \mathbb{R}^l_+$, so as to maximize $Eu \mid \mathcal{G}$ subject to the budget constraint $p \cdot x(\omega) \leq p \cdot e$, where the price vector $p \in \Delta = \{p \in \mathbb{R}^l_{++} : \sum_{i=1}^{l} |p_i| = 1\}$. Note that x depends on the state of the world and must be $\mathcal{G}$ - measurable.

The following result indicates that this topology for information is appropriate for economics: Let $\mathcal{G}_n \to \mathcal{G}$ in $(\mathcal{G}^{**}, d)$. Then $Eu|\mathcal{G}_n \to Eu|\mathcal{G}$ in probability and in the $\mathcal{L}^1$-norm; these distributions of conditional expected utility converge weakly (as probability measures on $\mathcal{U}$). Furthermore, (if $e_n \in \mathbb{R}^l_{++}$, $e_n \to e \in \mathbb{R}^l_{++}$), the distribution of $x(\cdot, \cdot, Eu|\mathcal{G}_n, e_n)$ (the demand functions, which depend on $\omega \in \Omega$, as functions of $p \in \Delta$) converge weakly to the distribution of the demand $x(\cdot\,;\cdot, Eu|\mathcal{G}, e)$ - as probability measures on $C^0(\Delta, \mathbb{R}^l)$, endowed with the compact - open topology. For this topology on information, the value of information is a continuous function of the information σ - field.

Markets with Satiation : "To each according to his abilities" (joint work with J. Drèze)

By Robert Aumann
Hebrew University, Jerusalem

The utility functions in this investigation need not be monotonic, and do have absolute maxima. In this situation it is known that competitive equilibria need not exist (Drèze and Müller, JET, 1980), Shapley values predict competitive equilibria in the case of ordinary markets, so one may expect that they suggest an appropriate extension of competitive equilibria in this case. It turns out that if a market is replicated m times and $m \to \infty$, then the value allocations tend to an allocation associated with prices and dividends, where the dividends are additional amounts added to the budget of each trader. These dividends depend only on the net trade set of each agent - on his ability to trade - and not on his utilities. Applications to fixed price markets are described.

Characterization Results for Incentive Compatible Games (joint work with G. Heal)

By Graciela Chichilnisky
University of Essex

We consider the problem of designing games to model incentives for the revelation of the 'true' characteristics of the agents, and for the 'implementation' of social functions that aggregate individual into social preferences. The results are of two sorts: the first class looks for a complete characterization of all such games, under standard assumptions on preference and strategy and outcome spaces. The second class of results gives minimal, necessary and sufficient conditions, on strategy and outcome spaces for the existence of some straightforward game. Two concepts of equilibrium are studied: NASH, and dominant strategy equilibria.

The first class of results is that a game form $g : S^k \to A$, S strategy space, A outcome space, is straightforward if and only if it is <u>locally simple</u>. Locally simple functions are coordinatewise locally constant, or else have an outcome which is identical to one of the strategies of the players, a.e. Similar results hold for Nash implementable games. It is proven that locally simple games are nowhere dense in the class of all continuous games, so their existence is not robust.

The second set of results shows that a necessary and sufficient condition for the existence of some straightforward game $g : S^k \to A$ (nontrivial) is that S and A be <u>contractible</u> spaces, i.e. there exists a continuous function $f: S \times [0,1] \to S$: $f(x,0) = x \ \forall x$ in $S, f(x,1)=x_0$, some x_0 in S. Since this contractibility condition has recently been shown to be also necessary and sufficient for a resolution of the social choice paradox, the incentive problem is in this sense equivalent to the social choice problem.

Quasi - homothetic preferences and the measurement of individual welfare (joint work with James C. Moore)

By John S. Chipman
University of Minnesota

Three standard welfare measures - the Hicksian compensating variation and equivalent variation and the Dupuit - Marshall consumer's surplus - are analyzed with respect to their validity as indicators of individual welfare (more precisely, as particular cardinal indirect utility functions) over appropriate subsets of budgets (price-income pairs).

The equivalent variation is already a valid indirect utility function. Being a well-defined cardinal indicator, it also induces a quaternary ordering of the form "the preference of (p^1,m^1) over (p^2,m^2) is at least as strong as the preference of (p^3,m^3) over (p^4,m^4)", it does so on the basis of expansions reckoned in terms of a base budget (p^o,m^o). It is of interest to ask what are the necessary and sufficient conditions on consumer preferences for the induced quaternary ordering to be independent of the particular base budget chosen. It turns out that it is necessary and sufficient that preferences be quasi-homothetic in Gorman's sense - such as to yield affine consumption-income path (Engel curves) in the positive orthant of commodity space.

With regard to the compensating variation and consumer's surplus, it is shown that if budgets (p,m) are restricted to a set of the form $\alpha(p) + \beta m = 1$, where α is positively homogeneous of degree 1 and β is a constant, then a necessary and sufficient condition for the validity of both these welfare measures is, again, that preferences be quasi-homothetic.

This generalizes previous results for the case of homothetic preferences ($\alpha(p) = 0$, $\beta>0$) and preferences which are parallel with respect to commodity k, ($\alpha(p) = p_k$, $\beta=0$), which are both special cases of quasi-homothetic preferences.

The Dynamics of Capacity Adjustments in a Competitive Economy (joint work with M. Quinzii)

By Jean Gabszewicz
CORE, Louvain-la-Neuve

This work is an attempt to explore the dynamics of capacity adjustments in a competitive economy, and compare the trajectories of these adjustments under instantaneous price flexibility and persistent wage rigidity.

Remarks on Overlapping Generation Models and Dynamic Models in General

By David Gale
University of California, Berkeley

It is argued that, while the O.G. Model seems like a promising framework for studying economic dynamics, it is not economically reasonable to combine this with perfect foresight. To illustrate this it is shown that for the simplest cases of three-period lines and log linear utility functions, the price of goods today depends on what happens at $t = \infty$. The example is a slight generalization of the original example treated by Samuelson. It is then shown that for a one good model if one replaces perfect foresight by myopia in which people expect todays interest rate to persist tomorrow, that equilibrium solutions exist and converge to the steady state. Finally it is observed that if one tries to extend the myopic model to one with many goods that then are more variables than equations so that the system is underdetermined. The suggestion is that, as opposed to the static case, this may be the natural behaviour for dynamic models and one should not seek to find conditions in which both prices and interest rates are determined endogenously.

Value of Information in Two-Person Organizations (joint work with N. Stokey)

By Jerry Green
Harvard University

Statistical decision theory studies the choice of an action based on an observation $y \in Y$. Observations are statistically related to states $\vartheta \in \Theta$ that are relevant for payoffs u. An information structure is the family of probability measures $\Lambda = \{\lambda(y|\vartheta)\}_{\vartheta \in \Theta}$. The value of Λ is the maximum expected payoff attainable by choosing actions as functions of y. An information structure Λ is said to be more informative than Λ' if its value is higher for all payoff functions. This partial ordering has been characterized by Blackwell (1951) and others.

The goal of this research is to develop analogous partial orderings for decisions undertaken by two-person organizations in which the players are specialized in their roles. The first, called the "agent", receives y and transmits $y' \in Y$, perhaps not truthfully, to the second, called the "principal" who chooses the action as a function of y'. Their payoffs may be different. This situation is modelled as a two-person game in which the strategy spaces are, respectively, transmission functions and action functions, with randomization allowed in both cases.

We say that "Λ is better for the agent than Λ'" if his payoff is higher at the equilibrium of the game with Λ for all payoff functions of the two players. Similarly for the principal.

The first case studied is that where the principal is a Stackelberg leader.

<u>Theorem</u> : There is no pair (Λ,Λ') for which Λ is better for the agent than Λ'.

That is to say, information structures can never be ranked unambiguously.

For the principal we need a definition:

Λ is said to be a <u>success-enhancing improvement</u> of Λ'

$$\text{if} \quad \lambda(y_o|\vartheta) = \alpha \qquad \forall \vartheta \in \Theta$$

$$\lambda'(y_o|\vartheta) = \alpha' \qquad \forall \vartheta \in \Theta$$

$$\lambda'(y|\vartheta) = \frac{1-\alpha'}{1-\alpha}\,\lambda(y|\vartheta) \quad \forall y \neq y_o, \quad \forall \vartheta \in \Theta, \alpha < \alpha'$$

In words, the probability that the experiment is completely uninformative (y_o received) is lower at Λ than at Λ', and otherwise the experiment is unchanged.

Theorem : Λ is better for the principal than Λ' if and only if Λ is a success-enhancing improvement of Λ'.

The second case studied is when the players choose their strategies simultaneously. It is necessary to give the space of information structures a topology, as the results are only local. They also apply only to a special type of equilibrium, called essential, which are generally shown to exist.

Theorem : If Λ is a sufficiently small improvement of Λ', then the agent's expected payoff is higher at Λ than at Λ'.

In contrast to the Stackelberg case, we have the complete ordering of Blackwell here.

Theorem : If Λ is a sufficiently small success-enhancing improvement of Λ', then the principal's payoff is higher at Λ than at Λ'.

For the principal, the results are very close. The necessity of the success-enhancing condition is, however, an open question.

The Number of Blocking Coalitions for Allocations Not Being Pareto Optimal

By Birgit Grodal
University of Copenhagen

The results are closely related to previous results by Mas-Colell (JME 1978) and Yamazaki, Greenberg, Weber (Bonn DP 1980-81) on the number of blocking coalitions for Pareto optimal allocations.

Let $N = \{ e \in \mathbb{R}^1_+ \mid e \leq s(1,..,1)\}$ where $s \in \mathbb{R}_+$ and

$X = \{ x \in \mathbb{R}^1_+ \mid x \leq K(1,...,1)\}$

where $K > 41 \cdot s$. Moreover let $\mathcal{P}$ be the set of monotone preference relations on X. We consider economies where agents have initial sources in N, X as consumption set and preference relation in $\mathcal{P}$.

Let $\mathcal{E} : I \to \mathcal{P} \times N$ be a finite economy and $\underset{\sim}{x}$ an allocation for $\mathcal{E}$.

We define $\mathcal{M}(\underset{\sim}{x}) = \inf \{\varepsilon \mid \exists \underline{x} : I \to X : \underline{x}(i) \succ_i \underset{\sim}{x}(i) \ \forall i$ and $\sum_{i \in I} \underline{x}(i) \leq \sum_{i \in I} e(i) - n\varepsilon(1,\dots,1)\}$ where $n = \# I$

$\mathcal{M}(\underset{\sim}{x})$ measure how far $\underset{\sim}{x}$ is from being Pareto optimal (PO.)

($\mathcal{M}(\underset{\sim}{x}) = 0 \Leftrightarrow \underset{\sim}{x}$ P.O.).

<u>Def.</u> : A coalition $C \subset I$ can improve upon $\underset{\sim}{x}$ if $\exists\ \underline{x} : C \to X$ s.t. $\underline{x}(i) \succ_i \underset{\sim}{x}(i) \ \forall i \in C$, and $\sum_{i \in C} \underline{x}(i) = \sum_{i \in C} e(i)$.

<u>Proposition 1</u>

For any v , $0 \leq v < 1$ and any S, K there exists H s.t. for any finite economy $\mathcal{E} : I \to \mathcal{P} \times N$ and any allocations x for $\mathcal{E}$

if

$$\mathcal{M}(\underset{\sim}{x}) > \frac{H}{\sqrt{n}} \text{ then } \frac{\#\{C \subset I \mid C \text{ can improve upon } \underset{\sim}{x}\}}{\#\{C \mid C \subset I\}} > v$$

This proposition also holds true if one considers any a, $1 \geq a > 0$ and restricts attention to coalitions C with m agents, where $\frac{m}{n} > a$.

Now moreover assume that preferences are C^2 and strictly convex and endow these with the topology of C^2 uniform convergence on compact sets. We shall now assume that agents have preferences in a compact subset of these preferences $\bar{E}$.

For each economy $\mathcal{E} : I \to \bar{E} \times N$, and any allocations $\underset{\sim}{x}$ for $\mathcal{E}$

we define a price system $p(\underset{\sim}{x})$ by considering the supporting hyperplane to the set:

$$Z(\underset{\sim}{x}) = \sum_{i \in I} \{x - e_i \mid x \in X \text{ and } x \succsim_i \underset{\sim}{x}(i)\} \text{ in the point}$$

$$\mu(\underset{\sim}{x})n(-1,..,-1) \text{ where } n = \#I.$$

$$\text{Define } \delta(x) = \sum_{i \in I} \frac{1}{\#I} (p(\underset{\sim}{x}) \; (\underset{\sim}{x}(i) - e(i))^2.$$

Proposition 2

For any $v < 1/2$ there exists an H such that for any finite economy $\mathcal{E} : I \to \bar{E} \times N$ and any allocation $\underset{\sim}{x}$ for $\mathcal{E}$ if

$$\delta(x) > \frac{H}{\sqrt{n}} \quad \text{then} \quad \frac{\#\{C \subset I \mid C \text{ can improve upon } \underset{\sim}{x}\}}{\#\{C \mid C \subset I\}} > v$$

This proposition also holds true if one considers any a, $0<a<1/2$ and restricts attention to coalitions C with m agents, where $a < \frac{m}{n} < 1-a$.

Welfare losses due to imperfect competition: Asymptotic results for Cournot-Nash equilibria with free entry (joint work with O. Hart)

By Roger Guesnerie
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In the past few years a number of authors have studied the properties of Cournot-Nash equilibria in large economies. One result which has been established is that under fairly general conditions, as the size of an economy tends to infinity the Cournot Nash equilibria of the economy converge to perfectly competitive equilibria of a well defined limit economy. One question which does not appear to have been addressed concerns the rate at which this convergence takes place. As indicated by the title, we measure this rate of convergence here by some traditional index of welfare loss (of Allais-type). We show that the asymptotic behaviour of welfare loss is sensitive to whether firm's average cost curves are U-shaped or whether they are everywhere declining. In the former case total welfare loss tends to zero as the size of the economy tends to infinity where in the latter case total welfare loss is unbounded and is in fact of the same order of magnitude as the square root of the size of the economy. We finally give arguments suggesting that our results on the order of magnitude are rather robust.

Incomplete Information: Non-Zero-Sum Repeated Games

By Sergiu Hart
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We give a characterization of all equilibrium points of two-person non-zero-sum infinitely repeated games of incomplete information, in the standard one-sided information case, where one player has more information than the other one.

It is shown that all equilibria are equivalent to a special class of equilibria; these consist of a "master plan", which each player follows so long as the other one does it too, and of "punishments", which come into effect after a deviation from the master plan has been detected. The master plan includes a sequence of "communications" between the players, the purpose being to settle eventually on an outcome that is jointly feasible, individually rational (minmax), and independent of the information ("non-revealing"); this outcome is then obtained by the two players, using the corresponding frequencies. The communications are of two sorts: "signalling", where the informed player makes use of his information and thus reveals some of it to the other player; and "jointly controlled lotteries", where the two players make together a randomization (and no player can change its probabilities). Finally, the punishment keeps the player that deviated at his individually rational level.

Formally, the concept of a bi-martingale is introduced. Given two compact subsets $\mathcal{X}$ and $\mathcal{Y}$ of some Euclidean spaces, an $(\mathcal{X} \times \mathcal{Y})$ - valued martingale $\{(X_n,Y_n)\}_{n=1}^{\infty}$ is a bi-martingale if (X_1,Y_1) is constant and, for each $n \geq 1$, either $X_n=X_{n+1}$ or $Y_n=Y_{n+1}$ (a.s.). For a set $G \subset \mathcal{X} \times \mathcal{Y}$, let G be the set of all points (x,y) such that there exists a bi-martingale $\{(X_n,Y_n)\}_{n=1}^{\infty}$ converging a.s. to (X_∞,Y_∞), and satisfying $(X_1,Y_1)=(x,y)$ and $(X_\infty,Y_\infty) \in G$ (a.s.).

The main result can be stated as follows: let G be the set of jointly feasible, individually rational and non-revealing payoffs in the one-shot game. Then the set of payoffs of all equilibria in the repeated game is precisely G .

On the law of demand

By Werner Hildenbrand
University of Bonn

Let $f(p_1,...,p_l,w)$ be a demand function which is derived from a preference relation $\precsim$ on R^l_+ .

Assume many agents having the same preference relation but they differ in income w. Let ρ be a density of the income distribution. Define the mean demand F

$$p \in \mathbb{R}^l_+ \;, \; p >> 0 \;, \; F(p) = \int_0^\infty f(p,w)\rho(w)\,dw$$

Problem:

Under what assumptions on $\precsim$ and ρ has $F(\cdot)$ properties like the "law of demand", e.g., $(p-q)\cdot(F(p) - F(q)) \leq 0$ for every $p \neq q$ (monotonicity).

Theorem:

Let $\precsim$ be smooth and regular and let ρ be continuous non-increasing on $[0,b]$, $b>0$. Then the Jacobian matrix $JF(p)$ of F at p is negative definite, i.e.

$$\sum_{h=1}^{l} \sum_{i=1}^{l} v_h v_i \partial_i F_h(p) < 0 \quad \text{for every } v \in \mathbb{R}^l \;, \; v \neq 0$$

Corollary

1) F is monotone (thus one-to-one) ; if f satisfies a boundary condition (desirability of commodities) then F is a diffeomorphism of $P= \{x \in R^l \setminus x>>0 \}$ onto itself.

2) F satisfies the weak axiom of revealed preference, i.e., $p \neq q$, if $qF(p) \leq \bar{w}$ then $pF(q) > \bar{w}$, ($\bar{w}$ mean income).

3) if $l>2$, then F does not satisfy, in general, the strong axiom of revealed preference.

4) There is a "non-transitive representative consumer".

Excess Supply Equilibria and IS-LM Analysis

By Reinhard John
University of Bonn

The determination of employment, aggregate production and the interest rate in an economy is usually described by the Hicksian IS-LM-diagram. It is shown that this model can be derived from a simple four commodity temporary equilibrium model (one good, labor, money, and a short term bond). To be more precise, the equilibrium values of output, employment, and interest are represented as an excess supply equilibrium at fixed money prices and money wages. This is defined by an allocation of output, an allocation of employment, and an interest rate, which have the following properties:

1) The output of each firm is not greater than its desired supply at the given prices.

2) The employment of each consumer is not greater than his desired labor supply.

3) Supply equals demand on each market, where aggregate supply and demand functions are derived from microeconomic decision behaviour which takes the output and employment constraints into account.

On the demand function generated by a smooth and concavifiable preference ordering (joint work with L. Hurwicz)

By Yakar Kannai
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Rehovot, Israel

The Giffen effect has a puzzling effect. Let u be a C^2 function defined on R^n_+ with positive partial derivatives and strictly convex level sets. Interpreting u as utility function, we consider the demand function $x(p,w)$ of an individual whose preferences are represented by u, who has an endowment vector w and faces the price vector p. It is well known that if the Gaussian curvature of the indifference surface is nonzero at $\bar{x}$, then x is differentiable at prices $\bar{p}$ for which $x(\bar{p},w) = \bar{x}$. If u is also concave (so that the preference ordering is concavifiable) then

(i) $\overline{\lim_{p \to p_o}} \frac{\partial x_i}{\partial p_i}(p,w) < \infty \qquad 1 \le i \le n$

(ii) If n=2, then

$$\lim_{p \to p_o} \frac{\partial x_i}{\partial p_i}(p,w) = -\infty$$

Here p_o is a price system such that the Gaussian curvature of the indifference surfaces vanishes at $x(p_o,w)$ (so that x is not differentiable at p_o). These results show that there is no "infinite slope Giffen effect".

A Substitution Theorem for General Linear Models (joint work with D. Hinrichsen)

By Ulrich Krause
Universität Bremen

It is shown how the well-known nonsubstitution theorem has to be replaced by a substitution theorem if general joint production is admitted. Consider a set $\mathfrak{J}$ of processes producing n goods by means of these goods and 1 primary factor (labor). Let $c_i \in \mathbb{R}^n$, $l_i \in \mathbb{R}_+$ be net output and labor input of $i \in \mathfrak{J}$. For y in the net output cone N corresponding to $\mathfrak{J}$ the minimal labour input to produce y (exactly) is given by $\lambda(y) = \inf \{ \sum_i x_i l_i \mid \sum x_i c_i = y, x_i \geq 0 \}$. The following assumptions (beside constant returns to scale) are made: $\mathfrak{J}$ productive, dim N=n, $l_i = 0$ only if $c_i \leq 0$, the cone generated by the (c_i, l_i), $i \in \mathfrak{J}$ is closed in $\mathbb{R}^{n+1}$. Employing a theorem on unique marked representation in convex sets the following substitution theorem is proved: There exists a collection $\mathfrak{C}$ of finite subsets $I \subset \mathfrak{J}$ (instead of a single subset as it is the case with non-substitution) having the following properties:

(i) $\{c_i \mid i \in I\}$ linear independent, I productive, $|I| \leq n$ for every $I \in \mathfrak{J}$

(ii) $N \cap \mathbb{R}^n_{++} = \bigcup \{N(I) \cap \mathbb{R}^n_{++} \mid I \in \mathfrak{C}\}$, N(I) being the net output cone corresponding to I.

(iii) If $y = \sum_{i \in J} x_i c_i \in N(I)$, $(I, J \in \mathfrak{C})$, then $x_i > 0$ only for $i \in I$ and $\lambda(y) = \sum_{i \in I} x_i l_i = y \lambda(I)$ where $\lambda(I)$ is a solution of $c_i \lambda \leq l_i$ for $i \in \mathfrak{J}$, $c_i \lambda = l_i$ for $i \in I$.

(iv) The collection of the cone N(I), $I \in \mathfrak{C}$, defines a certain simplicial subdivision of N with respect to which the

mappings $\lambda(\cdot)$, $x_i(\cdot)$ are piecewise linear on N. Furthermore, if $\mathfrak{J}$ is finite, then $\mathfrak{C}$ is finite and for every $I \in \mathfrak{C}$ $\lambda(I)$ is uniquely determined and $|I|=n$.

Loosely speaking, the theorem says, that in the presence of joint production non-substitution holds only piecewise.

A General Demand System

By Wilhelm Krelle
Universität Bonn

Given an arbitrary demand system (o) $x_i=f_i(p_1,\dots,p_n,y)$, $i=1,\dots,n$ with the usual properties: nonnegativity constraints: $x_i \geq 0$ if $p_i \geq 0$ and $y \geq 0$, adding up constraint: $\sum_i p_i f_i = y$ and homogeneity constraints: $x_i = f_i(\lambda p_1,\dots,\lambda p_n,\lambda y), \lambda > 0$. Assume that the system can be solved for the $p_1,\dots,p_n$ to get the unique solution $p_i=g_i(x_1,\dots,x_n,y)$, when the constraints hold again. This system may be brought into the form

$$\frac{x_i p_i}{y} = \varphi_i(x_1,\dots,x_n)$$

where $\sum_i \varphi_i = 1$. In the case of a utility maximizing household this system assumes the form

$$(*) \quad \frac{x_i p_i}{y} = \frac{x_i \;\partial U/\partial x_i}{\sum_j x_j \;\partial U/\partial x_j}$$

, where $U(x_1,\dots,x_n)$ is the utility function with the usual properties. Expand the RHS of (*) by a Taylor expansion up to the quadratic term:

$$(**) \quad \frac{x_i \;\partial U/\partial x_i}{\sum_j x_j \;\partial U/\partial x_j} = a_{io} + \sum_j a_{ij}x_j + \sum_j b_{ij}x_j^2 + \sum_{r<s} c_{irs}x_r x_s, \; i=1,..,n$$

where the adding up constraints are:

$$\sum_i a_{io}=1,\ \sum_i a_{ij} = \sum_i b_{ij} = \sum_i c_{irs} = 0$$

(*) with (**) is called the quadratic approximation to the general demand in case of an utility maximizing household.- This system could have been exactly derived from a utility maximizing household, if the integrability conditions are met. These are:

(1) $\sum_i a_{io} = 1$

(2) For all i=1,...,n $\exists\, c_i \in \mathbb{R}$ such that $\dfrac{a_{ii}}{\sum_{i \neq j} a_{ij}} = -\dfrac{a_{ki}}{a_{ko}} =: c_i, k \neq i$
$k \neq i$

(3) $\exists\, \alpha_1,\ldots,\alpha_n \in \mathbb{R}$ such that for all i,j,k=1,...,n:

$b_{ij}=\alpha_j a_{ij}$, $c_{ijk}=\alpha_j a_{ik}+\alpha_k a_{ij}$.

Thus such a household always exists in the case of n=2. The integration of the partial differential equation system for the linear case ($b_{ij}=c_{irs}=0$), (disregarding boundary cases) yields the utility function

$$U(x_1,\ldots,x_n) = F\left(\prod_k (x_k)^{-a_{ko}} - \sum_j c_j \left[\prod_{k \neq j} \left(\frac{x_k}{x_j} \right)^{-a_{ko}} \right] \right)$$

F an arbitrary, monotone , continously differentiable function. It is easy to derive the system of demand functions from it.- It is unknown whether the integration can be explicitly carried out in the quadratic case,- The following propositions have been proved.

<u>Proposition 1</u> : A system of continuous demand functions (O) may be approximated by a system (*) and (**) where $b_{ij}=c_{irs}=0$ to any desired degree of accuracy in a neighborhood of a given point $(\bar{p}_1,\ldots,\bar{p}_n,\bar{y})$.

Proposition 2 : In the quadratic case (b_{ij} and c_{irs} not all zero) also the partial derivatives

$$\left.\frac{\partial f_i}{\partial p_j}\right|_{\bar{p}}$$

may be approximated by the partial derivates of the demand system originating from this quadratic form to any desired degree of accuracy in the neighborhood of $(\bar{p},\bar{y})$.

Thus a representative household exists as an approximation. Empirical estimations of demand functions for firms and households have been presented which show that in many cases forecasts which are made under the assumption of the existence of a representative household or a representative firm have smaller % deviations from reality than those not using this concept.

Recent results on super-additive solutions

By Michael Maschler
Hebrew University, Jerusalem

We consider Nash bargaining games of two players with a conflict point at (0,0) and the feasible set S is a not empty compact, convex, and comprehensive subset of $\mathbb{R}^2_+$. The super-additive solution can be characterized from two sets of axoims. The first set, which we shall call "set A", is dealing directly with the space of these games. The second -set B - treats each game separately and, in fact, it is a set of axioms that characterizes a negotiation process that leads the players from the origin to the solution. (The second set of axioms does not require that S is convex, but then one looses the continuity property of the solution).

If we omit the symmetry axiom we get a two-parameter family of solutions $\bar{u}(S)$, defined by

$$\int_{p(s)}^{\bar{u}(s)} |du_1|^{\vartheta} |du_2|^{1-\vartheta} = \lambda \int_{p(s)}^{q(s)} |du_1|^{\vartheta} |du_2|^{1-\vartheta} \qquad 0 \le \vartheta \le 1$$

where $S \cap R_+^2 = ((0,0),(0,p(s)) \cup ((0,0),(q(s),0))$

In the symmetric case $\lambda = \vartheta = 1/2$. The parameter λ can be interpreted as an index of the bargaining ability of the two players. Apparently, the parameter ϑ represents an aggregation of players having the same utility function to player 1 and a similar aggregation for player 2.

The axioms in set A are inconsistent if we come to n-person Nash bargaining games, $n \geq 3$. Fortunately, set B yields a solution for many games. The axioms concern a path $u(t)$, $u(0)=(0,0)$ that is supposed to connect the origin to the solution point $\bar{u}(s)$ on the Pareto optimum of S. Specifically they are:

(1) For every two players i and j, $i \neq j$

$$\frac{\dot{u}_i}{\dot{u}_j} = F(p_{ij}^i, p_{ij}^j),$$

$$\text{where} \quad p_{ij}^i(t) = \frac{-\dot{v}_i^i}{\dot{v}_j^i} = \frac{-\dot{v}_i^i}{\dot{u}_j}, \quad p_{ij}^j(t) = \frac{-\dot{u}_i}{\dot{v}_i}$$

and $v^i = v^i(t) = (u_1, \ldots, u_i^i, \ldots, u_n)$

and $v_i^i = \text{Max}\{x \mid (u_1, \ldots, x_1, \ldots, u_n) \in S_n\}$

(2) $\alpha \geq 0 \Rightarrow F(\alpha x, \alpha y) = \alpha F(x,y)$.

(3) $F(x,y)\ F(1/y,\ 1/x) = 1.$

At present we know that a solution is obtained if S has a C^2-north east boundary with positive gradients at each point. We also know that the axioms yield a solution if S is a polyhedron. We are still working on other cases.

Walrasian Equilibria as Limits of Mixed Strategy Non-Cooperative Equilibria

By Andreu Mas-Colell
Harvard University

Consider an exchange economy $\mathcal{E}$ with a continuum of firms and let (p^*,y^*) be a Walrasian equilibrium, where p^* is the price vector and y^* the aggregate production. Suppose that, in the appropriate sense (p^*,y^*) is regular, i.e. nondegenerate. Individual firm technologies exhibit set-up cost, but are, otherwise, classical.

Let now $\mathcal{E}_n \to \mathcal{E}$ be a sequence of economies with a finite number of firms converging to $\mathcal{E}$ (in a natural sense). In $\mathcal{E}_n$ firms are significant relative to demand and therefore, can affect price. As in the earlier work of Novshek and Sonnenschein we adopt as solution concept the quantity setting Cournot non-cooperative equilibrium with mixed strategies.

We ask the following lower hemicontinuity question: is (p^*,y^*) the limit of a sequence (p_n,y_n) of mixed noncooperative equilibrium for $\mathcal{E}_n$? This is asked in order to clarify the predictive power of Walrasian equilibrium theory.

<u>Theorem</u> : If the aggregate production possibility set of the aggregate economy (which is convex) exhibits decreasing returns to scale (in whatever degree) then there is N and for each n>N a mixed non-cooperative equilibrium y_n for ε_n such that:

(i) $y_n \to y$

(ii) only marginal (i.e. zero profits) firms use mixed strategies.

(iii) The fraction of firms which are marginal tends to zero.

A corollary of the theorem is the existence of equilibrium for n>N. In contrast with the earlier work of Novshek and Sonnenschein (who obtain the same conclusion) we do not allow for strict free entry but impose no substantive restrictions on demand in order to get the existence of the sequence.

The Dynamics of Oligopoly
(joint work with Jean Tirole)

By Eric Maskin
Massachussetts Institute of Technology

We study the perfect equilibria of infinite horizon, alternating move duopoly games. Firms maximize their discounted sum of profits. The alternating move specification captures the idea that altering one's move and reacting to one's opponent takes time. A firm's strategy is assumed to be a function only of its opponent's preceding move. Restriction to such strategies is justified by their simplicity and by the fact that, at least in some of our models, there are limits of the finite horizon equilibrium strategies as the horizon grows. We study games both where moves are quantities and where they are prices and discuss the circumstances where these alternative specifications are appropriate.

In the quantity-setting case, we show that the equilibrium steady-state quantities are always higher than in the corresponding static Cournot game. Moreover, these quantities are increasing functions of the firms' discount factors. Indeed, in the case where fixed costs are so large that only one firm can operate profitably, the steady-state quantity approaches the zero-profit level in the unique symmetric equilibrium as the discount factor approches 1 . In the model with constant costs, we introduce a cost of adjustment and show that equilibrium output tends to the static Cournot level when adjustment costs become large.

In the price setting models, by contrast, we show that an increasing discount factor enhances firms' opportunities to collude implicitly. In one kind of equilibrium, a steady-state price above marginal cost is enforced by the threat of a price war if a firm attempts to untercut the other and by the threat that the other firm will not follow if a firm attempts to increase its price. Thus, equilibria resemble those of the traditional "kinked demand curve" model. Another kind of equilibrium that we exhibit resembles a classical Edgeworth cycle. We also show that, in price-setting models, market shares emerge naturally as strategic variables when firms' costs differ. A firm with high marginal costs can induce a low marginal cost firm to accept a higher market price than otherwise by allowing it a greater share of the market.

Bargaining as a Non-cooperative Implementation Problem

By Hervé Moulin
University of Paris

The "successive proposals with a geometric stopping rule" game consists of a bargaining procedure where the agents, successively and randomly called upon, make proposals that the other agents must unanimously accept. Each rejection induces a small probability of cooperative breakdown that the referee enforces. The perfect equilibrium of the above game form is computed and it is shown that when the probability of breakdown goes to zero, the corresponding equilibrium payoffs converge towards the Nash bargaining solution. Another procedure, called "auctioning fractions of dictatorship" is also studied which noncooperatively implements the Kalai-Smorodinsky arbitration payoffs.

General Equilibrium with Price Setting Firms (joint work with E. Dierker and R. Guesnerie)

By Wilhelm Neuefeind
Washington University, St. Louis

An economy is considered which consists of price taking agents (utility maximizing consumers and profit maximizing producers) as well as of price setting firms. These latter firms are given input prices and output targets as parameters, they minimize costs, and set prices according to a certain rule. Price setting firms are not required to maximize profits, nor to generate non-negative profits.

Conditions on the price setting rule are explored which warrant the existence of decentralizing parameters (i.e. prices for price taking agents and input prices and output targets for price setting firms) such that the induced aggregate demand and supply vectors coincide. An existence theorem is given.

Walrasian Equilibria as Limits of Pure Strategy Noncooperative Equilibria (joint work with H. Sonnenschein)

By William Novshek
Stanford University

This paper concerns the connection between the Walrasian equilibria of a limit economy, in which firms are infinitesimal relative to the demand, and the (quantity setting) noncooperative equilibria of approximating finite economies, in which firms are significant relative to demand. We allow for nonconvex production sets and the number of active firms in an equilibrium is determined by the possibility of profitable activity. We include in our analysis the case in which returns of scale are decreasing in the aggregate, i.e. the employment of a production set is associated with the use of a scarce factor. A condition (loosely analogous to downward sloping demand in the partial equilibrium case of constant returns of scale aggregate production) on the Walrasian equilibrium is necessary for that equilibrium to be a limit of pure strategy noncooperative equilibria. The condition is also sufficient to guarantee the existence of a robust sequence of pure strategy noncooperative equilibria which converges to the Walrasian equilibrium though it is not sufficient to guarantee existence of pure strategy equilibria in the tail of any sequence of economies converging to the limit economy.

Walrasian Equilibrium Indeterminacy and Aggregate Policy (joint work with J.D. Geanakoplos)

By Herakles Polemarchakis
C.O.R.E., Louvain-la-Neuve

In economies extending infinitely into time, local uniqueness and Pareto optimality of the Walrasian equilibria may fail; they do indeed in a model of overlapping generations. On the one hand this result is negative and invites the consideration of additional structures which restore local uniqueness and Pareto optimality; on the other hand it makes possible the analysis of comparative statics and of the behaviour of macroeconomic aggregates in an equilibrium framework.

Partnerships and Moral Hazard

By Roy Radner
Bell Laboratories, Murray Hill, N.J.

This paper investigates partnerships with moral hazard. in which (1) The output of an organization depends jointly on the efforts of the members (partners) and on a stochastic environment,

(2) individual efforts cannot be accurately measured, and,

(3) the output is shared among the partners according to a well-defined rule.

In the corresponding (one-period) game model, equilibria of the game are typically inefficient. If the game situation is repeated indefinitely, and the partners do not discount future expected utility, then there are equilibria of the repeated game (i.e. self-enforcing behaviours) that are efficient. An open question is (Q): is it the case that the less the partners discount future utility the closer they can get to efficiency in equilibria of the repeated game? Analogous theorems can be proved for the principal agent relationship, for which question Q can be answered in the affirmative.

L.P. - Games with sufficiently many players

By J. Rosenmüller
Universität Bielefeld

An integer valued measure m $(=(m_1,\dots,m_n))$ on a finite set $\Omega = \{1,\dots,n\}$ is said to be nondegenerate w.r.t. some system of subsets of Ω, say $\underline{Q}$ if it is uniquely determined by its values on the elements of $\underline{Q}$, i.e., the linear system of equations in variables $x_1,\dots,x_n$ given by

$$\sum_{i\in S} x_i = m(S) \;\Big|\; S \in \underline{Q}$$

has the unique solution m. E.g. if $\underline{Q} = \{S \mid m(S) = \lambda\}$ then $\underline{Q}$ may be interpreted as a system of minimal winning coalitions of a homogeneous weighted majority game and to find conditions for "n.d." amounts to solving certain number-theoretical problems. Now, consider an "L.-P. game" (Linear production, linear program) given in characteristic function form via

$$v(S) = \max \{cx \mid Ax \leq b \;,\; x \in \mathbb{R}^l_+\}$$
$$= \min \{yb(S) \mid yA \geq c \;,\; y \in \mathbb{R}^n_+\} \qquad (S \subseteq \Omega)$$

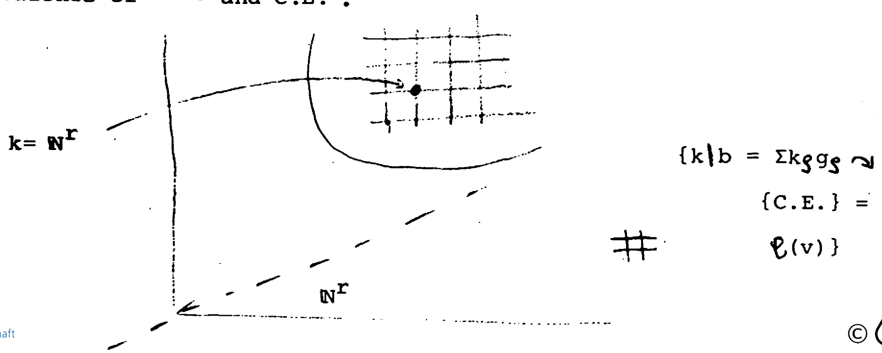
where A:a matrix, c:a vector and b( ):a vector valued measure (all nonnegative and nontrivial).Pick an optimal dual Ω-solution (S=Ω in the dual problem) then $\bar{y}b(\cdot)$ is a core element of v (the"competitive equilibrium") (Owen, Billera-Raanan) Now, if

$Q_o = \{z \mid \bar{y}$ is optimal for A,c,z$\} \subseteq \mathbb{R}^m_+$ then the decisive system is

$$\underline{Q} = \{S \subseteq \Omega \mid b(S) \in Q_o \ni b(S^c)\} \;,$$

that is, if "b n.d. $\underline{Q}$" (the vectorvalued measure b uniquely defined by its values on $\underline{Q}$) then (assuming $\bar{y}$ unique) $\mathcal{C}(v)$={C.E.}. If we introduce types of players and g_ϱ is the initial endowment of type ϱ , k_ϱ =#{agents of type ϱ}, then b n.d.$\underline{Q}$ iff there are r linearly independent elements of the $(g_\varrho)_{\varrho=1,..,r}$ - lattice in kQ_o ("Q_o blown up by k") .

Minkowski's estimates concerning the lattice constants of a convex body may therefore be used in order to design areas in N^r such that, within these areas the vectors $k \in \mathbb{N}^r$, representing the distribution of players over types, guarantee the equivalence of core and C.E. .



Arbitrage and Mean-Variance Analysis on Large Asset Markets (joint work with Gary Chamberlain)

By Michael Rothschild
University of Wisconsin

We examine the implication of arbitrage in a market with many assets. The absence of arbitrage opportunities implies that the linear functions that give the mean and cost of a portfolio are continuous; hence there exists unique portfolios that represent these functions. The mean variance efficient set is a cone generated by these portfolios. Ross (JET 1976) showed that if there is a factor structure, then the distance between the vector of mean returns and the space spanned by the factor loadings is bounded as the number of assets increases to ∞. We show that if the covariance matrix of asset returns has only K unbounded eigenvalues then the corresponding eigenvectors converge and play the role of factor loadings in Ross's result. Our eigenvalue condition can hold even though conventional measures of the approximation error in a K factor model are unbounded. We also resolve the question of when a market with many assets permits so much diversification that risk-free investment opportunities are available.

Neighborhood Systems for Integer Programs

By Herbert Scarf
Yale University

Let $A = \begin{bmatrix} a_{o1} & \cdots & a_{on} \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}$, assume that for each i

$\sum a_{ij} h_j = h_i$ contains at most one lattice point $(h_1, \ldots, h_n)$ and that for any $b = (b_o, \ldots, b_m)'$, the inequalities $Ah \geq b$ contain finitely many lattice points. We are concerned with the integer programs

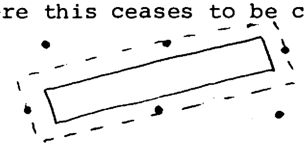
$$\max \sum a_{oj} h_j$$

$$\sum a_{ij} h_j \geq b_i \quad (i=1,\ldots,m) \text{ and } h \text{ integral.}$$

For each lattice point in R^n, h, let N(h) be a finite set of other lattice points called the neighborhood of h. We assume that neighborhoods translate and that $h \in N(k) \Rightarrow k \in N(h)$.

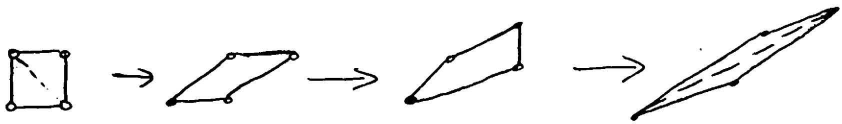
Theorem : There is a unique, minimal neighborhood system, depending on A, for which a local maximum - with respect to that neighborhood system - is a global maximum to the above integer program, for any right hand side.

The neighborhood system is found by placing the m+1 linear inequalities $\sum a_{ij} h_j \geq \xi_i$ in such a way that the constraint set contains no lattice points and then relaxing the inequalities up to the point where this ceases to be correct.



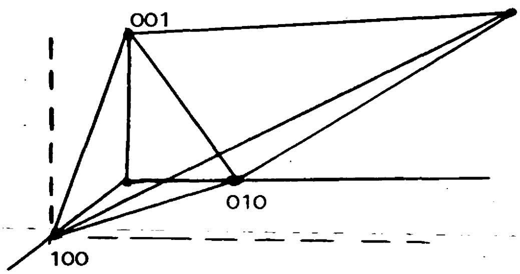
When this is done we obtain a set of lattice points whose convex hull contains no other lattice points. Any two lattice points which are in a common polyhedron obtained by an arbitrary relaxation from a lattice free region are defined to be neighbors.

The collection of polyhedra form a pseudomanifold, equivalent to $\mathbb{R}^n$, and embedded in $\mathbb{R}^n$. A form of Sperner's Lemma can then be used to solve the resulting integer program. The method can be carried out effectively if the complete set of relaxations associated with A can be found. If A is 4x2, they are of the following form



with two pairs of triangles at the end of the series. This knowledge permits one to construct a polynomial algorithm for the 2 variable problem.

If A is 4x3 we are led to a collection of tetrahedra in $\mathbb{R}^3$ with integral vertices and which contain no other lattice points. Recently, Roger Howe of Yale University has shown that such a tetrahedron can, up to a unimodular transformation, be brought to the form



1,p,q with
(p,q)=1

In other words there is a plane in lattice space, say $h_1=0$ so that such a tetrahedron lies between $h_1=0$ and $h_1=1$. It can be shown that the same lattice plane has this property for all relaxations of the same set of 4 linear inequalities from a lattice free region. This tells us that all of the neighbors of a point (h_1,h_2,h_3) have some first coordinate equal to h_1-1, or h_1 or h_1+1 (after a suitable unimodular transformation).

Subjective Probability without Additivity

By David Schmeidler
University of Tel-Aviv

Let Y denote the set of distributions with finite support over X and let Σ denote an algebra of subsets of S. $L = \{f:S \to Y \mid f \text{ is simple and } \Sigma\text{-measurable}\}$. $\succsim$ is a weak order over L, its restriction to constant functions in L induces a weak order, $\succsim$ over Y. Suppose that $\succsim$ satisfies:

(i) $f \succ g$: $\forall h \; \exists \alpha, \beta$ s.t. $\alpha, \beta \in (0,1)$, $\alpha f+(1-\alpha)h \succ g \succ \beta f+(1-\beta)h$.

(ii) $E \in \Sigma$, $f(s)=y$ $s \in E$, $g(s)=z$ $s \in E$, $f(s)=g(s)$ $s \notin E$,
$f \succ g \Rightarrow y \succ z$.

(iii) If for all (ordered) $(s,t) \in S \times S$: $f(s) \succ f(t) \Rightarrow$
$g(s) \succsim g(t) \wedge h(s) \succsim h(t)$; $g(s) \succ g(t) \Rightarrow f(s) \succsim f(t) \wedge$
$h(s) \succsim h(t)$ and $h(s) \succ h(t)$ .... . Then $f \succ g \Rightarrow \alpha f+(1-\alpha)h \succ$
$\alpha g+(1-\alpha)h$ for $\alpha \in (0,1)$.

<u>Theorem</u>: There exists a unique set function v on Σ with $v(\emptyset) = 0, v(S)=1$ and a unique NM utility on Y s.t. $f \succ g$ iff $\int u(f(s))\,dv > \int u(g(s))\,dv$, where $\int a(s)\,dv$ for positive valued $a(\cdot)$ is defined as $\int_0^\infty a^*(\alpha)d\alpha$ and $a^*(\alpha)=v(a > \alpha)$.

Ergodic Theorems for Subadditive Superstationary Families of Random Sets

By Klaus Schürger
Universität Bonn

Let $\mathcal{C}=\{C\subset R^d : C \text{ compact, nonvoid}\}$,

$co\mathcal{C}=\{C\in\mathcal{C} : C \text{ convex}\}$.

Let $\varrho(\cdot,\cdot)$ denote the Hausdorff metric on $\mathcal{C}$ (so $(\mathcal{C},\varrho)$ is a Polish space). A random set (r.s.) is a measurable mapping Y from some probability space $(\Omega,\mathfrak{A}, P)$ into $(\mathcal{C},\varrho)$ ("measurability" is referring to the Borel sets in $(\mathcal{C},\varrho)$). We consider families $X=(X_{s,t})$ of $co\mathcal{C}$-valued r.s. $X_{s,t}$ $(0\le s<t,\ s,t$ integers). X is called subadditive provided $X_{s,t}\subset X_{s,u}+X_{u,t}$ $s<u<t$ ("+" denoting the Minkowski sum.) Now conceive X as a matrix

$$X = \begin{pmatrix} X_{o,1} & X_{1,2} & X_{2,3} & \cdots \\ X_{o,2} & X_{1,} & X_{2,4} & \cdots \\ \cdots & \cdots & \cdots & \cdots \end{pmatrix} : \Omega \to (\mathcal{C}^N)^N =: \mathcal{M}$$

($\mathcal{M}$ is given the product topology making $\mathcal{M}$ Polish). Let "$\subset$" denote the elementwise set theoretic inclusion on $\mathcal{M}$ (i.e. $\forall Y,Z\in\mathcal{M}: Y\subset Z$: iff $Y_{i,j}\subset Z_{i,j}$). "$\subset$" is a closed partial order relation on $\mathcal{M}$. If P_1,P_2 are any probability measures on $\mathcal{M}$, let $P_1\prec P_2$ ("stochastically smaller") iff

$$\int_{\mathcal{M}} f dP_1 \le \int_{\mathcal{M}} f dP_2 \quad \text{for all } f:\mathcal{M}\to R$$

which are measurable, bounded and increasing. Let $T:\mathcal{M}\to\mathcal{M}$ be the "shift" defined by $(TY)_{i,j} = Y_{i,j+1}$, $Y\in\mathcal{M}$. $X=(X_{s,t})$ is called superstationary iff distribution of $TX\prec$ distribution of X. Based on an ergodic theorem due to Abid (1978) (generalizing ergodic theorems of Kingman (1968) and Krengel (1976)) we can prove : Let $(X_{s,t})$ be a subadditive superstationary family of co $\mathcal{C}$ -valued r.s. Under certain integrability conditions, there exists a r.s. Y such that

$$\lim_{t\to\infty} \frac{1}{t} X_{o,t} = Y \text{ a.e.}$$

and
$$\lim_{t\to\infty} \int_{\Omega} \delta\left(\frac{1}{t} X_{o,t}, Y\right) dP = 0.$$

Under additional assumptions Y can be chosen constant.
If the r.s. $X_{s,t}$ are merely assumed to be $\mathcal{C}$-valued, the above mentioned result fails, in general, to hold.

Bureaucratic Budget Bargaining as a Game of Incomplete Information

By Reinhard Selten
Universität Bielefeld

Niskunen's theory of bureaucracy assumes that the bureau can fully exploit the sponsor's ignorance of its costs. It seems to be most adequate to look at the relationship between bureau and sponsor as a bargaining game with incomplete information. In order to simplify the analysis attention is focused on a special case of budget bargaining on a new service whose amount is exogenously fixed.

In a very simple bargaining model, the "sponsor commitment model", the sponsor first commits himself to a budget which the bureau either accepts or rejects. Another more realistic "sequential model" assumes that the bureau has to make proposals; if the sponsor rejects a proposal, the bureau can make a new one.

The bureau faces a small cost of making a proposal. The game ends, if the sponsor accepts or if the bureau decides not to make any further proposals.

Under reasonable restrictions on the nature of the equilibrium point (uniform perfectness and an additional simplicity requirement) it turns out that the sponsor commitment model and the sequential model come to essentially the same conclusion.

Finally the question of optimal institutional design of the bargaining procedure is considered from the sponsor's point of view. It is shown that it would be optimal to present the bureau with a choice between a number of simple lotteries each involving one budget to be accepted with an associated probability. Since the expected budget must be equal for all lotteries, it is easy to compute the optimal combinations of lotteries for specific examples.

Game Theoretic Approach of the Money Rate of Interest

By Martin Shubik
Yale University

The money rate of interest may be regarded as a tax which can be used to finance the transfer of capital stock which is not transferred by gift. It is possible to construct a closed multiperiod noncooperative game with two banks and two types of money - an outside - or central bank of issue and an inside - or commercial bank of deposit, and solve for equilibria.

With the appropriate conditions on production technology and initial endowments a stationary state equilibrium will exist with a positive money rate of interest for outside money and a generally different rate for inside money.

The commercial bank is sufficient to vary the money supply but not necessary. If the central bank is willing to serve as a continuous bank of issue and deposit then only one bank is needed. But both capital stock and the needs of trade must be financed.

The model has only spot markets for goods. Exogenous uncertainty is not considered as it introduces further difficulties which will be considered separately. The simplest game has 7 stages (Fig. 1)

Let there be n types of traders, g types of firms, τ periods

A typical trader of type i has an initial endowment of a^i;

utility of $\Phi_i(x_1^i,\dots,x_\tau^i,b_\tau^i) = \mathcal{S}_i(x_1^i,\dots,x_\tau^i) + \mu_i \min[0,b_\tau^i]$

There are stocks $A_1,\dots,A_m$ available to be bought. The government announces ϱ the outside money rate of interest, η the inside rate, $\mu_1,\dots,\mu_n$ bankruptcy penalties, $\pi_1,\dots,\pi_m$ salvage value or buy back prices. Let b_τ^i = net assets at end, $\hat{b}_t^i$ = net assets not counting outside debt and final asset credit. Let k_τ^i = assets left over then $\{(1+\varrho)^\tau - 1\}\, m = \pi k$ and if e_t = net earnings of inside bank in period t $\left|\ \sum_{t=1}^{\tau} (1-\eta)^{\tau-t} e_t = \{(1+\varrho)^\tau - 1\}\, m\right.$ (e_t will include float loss)

<u>Remark 1</u> : A stationary equilibrium with $\varrho > 0$ exists

<u>Remark 2</u> : All m is used to finance bank stock

<u>Remark 3</u> : Growth can be directed

<u>Remark 4</u> : Consider T generations

Model I $\varphi \quad \varphi \quad \varphi \quad \varphi$

Model II $\varphi + \varphi + \varphi + \varphi$

Model I ϱ = const as $T \to \infty$

Model II $\varrho \to 0$ as $T \to \infty$

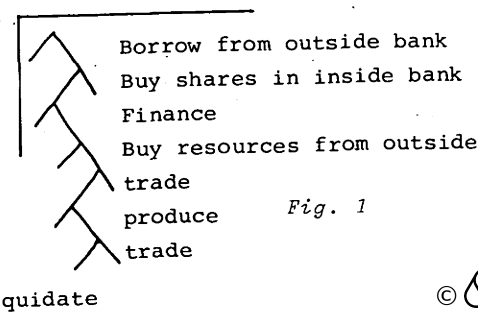


Fig. 1

Voluntary Trading and Pareto Efficiency

By Joaquim Silvestre
Universitat Autònoma de Barcelona

Theorem : Consider an Arrow-Debreu productive economy with given endowments and profit shares. Assume that preferences and technologies are smooth. Let w^* be an allocation where all consumption vectors are interior to the consumption set and which (1) : is Pareto efficient, and (2) : at some price vector $p^* >> 0$ all agents maximize their objective function relative to smaller (in absolute value) trades. Then there exists a vector $\hat{p}$ such that $(\hat{p}, w^*)$ is a competetive equilibrum for $\mathcal{E}$ ($\hat{p}$ and p^* may differ only in the coordinates corresponding to non-traded commodities.

Corollary : Unless interiority or differentiability are violated (A) : Non-Walrasian Fixprice Allocations - in the sense of Drèze, Benassy, Younès - will not be Pareto efficient; (B) : Non-Walrasian Budgét Constrained Pareto Optima - in the sense of Balasko and Keiding - will force some agents to trade more than they wish.

The interiority and smoothness assumptions are indispensabl Some economically interesting instances of voluntary, Pareto efficient allocations which are not Competitive Equilibria for $\mathcal{E}$ may arise when either some consumer is selling all his endowment of a commodity (interiority violated) or some firm is operating at capacity limits (smoothness violated).

Price-Dispersed Preferences and Mean Demand (joint work with E.&H. Dierker)

By Walter Trockel
Universität Bonn

Problem : Classify consumption sectors in terms of distributions on the space of consumer characteristics which yield a C^1 mean demand function of prices although individual preferences may be non convex.

Result : The theorem to which the talk referred to (Dierker, Dierker, Trockel (1981)) states a class of consumption sectors for which mean demand is C^1.

The talk confined to one of the basic concepts: price-dispersion of preferences.

R^l commodity space, $l \geq 2$ divisible commodities

$X = \text{int}\, \mathbb{R}^l_+$ consumption set of all agents

$S = \{p \in R^l \mid p \gg 0,\ p_1 = 1\}$ price space

$(w, \lesssim)$ describes a consumer, $w \in]0,\infty[$ being wealth, $\lesssim \in \mathcal{P}$ being a preference relation on X.

$\varphi(\lesssim, p, w) = \{x \in X \mid y \not\lesssim x \Rightarrow py > w\}$ demand set of $(\lesssim, w)$ at price system p.

Specification of the topology on $\mathcal{P}$ is not needed for discussion of price dispersion. However assume $\mathcal{P}$ is Polish.

Define multiplication in S coordinatewise. This makes S a locally compact commutative group admitting a Haar measure on it. Look at the following continuous actions of S on X and on $\mathcal{P}$

$S \times X \to X : (q,x) \mapsto x^q = (x_1 q_1, \ldots, x_{l-1} q_{l-1}, x_l 1)$

$a : S \times \mathcal{P} \to \mathcal{P} : (q, \precsim) \mapsto \precsim_q$ where $x \precsim y \Leftrightarrow x^q \precsim_q y^q$

key consequence for demand set is

$$q \circ \varphi(\precsim, q \cdot p, w) = \varphi(\precsim_q, p, w)$$

demand behaviour of all preferences in the orbit $S \circ \precsim$ at one price system is determined by the demand behaviour of $\precsim$ at all prices.

The measurable, surjective action a defines a disintegration for any Borel probability γ on $\mathcal{P} \times S$, i.e.

$$\gamma = \int_{\mathcal{P}} \int'_{\precsim} \gamma \circ a^{-1} (d \precsim)$$

where $\int'_{\precsim}$ is the conditional probability given $\precsim$.

<u>Definition</u> : A probability μ on $\mathcal{P}$ is called <u>price-dispersed</u> iff μ is the marginal distribution of γ on $\mathcal{P}$, where $\gamma = \int_{\mathcal{P}} \int'_{\precsim} \gamma \circ a^{-1} (d \precsim)$ is such that

$$\int'_{\precsim} << X \text{ for } \gamma \circ a^{-1} \quad \text{a.e.}, \precsim \in \mathcal{P}$$

Together with other conceptual assumptions this dispersion yields a C^1 mean demand function in the framework of smooth preferences. Price-dispersion together with similarly defined wealth dispersion yields C^0 mean demand independent of the specification of the Polish preference space $\mathcal{P}$.

The Equivalence of Superadditivity and Balancedness in the Proportional Tax Game (joint work with Joseph Greenberg)

By Shlomo Weber
University of Haifa

Under the regulation that the single public good is to be financed through the tax of the single private good, it is proved that a weak notion of superadditivity is equivalent to the balancedness of the induced game.

More precisely : consider an economy (which was investigated by Guesnerie and Oddou (1979 Economics Letters and 1981, Journal of Economic Theory) with a set $N = \{1,2,...,n\}$ of individuals, a single private good α, and a single public good β. Individual $i \in N$ is endowed $w^i \in R_+$ units of the private good (and with no public good) and his preferences are represented by the utility function $u^i(\alpha,\beta) : R_+^2 \to R^+$, which is assumed to be continuous, quasi-concave and nondecreasing in β. The public good is produced according to the concave, continuous and nondecreasing production function $\beta = f(\alpha)$. We assume that if the group of individuals (a coalition) decides to produce the public good jointly, each member contributes for this end the same percentage of his initial endowment. We can therefore define the "indirect utility function" of individual i who belongs to a coalition S in which the tax rate is t, $0 \leq t \leq 1$, to be

$$\varphi^i(t,s) \equiv u^i\left((1-t)\,w_i,\ f(tw(s))\right), \text{ where } w(s) = \sum_{j \in S} w^j$$

Then we can induce the associated non-side-payment game V and our main theorem is:

V is balanced for any number of individuals provided it is superadditive in some very weak sense.

An Axiomatic Characterization of Welfare Measures

By Jakob Weinberg
Universität Bonn

Following ideas introduced by Pigou, the class of Economic Welfare Measures (EWM) is introduced. An EWM is defined by associating with every state s of an economy a price vector $p(s)$, and the corresponding wealth is given by evaluating the state with these prices, i.e. minimal social cost. This concept is made precise within an Arrow-Debreu economy. Examples are the coefficient of resource utilization, and the (modified) Equivalent Variation (EV) measure. The first theorem shows that this class contains all social Welfare Functions which are consistent with the Pareto ordering, a class widely applied in public finance. This implies that every such SWF has some underlying economic meaning, which, however, is not revealed. Analysing the properties of examples, a set of 3 axioms can be introduced. These axioms rule the reaction of an EWM to changes of the characteristics of the economy. Since an EWM generalizes in some sense the concept of potential compensations, underlying the Hicksian EV, a fourth axiom is introduced. This axiom requires the feasibility of these compensations. It is shown that this additional axiom characterizes Debreu's coefficient of resource utilization.

Price Discrimination Based on Imperfect Information

By Hans Wiesmeth
Universität Bonn

Suppose that on a monopolistic market the producer can not apriori separate various groups of consumers to discriminate against them. Suppose further that information gathering is not costless on the side of the consumers. The problem is then whether the monopolist can increase his profits via a self-selection-mechanism by simply setting different prices for his commodity in different shops. One can handle this question in the framework of a model originally set up by S. Salop (RES, 1977):

Consumers are characterized by unit search costs, c, and their money valuation , v, of the commodity, where $(c,v) \in [0,z] \times [0,\bar{v}]$ Consumers' characteristics are distributed in $[0,z] \times [0,\bar{v}]$ according to a density function g(c,v). Consumers know the distribution funtion F of the monopolist's prices without knowing a priori which store charges which price. They buy at most one unit of the commodity. Under the assumptions of the standard search model, consumers minimize expected total costs $\pi(c)$ given by the sum of expected purchase price p(c) and expected total search costs. π proves to be non-decreasing and concave.

Within this framework, one can show the following theorem (neglecting some technical assumptions):

<u>Theorem</u> : There exists a price dispersion yielding higher profits than the monopoly price p if and only if

$$\exists \delta \in [0,z] : \int_0^{\delta} [p^* \cdot g(c,p^*)(1-\frac{c}{\delta}) - \int_{p^*}^{\bar{v}} g(c,v)\,dv]\,dc > 0$$

The proof of the theorem furthermore shows that this condition is equivalent to the existence of a two-part-tariff, yielding higher profits than the monopoly price. This means that the monopolist can - if at all - increase his profits by experimenting with rather simple price dispersions, without knowing the optimal one. One can further investigate relations between statistical properties of the set of consumers' characteristics and the existence problem. Among others one obtains the following results:

(i) If search costs and money valuations are statistically independent, then the monopoly price is the optimal price dispersion.

(ii) A negative correlation between search costs and money valuations does not exclude the existence of a more profitable price dispersion (in comparison to the monopoly price).

This last result is interesting, because one intuitively relates the existence problem to a positve correlation between search costs and money valuations.

Specific Training, Collective Bargaining, Organization of Production and Involuntary Unemployment

By Yves Younès
CEPREMAP, CNRS, Paris

The starting point is the remark that in many firms, there is knowledge specific to the firm which cannot be codified totally by engineers which is easily transmissible if there are relatively few new entrants in the firm but transmissible with difficulty if there are massive quits and thus many new workers. Thus given the number of workers productivity is an increasing and strictly concave function A of the proportion of experienced workers given the fact that a worker becomes experienced after working one period in a given firm. If workers bargain collectively in each firm, there are two situation:

-- if there is disagreement, all workers are outside and the expected value of their income depends on the rate of unemployment and the distribution of wage rates in other firms and the boss hires new workers and after one period pays them according to some wage profile.

-- if there is agreement the problem arises of deviding the extra bonus between the capitalist and the workers.

I. Justification of collective action

Inside each firm, there is thus a game situation with linearly transferable utilities. To each partition B of players inside each firm it is possible to calculate according to the Nucleolus what people get (in each group): $\tilde{x}_B(i)$. In order to find the equilibrium partition $(B^*, \tilde{x}^* = \tilde{x}_{B^*}(i))$, we define a new game. We define $\underline{x}_i(S) = \min_{s \in B} \tilde{x}_B(i)$ as the level of utility that i can guarantee to himself if he joins in group S. Define $e(S, x_B) = \min_{i \in S} [\underline{x}_i(S) - \bar{x}_B(i)]$.

We apply the nucleolus to this excess function to get an equilibrium. In our case, all workers will act collectively if A is strictly concave.

II. There will be involuntary unemployment in this situation. If the wage profile is a non-increasing one in each firm, the gap between equilibrium wage rate in firm j and what a worker expects to have if he leaves the firm can be created only by unemployment. The gap arises because we apply the nucleolus to a situation in which the boss looses something if all experiencdd workers leave.

III. Admissible wage profiles

If function A is "enough concave", only non-increasing wage profiles are non manipulable by the manager who has the right to fire workers. If A is linear, there are non-manipulable increasing wage profiles and equilibrium does no more require unemployment.

The Notions of TYPE and CONSISTENCY in Games with Incomplete Information (joint work with J.F. Mertens)

By Shmuel Zamir
C.O.R.E., Louvain-la-Neuve

The situations modeled by games with incomplete information are conflicts in which the involved players do not have a full knowledge about the data (i.e. the action spaces and the outcome and utility functions). We assume that each of the players has some subjective beliefs about anything which is relevant to the conflict and about which he is uncertain.

The first natural attempt to model such a situation as a game leads to what is known as "Infinite Hierarchy of Beliefs": the beliefs of a certain player on the parameters of the game, his beliefs on the beliefs of the other players on the parameters, his beliefs on the beliefs of the other players on his beliefs on the parameters etc.

To overcome this difficulty, Harsanyi (1967-1968) introduced the notion of <u>TYPE.</u> This is supposed to be an entity which pools <u>all</u> the characteristics, in particular it should include his beliefs of all levels.

Can the set of "types" be well defined mathematically? The first result of this paper is to answer this question affirmatively:

<u>Theorem</u> : Given a compact set S (possible parameter values) and a positive integer n (number of players) there is a compact space $\mathcal{Y}$ and a compact space T s.t.

(1) $\mathcal{Y} = S \times \bigtimes_{i=1}^{n} T^i$

(2) T^i = Space of probability measures on $[S \times \bigtimes_{j \neq i} T^j] \ \forall i$,

where $\forall$ i T^i is a copy of T. The equalities are up to homeomorphisms.

$\mathcal{Y}$ and T are respectively the Universal Beliefs Space and the Universal Type Space, generated by S.

We study the structure of the Universal Beliefs Space and introduce the notion of Beliefs Subspace of $\mathcal{Y}$. We prove that any incomplete information situation based on S can be identified with a Beliefs Subspace of $\mathcal{Y}$.

Next we prove that any Beliefs Subspace of $\mathcal{Y}$ (in particular $\mathcal{Y}$ itself) can be approximated by a finite Beliefs-Subspace.

Finally we study the question of consistency, namely: In which points $y \in \mathcal{Y}$ the subjective beliefs of the players may be derived as posterior probability distributions (given each player's type) from some <u>prior</u> probability distribution and what is this prior distribution in this case.

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OPEN PROBLEMS

Robert Aumann

Hebrew University of Jerusalem, Israel

1.) Find a bargaining procedure for the Maschler-Perles superadditive bargaining solution, in the spirit of the procedures presented here by H. Moulin for the Nash and Kalai-Smorodinski solutions.

2.) Do all infinitely repeated games of incomplete information with complete information on one side possess an equilibrium point?

David Gale

University of Berkeley

Each of three players has an atomless measure on the unit circle, μ_1, μ_2, μ_3.

Is it always possible to divide the circle into three areas $\alpha_1, \alpha_2, \alpha_3$ such that the division is

(a) Pareto optimal , meaning there is no other division $\beta_1, \beta_2, \beta_3$, such that $\mu_i(\beta_i) > \mu_i(\alpha_i)$ $i=1,2,3$, and

(b) envy free meaning $\mu_i(\alpha_i) \geq \mu_i(\alpha_j)$ $i \neq j$ for all i,j.

Does then exist a price equilibrium meaning a measure π on the circle and division $\alpha_1, \alpha_2, \alpha_3$ such that

$\pi(\alpha_i) = \frac{1}{3}$ for all i and

$\mu_i(\alpha_i) \geqq \mu_i(\beta)$ for any β such that $\pi(\beta) \leqq \frac{1}{3}$.

Jean Gabszewicz
CORE, Louvain-la-Neuve

In the "marriage market", there are n women and n men who must be coupled so as to get a "stable configuration". Gale and Shapley have shown that there exists such a stable configuration, where one cannot find two couples who would improve upon by exchanging partners.

In the "family market", there are n women, n men , and n children; each person has an ordering on pairs of partners, differing from their own type (for instance, each man has an ordering on pairs woman-child). The question is : is it possible to find a stable configuration of families for this extended case?

Yakar Kannai
Weizman Institute of Science, Israel

1.) The strong axiom of revealed preferences, when applied to revealed demand, is sufficient for demonstrating that the choices are derived from a convex preference ordering (i.e. an ordering representable by a quasi - concave utility function). Problem: Find a stronger axiom of revealed preferences, which would imply that the underlying preference ordering is concavifiable (i.e., representable by a concave utility function).

2.) The 0-th, 1-st, and 2-nd derivatives have obvious geometric meanings; everybody knows what it means for a function to be positive, monotone increasing, or convex. Problem: Is there a simple interpretation of the n-th derivative for $n \geq 3$, or is there a qualitative difference between $n \leq 2$ and $n \geq 3$?

A clue for the second direction: T. Popoviciu calls a function f defined on a subset E of the real line n-convex if for all $x_0, x_1, \ldots, x_n$ with $x_i \in E$, $0 \leq i \leq n$, it is true that $\sum_{i=0}^{n} L'(x_i) f(x_i) \geq 0$, where $L(x) = \prod_{i=0}^{n} (x-x_i)$. Popoviciu remarks that while it is true that a function f, n-convex in E, can be extended so as to be n-convex in the convex hull of E, if $n \leq 2$, this is no longer the case if $n \geq 3$ (even if E is a finite set).

Michael Maschler
The Hebrew University of Jerusalem, Israel

1.) Generalize the definition of the nucleolus to games without side payments.

2.) Given a multi-valued or a single valued dynamic system, various ways are known to determine if a certain point (set) is stable under this process. Suppose we found many stable points (sets) for a given system, how do we recognize if we have found all of them?

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3.) Many procedures are open for a government man to perform a mixed strategy. For example, he can appoint a committee to recommend a decision. This and other methods are mixed strategies based on subjective probabilities, namely, the beliefs of this man. Inform me of methods to perform mixed strategies based on objective events which would be acceptable to a government man.

David Schmeidler
Tel-Aviv University, Israel

Let f and g be two continuous functions from strictly positive normalized price vectors to strictly positive consumption vectors. Suppose that f and g also satisfy

(i) $p \cdot f(p) = p \cdot g(p)$ for every price vector

(ii) g is bounded

(iii) the Euclidean norm of $f(p_n)$ diverges to $+\infty$ if p_n converges to a price vector which is not positive for all commodities.

Does there exist an economy with agents characterized by initial endowments w_t and preferences $\succsim$ such that the derived individual excess demand functions e_t satisfy for all price vectors p:

$f(p) = \sum_t e_t(p)^+$ and $g(p) = -\sum_t e_t(p)^-$? $((2,3,-4)^+=(2,3,0))$

More generally, what additional condition on f and g are necessary (or sufficient) for such a decomposition?

Remark: This decomposition does not trivially follow from Sonnenschein-Mantel-Debreu Theorem.

Martin Shubik
Yale University

1.) Sidepaymentness

A sidepayment game is one where all Pareto optimal surfaces and subsurfaces are flat. Is there any useful characterization of "sidepaymentness"? For example for any n we can normalize the largest payoff to i as (0,..0,1,0..)

For n=2

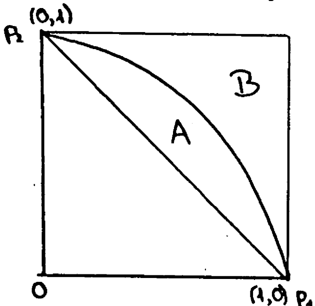


Figure 1

one index given as an example is:

$$S = 1 - \frac{A}{A+B}$$

where S=1 is sidepayments

For n=3 this index will not suffice in the sense that even though S=1 implies $v(\overline{123})$ is flat, the smaller coalitions may not be, as is shown in Figure 2

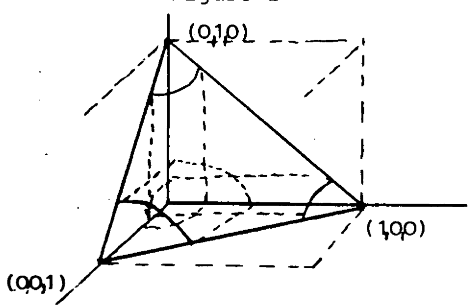


Figure 2

How sidepayment like is this game? One might argue that from the point of view of welfare theory this is a sidepayment game as for joint welfare, the distribution and production problems are separated.

For market games a flat Pareto optimal surface does not imply a unique C.E. even though there is an overall constant welfare

measure. Thus n-person "sidepaymentness" does not imply uniqueness of the C.E. for a market game but a full sidepayment characteristic function does. A sidepayment equivalent nonsidepayment game is one for which n independent order preserving transformations will flatten the 2^n-n-1 Pareto subsurfaces - This is clearly a considerably strong condition.

The reason why I suspect that this question might be of some economic interest is in the construction of welfare measures concerned with redistribution and lump sum payments.

For market games where price systems exist one can further ask: do the Lagrangian multipliers associated with each C.E. give hyperplanes to provide the best (in the sense of least distorted).

2.) Splitting of the core

For simplicity consider the Edgeworth box with 3 C.E.. Under replication there will be some k at which the core which is originally connected splits into at least two parts. Do the splitting points have any nice economic interpretation? Clearly a slight charge in initial endowments from say τ_1 to τ_2 may remove several C.E. yet enlarge the core hence if there is any economic meaning it depends heavily on the initial distribution.

In some sense the observation that a perturbation of the distribution of assets can both enlarge the core and cut down the number of C.E. stresses the conceptual difference between the core and the C.E hence the question might be a bad question - but I feel that for a specific exchange economy the splitting may in some sense indicate a global region of association around each equilibrium.

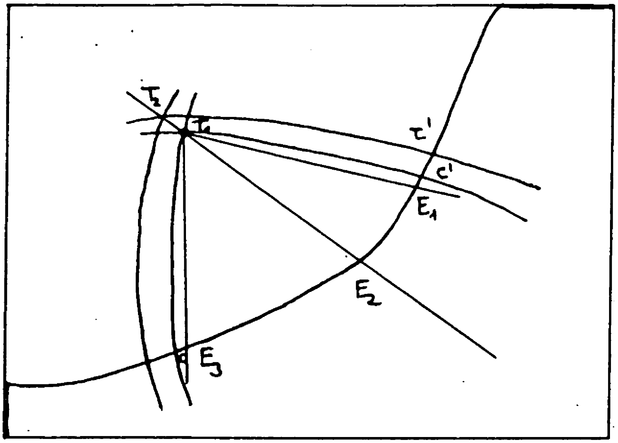
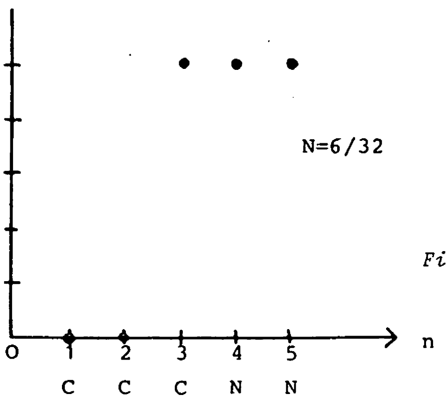


Figure 1

3.) <u>On the Corelessness of a game</u>

The major application of game theory to economics has involved the study of market games which are totally balanced n-person games or games for which all of the 2^n subgames have a core.

Restricting ourselves to sidepayment games with superadditive characteristic functions it appears that in some sense with respect to the core the polar opposite to the market game is a voting game such as the simple majority voting game. Limiting ourselves to the symmetric game we illustrate for n=5



$f(s)=0$ for $s \leq 2$

$f(s)=1$ for $5 \geq s \geq 3$

(this implies that there is a quorum at s=3 and for s=3 a vote to pass would require unanimity)

Figure 1

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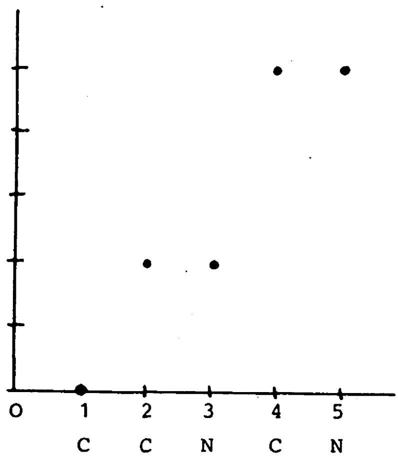
In Figure 1 the number under the coalition size indicates if the game of that size has a core. N signifies no, C signifies yes.

For the symmetric game we can distinguish two types of questions based on the statistics we use. (1) Counting only the different size of coalitions minimize the number with cores. (2) Weighting all combinations as equiprobable thus s can be formed in

$\frac{n!}{s!(n-s)!}$ ways minimize the number of cores. Figure 2 shows a game with n=5.



N=11/32

f(1)=0

f(s)=2 s=2,3

f(s)=5 s=4,5

Figure 2

We may further wish to distinguish the maximum number of coreless subcoalitions attainable working down from n. Figure 1 and Figure 2 show n=5, but only Figure 1 satisfies this condition. An example for n=13 is given in Figure 3

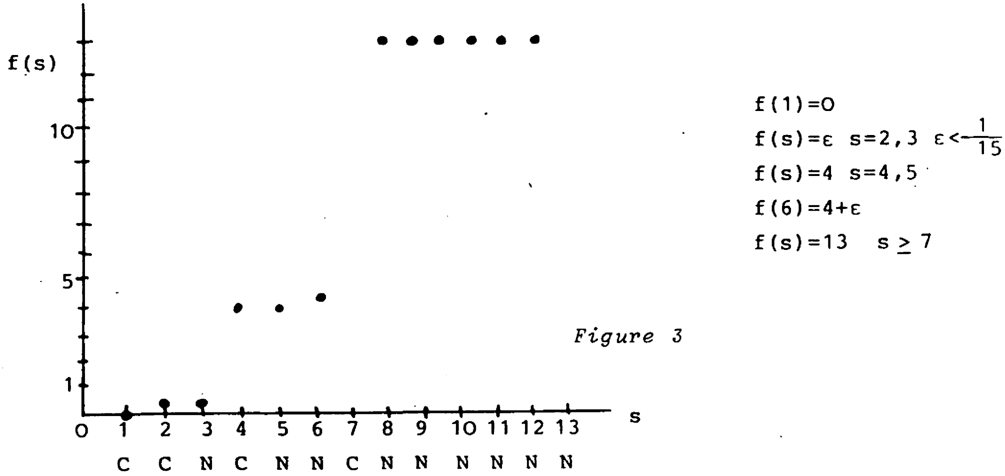


Figure 3

It is of interest to note that the largest sequence of coreless subgames is given by the simple majority voting game. For example for $f(S) = \begin{cases} 13 & 13 \geq s \geq 7 \\ 0 & 6 \geq s > 0 \end{cases}$ then all games for $s \leq 6$ have cores, but we can modify away some of these cores as shown in the example in Figure 3.

<u>Question 1</u> (a) For a given n what is the minimum number of coalition sizes with cores? (b) What is the minimum number of a 2^n-1 coalition with cores?

<u>Question 2</u> How do the bounds for the nonsymmetric game differ from the symmetric game?

The motivation of these questions is to gain a better understanding of "corelessness" and its relationship to voting games.

4.) <u>A Metatheorem on Solutions for Market Games with Money Trades</u>

One class of solutions to an n-person game with sidepayments in coalitional form yields an imputation $\alpha = (\alpha_1,\ldots,\alpha_n)$, where $\sum_{i=1}^{n} \alpha_i = v(N)$, which satisfies certain conditions improved upon the 2^n numbers of the characteristic function - the core for example requires that α satisfies a set of inequalities given by

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the characteristic function. The value requires an averaging over the characteristic function. The nucleolus and a point in the kernel or bargaining set can also be considered in terms of inequalities derived by relatively simple operations on the characteristic function.

Replication enlarges the characteristic function in a manner that is homogeneous of order 1 - this combined with superadditivity appears to "flatten out" the contribution of individuals to all large coalitions so that they in essence add the same to large coalitions.

There are two questions.

Question 1 Given C_2 on the utility functions of individuals is there a way of characterizing a set of operations that cover the core, value, nucleolus, kernel and bargaining set to show that (a) for replication (b) for a continuum of traders the approach to the same limit or the equivalence to the price system can be proved in one package.

Question 2 (very vague) Do we think that we have exhausted good one point (in the sense that an imputation in a set can be regarded as a solution) solutions or could we construct some form of search procedure on reasonable operators on v(S) to create other solution concepts? (My guess is that there will be no more solutions on v(S) of the importance of the core or value - I hope I am wrong).

5.) Cores and Values of Games of Status

Consider a game as follows: v(S) is a given characteristic function and the players must divide the points among themselves but the only thing that matters is "status" - who gets more.

Each player has a cardinal utility over being first, second, third, etc.

If two coalitions S and $\bar{S}$ face each other they each pick a strategy

α_i, $i \in S$ of dimension $|S|$

β_j, $j \in \bar{S}$ of dimension $|\bar{S}|$

or a mixed strategy where $\sum_{i \in S} \alpha_i = v(S)$ $\sum_{j \in \bar{S}} \beta_j = v(\bar{S})$

and hence the ordering of the n players can be calculated. This is a game of status.

Problem 1 characterize the core

Problem 2 characterize the value

Motivation: Especially in sociological and bureaucratic situations status is more desired than assets.

Problem 3 (open and somewhat vague) to characterize a satisfactory game of wealth and status where both count (first possibility-a two dimensional payoff where the final score is a function $\varphi_i(p_i(\alpha_1,\dots,\alpha_n),\alpha_i)$ the first component is the individual's rank and the second his wealth.

6.) Agenda Games

Consider a committee with n+1 individuals the (n+1) st being chairman who has a tie-breaking vote and may select the order in which m items of business will be voted on. Let individual i have utility a_{ij} for item j. Let item j have a cost to the committee of c_j. The committee has a total budget of B. The utility of o items is o. The final score for an individual is $\sum_j a_{ij}$ where j is in the accepted set of items.

$$\sum_{j=1}^{m} c_j > B$$

Once the committee has voted in favour of items whose cost is such that any further item will have total expense greater than

B they must stop.

The chairman's strategy is to select the order of the vote and his own vote (or the breaking vote). An individual's strategy is his voting policy on each item. Items are voted upon one at a time and accepted or rejected. After all m are voted on the game stops. If nothing passes, the payoffs are 0.

(Variation if the budget is underspent - after m votes the rejected items may be reconsidered).

Note The c_j are paid by the committee, hence do not enter individual utilities.

Example : Condorcet Paradox

		1	2	3
Voter	A	30	20	10
	B	10	30	20
Chairman	C*	20	10	30

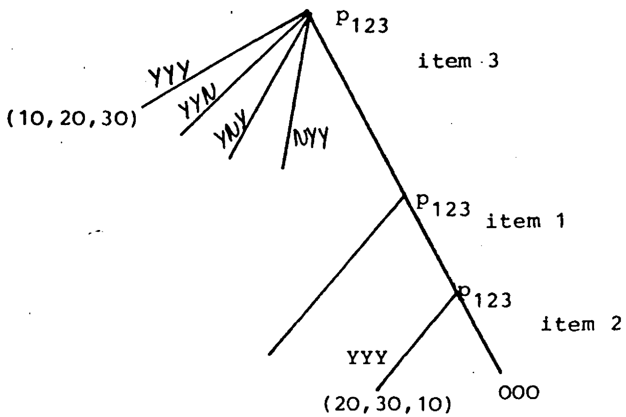
3 items

$c_1=c_2=c_3=15$

B = 20

Chairman selects order of vote 3,1,2

Solution concept Perfect equilibrium



By backward induction 2 will be chosen at the end over 0. At stage 2 the choice is between 1 or 2 hence 1 would be chosen. At stage 1 the choice is between 3 or 1 hence B votes for, A against and the chairman C votes for the item with payoff (10,20,30) - Note that without the agenda solution the players are symmetric.

Motivation : To capture the concept that the chairman may have some power in his ability to select agenda.

Comment : For any size this may become overly complex. In practice $n \leq 20$ and usually $n \leq 10$. However for small committees not only is knowledge of each other's preferences reasonable - but possibly a cooperative solution should be considered.

7.) "Sidepaymentness" and "Total Sidepaymentness"

A game Γ (v,N) is a sidepayment game if v(s) is a superadditive set function. But it is possible that the sub Pareto optimal surface is flat for some coalitions and not for others.

The following definition suggests itself. We could call a game Γ a sidepayment game if v(N) is a hyperplane and a totally sidepayment game if all v(S) are hyperplanes.

Question 1 Consider a game for which all v(S) for $1 \leq s \leq n-1$ are hyperplanes (where $|N|=m$) but v(N) is not. Suppose that this game is totally balanced. Can such a game always be represented by a market game?

Observation It is straightforward to show that there will be a class of games that are market games. Let there be n players- there are n goods. Player i has an endowment of 1 unit of good i (0,0,...,1,...,0)

Each player i has utility function of the form

$$\varphi_i(x_1^i,\ldots,x_n^i) = f_i(x_1^i,\ldots,x_n^i) + \sum_{j=1}^{n} \alpha_{ij} x_j^i + \beta_i$$

where $f_i(x_1^i,\ldots,x_n^i) = 0$ if any $x_j^i = 0$ and f_i is concave.

Question 2 If it can always be represented as a market game can an associated exchange economy have multiple equilibria or are the conditions strong enough for uniqueness.

Conjecture Yes (near example Shapley-Shubik JPE example with 3 equilibrium points).

Motivation for question There is an open problem concerning the representation of NSP totally balanced games as markets. This is a suggested simple class..

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