MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 6/1982

Mehrdimensionale konstruktive Funktionentheorie

7. bis 13.2.1982

Die Tagung fand unter der Leitung von Herrn Walter Schempp (Siegen) und Herrn Karl Zeller (Tübingen) statt. Im Mittelpunkt des Interesses standen Fragen der Darstellung, Approximation und Behandlung reeller Funktionen mehrerer Variablen. Da Problemstellungen dieser Art in den letzten Jahren zunehmend an theoretischer und praktischer Bedeutung gewonnen haben, wurde bei der Planung der Tagung ein ausgewogenes Verhältnis zwischen Themen aus der Theorie der multivariaten Funktionen und ihren numerischen und praktischen Anwendungen angestrebt. Dieses Ziel ist erreicht worden.

Im Bereich der multivariaten Approximationstheorie wurden u.a. H-Mengen, die Polynomapproximation in Sobolev-Räumen, Orthogonalsysteme in mehreren Variablen, mehrdimensionale Spline-Funktionen, gruppentheoretische Methoden, rationale Approximation in mehreren Variablen, positive lineare Operatoren und funktionalanalytische Aspekte der Radon-Transformation diskutiert. Schwerpunkte der numerischen Anwendungen waren vor allem neue Fehlerabschätzungen für Approximation und Kubatur, Boolesche Methoden in der mehrdimensionalen Interpolation und der Einsatz von Summationsverfahren. Die praktischen Anwendungen waren außerordentlich breit gestreut. Zu nennen sind hier vor allem Anwendungen in der Geodäsie, Geologie, Limnologie, Meteorologie, Radarortung und medizinischen Tomographie.

Die erzielten Ergebnisse werden in einem von den Tagungsleitern herausgegebenen und vom Birkhäuser Verlag, Basel-Boston-Stuttgart, in der ISNM-Reihe zu veröffentlichenden Tagungsband

Multivariate Approximation Theory, Vol. 2,

einer breiteren wissenschaftlichen Öffentlichkeit zugänglich gemacht. Mit diesem neuen Band wird die Reihe der zu den Oberwolfach-Tagungen gleichen Themas der Jahre 1976 und 1979 erschienenen Tagungsbände fortgesetzt:

Constructive Theory of Functions of Several Variables.
Lecture Notes in Mathematics 571 (Springer 1977)

Multivariate Approximation Theory.
ISNM 51 (Birkhäuser 1979)

Mit 48 Teilnehmern aus Belgien, England, Frankreich, Japan, Kanada, den Niederlanden, Schottland, Schweden, Spanien, Ungarn, den Vereinigten Staaten von Amerika, der Volksrepublik China und der Bundesrepublik war die Kapazität des Instituts so ausgeschöpft, daß eine Reihe weiterer Interessenten leider nicht mehr berücksichtigt werden konnte.

Dem Direktor des Mathematischen Forschungsinstituts, Herrn Prof. Dr. M. Barner, und seinen Mitarbeitern sei für die freundliche Aufnahme und zuvorkommende Hilfe sehr herzlich gedankt.

Vortragsauszüge

M.F. BARNSLEY:

Orthogonal Polynomials on Julia sets

Let $T: \mathbb{C} \to \mathbb{C}$ be a polynomial of degree N, let $T^o z=z$, and $T^n z=T(T^{n-1}z)$ for $n \in \{1,2,3,...\}$. The Julia set J of T is the set of points $z \in \mathbb{C}$ such that $\{T^n z\}$ is not a normal family. There exists a unique balanced T-invariant measure on J, denoted by μ. The orthogonal monic polynomials $\{P_l\}$ are introduced, where P_l is of degree l, and where $\int_J \overline{P_l(z)} P_m(z) d\mu = 0$ for $l \neq m$. Theorems concerning these polynomials are given. For the family of mappings $Tz=(z-\lambda)^2$ these polynomials are studied in detail. When $\lambda=2$ they are related to the Chebychev polynomials $\{T_n\}$ by $2T_n(x)=P_n(2x+2)$. When $\lambda > 2$ they generalize the Chebychev polynomials in a remarkable manner. Their support becomes a Cantor set of Lebesgue measure zero. The subsequence $\{P_{2^n}\}$ display the equal oscillation property on J. Applications to such problems as approximation theory on a snowflake are considered.

G. BASZENSKI:

Bemerkungen zur reduzierten Hermite-Interpolation

Es werden einige neue Elemente der reduzierten Hermite-Interpolation für Funktionen in zwei Variablen vorgestellt. Für diese Elemente werden mit Booleschen Methoden explizite Kardinaldarstellungsformeln hergeleitet. Ebenso erfolgt eine Herleitung von Fehlerformeln.

J. BOMAN:

On the range of the Radon transform and the closure of spaces of sums of plane waves

For $a \in R^n \setminus \{0\}$, $1 \leq p \leq \infty$, and Ω an open subset of R^n let $L^p(\Omega,a)$ be the set of functions in $L^p(\Omega)$ that are constant on all lines parallel to a. In the mathematical theory of image reconstruction from projections, usually called computerized tomography, the following question arizes. For given $a^k \in R^n \setminus \{0\}$, $k = 1,\dots,m$, decide whether the vector space

$$(*) \qquad L^p(\Omega,a^1) + \dots + L^p(\Omega,a^m)$$

is a closed subspace of $L^p(\Omega)$. If n = 2 the answer is yes for all Ω with Lipschitz continuous boundary. This statement is a special case of a theorem about so-called very strongly elliptic differential operators. If $n \geqq 3$, (*) need not be closed even if Ω is strictly convex and has C^∞ boundary.

H. BRASS:

Ein Beispiel zur Theorie der besten Approximation

Sei $Q = \{(x,y) \mid -1 \leqq y \leqq x \leqq 1\}$ und P_s die Menge der Polynome in zwei Variablen vom Grad s. Das Beispiel des Titels ist die beste Approximation im Tschebyscheffschen Sinne von

$$f(x,y) = x^p y^q$$

auf Q aus P_{p+q-1}. Es lassen sich einfache geschlossene Ausdrücke für Proxima angeben.

Bemerkenswert ist, daß die zugehörige Extremalsignatur besonders wenig Punkte hat. Damit hängt zusammen, daß die Menge der Proxima des Problems die Dimension $\frac{1}{2}(p+q)(p+q-1)$ hat, dies ist die größtmögliche Dimension bei Approximation aus P_{p+q-1}.

Verallgemeinerung auf mehr Variable bereitet keine Schwierigkeit.

W. DAHMEN:

On limits of multivariate B-splines and entire functions of affine lineage. (with C.A. Micchelli)

A function $\Lambda(x) \in L_1(\mathbb{R}^s)$ is shown to be the limit of multivariate B-splines (as the number of knots tends to infinity) iff $\psi(z) = \left(\int_{\mathbb{R}^s} e^{-z\cdot x}\Lambda(x)dx\right)^{-1}$ belongs to the s-variate Polya-Laguerre class

$$E_s = \{\psi : \psi(z) = e^{-(Az)\cdot z + b^o\cdot z}\prod_{j=1}^{\infty}(1+z\cdot b^j)e^{-z\cdot b^j}, b^j \in \mathbb{R}^s, \text{ A a real positive semi-definite } s\times s \text{ matrix } \sum_{j=1}^{\infty}\|b^j\|_2^2 < \infty\}.$$

Moreover, we show that $\psi \in E_s$ iff $\tilde{\psi}(t) := \psi(x+ty)$ belongs for any $x,y \in \mathbb{R}^s$ to the univariate class E_1. This is essentially a consequence of the more general result that any entire function $f(z)$ on $\mathbb{C}^s$ has real affine lineage (i.e. its zero set is a union of real hyperplanes) iff $\tilde{f}(t) := f(x+ty)$ has for any $x,y \in \mathbb{R}^s$ only real zeros. So, in particular, any polynomial p on $\mathbb{R}^s$ is a product of real linear factors, $p(x) = \prod_{j=1}^{m}(\xi^j\cdot x - t_j)$, $\xi^j \in \mathbb{R}^s$, $t_j \in \mathbb{R}$, iff its restriction to any line has only real zeros.

F.J. DELVOS:

Remainders for trivariate Boolean interpolation

In this lecture we will discuss some trivariate interpolation schemes which are related to the trivariate blending function interpolation of Gordon. In particular we will derive explicit remainders for these trivariate interpolation methods which may be considered as (extended) discrete trivariate blending function interpolation.

P. DEFERT:

Approximation by first degree multivariate polynomials

In 1972, G.D. Taylor gave geometrical properties of the minimal H-sets relative to the space P_n of first degree polynomials in n variables; he proved that the number h(n) of different minimal H-sets of P_n is quadratic in [n/2]. More recently, Carasso and Laurent introduced the new concept of chain of supports to overcome the failure of the classical exchange algorithm when Haar's condition is not satisfied.

We give geometrical properties of the chain of supports of the space P_n, as well as rules to build them recursively. Moreover, we prove that the number c(n) of different chains of minimal supports of P_n is an exponential function of n.

F. DEUTSCH:

"Which closed convex subsets of an incomplete inner product space are Chebyshev?"

The question in the title is answered for a class of convex cones which include the subspaces of finite codimension.

W. FREEDEN:

Integral formulas of the (unit) sphere and their applications

The purpose of the lecture is the study of integral formulas of the (unit) sphere and their application to problems in approximation theory and numerical analysis. The integral formulas and the approximating techniques essentially use the theory of Green's functions on the sphere with respect to the (Laplace-) Beltrami operator. The theory of spherical harmonics is the main tool.

First the spherical harmonics are introduced as the bounded eigenfunctions of the Beltrami operator Δ^* of the unit sphere Ω. Based on the concept of Green's functions, a class of integral formulas for the unit sphere is deduced from Green's surface identities.

A method is described to approximate integrals over the unit sphere using uniformly distributed sequences on Ω in the sense of Weyl (cf. E. Hlawka: Gleichverteilung auf Produkten von Sphären).

Finally spherical spline functions are defined by observing the specific properties of Green's functions. Spherical spline functions are used to interpolate and smooth data discretely given on the sphere. Extensions of Peano's theorem and Sard's theory of best approximation to the spherical case are mentioned.

The reason for developing spline techniques for the (unit) sphere is that in many geophysical problems we are confronted with the question of approximating experimental data or measured values.

M. GASCA (in collaboration with A. Lopez-Carmona and V. Ramirez):
<u>A generalized Sylvester's identity on determinants and its applications to interpolation problems</u>
Recently G. Mühlbach has extended to Chebyshev systems the Neville-Aitken algorithm and the Newton formula for polynomial interpolation. C. Brezinski has used an identity on determinants due to Sylvester to obtain in a simpler manner the same results for complete Chebyshev systems (and similar situation in the general interpolation problem).

In this paper a substantial generalization of that identity is obtained and applied to derive some recurrence interpolation formulae for the solution of the general interpolation problem. These formulae are analogous to, and often coincident with, those obtained by M. Gasca and A. Lopez-Carmona.

M. v. GOLITSCHEK:

Approximation of bivariate functions by functions of one variable

In my talk I want to survey recent results of a cooperation with E.W. Cheney studying the following problem.

Suppose that I and J are compact real subsets and $D \subseteq I \times J$ is a compact subset of R^2. Let f be a continuous function on D and $\{\varphi_i\}_{i=1}^{n}$, $\{\psi_j\}_{j=1}^{m}$ be Haar subspaces of continuous functions on J and I, respectively. Then find continuous functions g_i^* on I and h_j^* on J such that the infimum

$$\inf_{g_i, h_j} \| f(x,y) - \sum_{i=1}^{n} \varphi_i(y)\, g_i(x) - \sum_{j=1}^{m} \psi_j(x) h_j(y) \|$$

is attained where $\|\cdot\|$ denotes the supremum norm on D.

M.S. HENRY:

Multivariate approximation theory: Theoretical error estimates and calculation

Let $D = I \times J = [-1,1] \times [-1,1]$. Suppose $\{\varphi_i\}_{i=0}^{n} \subseteq C(I)$ and $\{\psi_j\}_{j=0}^{m} \subseteq C(J)$ are sets of basis functions with linear spans Φ_n and Ψ_m, respectively. Linear product approximation methods in two variables are described as follows. Let $L: C(I) \to \Phi_n$ and $M: C(J) \to \Psi_m$ be two linear operators. For $f \in C(D)$ and $y \in J$ let $(Lf_y)(x) = \sum_{i=0}^{n} b_i(y)\varphi_i(x)$, where $f_y(x) = f(x,y)$. If $b_i \in C(J)$, $i=0,\ldots,n$, let $(Mb_i)(y) = \sum_{j=0}^{m} b_{ij}\, \psi_j(y)$. Then $L: C(D) \to \operatorname{span} \{\varphi_i\, \psi_j\}_{i=0,j=0}^{n \quad m}$ is defined by $(Lf)(x,y) = [M \circ L](f)(x,y) = \sum_{i=0}^{n} \sum_{j=0}^{m} b_{ij}\, \psi_j(y)\, \varphi_i(x)$.

Uniform product approximation is a mildly non-linear counterpart to the linear product approximation concept. In this latter case L and M are replaced by best uniform approximation operators. Denote the best uniform product approximation by $(Pf)(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{m} a_{ij}\psi_j(y)\varphi_i(x)$. Now let $(\hat{P}f)(x,y) = \sum_{i=0}^{n}\sum_{j=0}^{m}\hat{a}_{ij}\psi_j(y)\varphi_i(x)$ be that such $\| f - \hat{P} \|_D = \inf_{c_{ij}} \| f - \sum_{i=0}^{n}\sum_{j=0}^{m} c_{ij}\psi_j\varphi_i \| = E_{n,m}(f)$.

For a proper choice of L above it is well known that $\| f - Lf \|_D = O[\log(n+1)\log(m+1)]E_{n,m}(f)$. For the same choice of basis functions it can be shown that $\| f - Pf \|_D = O[\log(n+1)]E_{n,m}(f$ Implications and extensions of the above results are the focus of this presentation.

J.C. MASON:

Minimal projections and near-best approximations by multivariate polynomial expansion and interpolation

Multivariate polynomial approximations are defined for spaces of real functions on hypercubes and spaces of complex (analytic) functions on various polydomains, based on classical expansion and interpolation projections. A variety of relevant results are presented which follow from generalisations of known univariate results. Specifically multivariate Fourier, Taylor, and Laurent projections are shown to be minimal projections in L_∞ on hypercubes, polydiscs, and polyrings, respectively, and all yield near-best approximations in both L_∞ and L_1 within relative distances of the order of $\prod \log n_k$ (where n_k is the degree of the kth variable). Related interpolation projections are shown to give near-best L_∞ approximations within relative distances of the same order. Furthermore, on polyellipses, expansions in Chebyshev polynomials of the first kind (in L_∞) and of the second kind (in L_1) yield similar results. Convergence in norm is established in all cases subject to appropriate conditions on the functions. Finally, two interesting bivariate approximation problems are discussed, which involve one complex variable and one spatial parameter.

A. LE MÊHAUTÊ:

<u>Construction of a surface of class $\mathcal{C}^k$ on a domain $\Omega \subset \mathbb{R}^2$, after triangulation</u>

Given a domain $\Omega \subset \mathbb{R}^2$, where we have randomly selected points t_i, i= 1,...,n, we want to construct a surface that interpolates a given function f and its derivatives at the t_i. After an automatic triangulation $\mathcal{T}$ of Ω , based on the t_i, we construct a surface S which is of class $\mathcal{C}^k$ on Ω , and is polynomial on each triangle T of $\mathcal{T}$. To do this, we have to construct, on each triangle, an approximation of f using what we call an Hermite finite element of class $\mathcal{C}^k$. In this way we obtain generalizations of some classical elements, such as Zienkiewicz, Argyris and Bell's elements. We present some numerical examples of such surfaces.

J. MEINGUET:

Sharp "a priori" error bounds for polynomial approximation in Sobolev spaces

In matter of quantitative error estimation for pointwise and mean-square polynomial approximation problems in $H^m(\Omega)$, we are naturally interested in key estimates of the general form

$$|f-Pf|_I \leq C_o |f|_{II},$$

where P is a linear projector onto polynomials, $|.|_I$ and $|.|_{II}$ denote seminorms involving appropriately related subsets of generalized partial derivatives of f, and C_o (the so-called optimal error coefficient) is a numerical constant (depending on Ω) to be estimated quantitatively.

Unlike the characterization of C_o, which actually requires to solve unduly complicated eigenvalue or boundary value problems, the practical determination of reasonably sharp bounds for C_o is quite feasible (this is even trivial as regards lower bounds). It turns out that realistic upper bounds can be obtained at a reasonable cost by manipulating (very carefully!) the remainder term in an averaged Taylor series (serving as standard representation formula in $H^m(\Omega)$, at least whenever $\Omega \subseteq \mathbb{R}^n$ is bounded and convex).

H.M. MÖLLER:

Eine einfache Methode zur Konstruktion numerischer Differentiations- und Integrationsformeln

Mein Vortrag auf der letzten Tagung in Oberwolfach über mehrdimensionale Approximationstheorie handelte von der Konstruktion von Kubaturformeln mit Hilfe der gemeinsamen Nullstellen von Polynomen, die entweder orthogonal oder Repräsentanten von Punktfunktionalen in einem geeigneten unitären Raum sind. Diese Methode wird übertragen auf die Diskretisierung von partiellen Differentialgleichungen, und es wird ein Programm vorgestellt, das für beliebige Integrale Kubaturformeln mit geringer Knotenzahl bei niedrigem Genauigkeitsgrad rasch berechnet.

F. MÓRICZ:

On the strong approximation by multiple orthogonal series

Let $\{\varphi_{ik}(x): i,k=1,2,\dots\}$ be an orthonormal system over a measure space $(X,\mathcal{F},\mu)$ and $\{a_{ik}\}$ a coefficient system for which $\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} a_{ik}^2 < \infty$. Then there exists an $f \in L^2(X)$ such that $f(x) \sim \sum_{i=1}^{\infty}\sum_{k=1}^{\infty} a_{ik}\varphi_{ik}(x)$. Set $s_{mn}(x) = \sum_{i=1}^{m}\sum_{k=1}^{n} a_{ik}\varphi_{ik}(x)$ and $\sigma_{mn}(x) = \frac{1}{mn}\sum_{i=1}^{m}\sum_{k=1}^{n} s_{ik}(x)$. The main result is the following

Theorem 1. If $\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} a_{ik}^2[\max(i,k)]^{2\gamma} < \infty$ and $\lambda > 1$, then

(i) $\max_{\{n:\ \lambda^{-1} \leq n/m \leq \lambda\}} |\sigma_{m,n}(x)-f(x)| = o_x(m^{-\gamma})$ a.e. for $0 < \gamma < 1$;

(ii) $\{\frac{1}{m}\sum_{i=1}^{m} \max_{\{k:\ \lambda^{-1} \leq k/i \leq \lambda\}} |s_{ik}(x)-f(x)|^2\}^{1/2} = o_x(m^{-\gamma})$ a.e. for $0 < \gamma < 1/2$.

Statement (ii) expresses a strong approximation property in the sense of Alexits. Another result:

Theorem 2. If $\sum_{i=1}^{\infty}\sum_{k=1}^{\infty} a_{ik}^2 i^{2\alpha} k^{2\beta} < \infty$ and $0 < \alpha, \beta < 1$, then

$$\sigma_{mn}(x)-f(x)=o_x(\max(m^{-\alpha},n^{-\beta})) \text{ a.e.}$$

These results extend those of Alexits, Leindler, Sunouchi and others from single series to multiple ones.

W. NIETHAMMER:

Interpolationsverfahren zur numerischen analytischen Fortsetzung

Gegeben seien die Potenzreihenentwicklung $f(z) = \sum_{i=0}^{\infty} u_i z^i$ einer im Nullpunkt holomorphen Funktion und ein Gebiet G ($0 \in G \neq \mathbb{C}$), in das sich f analytisch fortsetzen läßt. Untersucht werden konstruktive Verfahren zur Berechnung von f(z) an einer Stelle $z \in G$, die i.a. außerhalb des Konvergenzkreises der Taylorreihe liegt.

Zu einer gegebenen Knotenmatrix $\mathfrak{X} = (x_i^{(n)})_{n \geq i \geq 0}$ bildet man die Folge der allgemeinen Hermite-Interpolationspolynome $\{P_n(y)\}_{n \geq 0}$ für die geometrische Reihe $1/(1-y)$. Für gewisse $z \in G$ gilt $f(z) = \Omega_f(1/(1-Xz))$, wobei das lineare Funktional Ω_f durch $\Omega_f(x^i) = u_i$ $(i \geq 0)$ definiert wird. Deshalb kann man $\{\Omega_f(P_n(Xz))\}_{n \geq 0}$ als Näherungsfolge für f(z) auffassen. Bei geeigneter Knotenwahl lassen sich diese Folgen rekursiv berechnen. Es zeigt sich, daß diese

Verfahren auch als Anwendung eines Matrix-Summierungsverfahrens (in Reihe-Folge-Form) auf die Potenzreihe von f interpretiert werden können. Nach dem Satz von Perron-Okada ist es daher möglich, sich bei Konvergenzfragen auf die geometrische Reihe zu beschränken. Ist nun $T \subseteq \mathbb{C} (0 \in T, 1 \notin T)$ kompakt, so ist unter allen Verfahren - jeweils gegeben durch eine Knotenmatrix -, die f nach T fortsetzen, dasjenige optimal bezüglich T, dessen Knoten auf T gleichverteilt sind.

Die Ergebnisse wurden zusammen mit M. Eiermann erzielt.

T. NISHISHIRAHO:

Quantitative theorems on approximation processes of positive linear operators

We establish a theorem of Korovkin type for approximation processes of positive linear operators and give quantitative versions of this result. The most typical examples of these processes in question are summation processes of positive linear operators which can be obtained by very general summability methods including convergence, almost convergence and so on and the Bernstein-Lototsky-Schnabl functions on a compact convex subset of a locally convex Hausdorff vector space over the field of real numbers.

K. SALKAUSKAS:

Some relationships between surface splines and Kriging methods of multivariate interpolation

There are many points of contact between the theory of surface splines of Duchon, Meinguet et al. and the statistically based Kriging methods of multivariate interpolation. Some of these connections are explored in a mostly finite dimensional setting, without appealing to statistical ideas. It is shown that both techniques are Boolean sums of projectors constructed from certain conditionally positive definite functions.

W. SCHEMPP:

Drei statt einer reellen Variablen?

In der konstruktiven Funktionentheorie mehrerer Variablen wird häufig das Verfahren angewandt, mehrdimensionale Approximationsprobleme auf eindimensionale Probleme zurückzuführen. Der umgekehrte Weg, daß sich Eigenschaften von Funktionen einer reellen Variablen auf natürliche Weise aus der Sicht höherer Dimension ergeben, ist weitaus weniger geläufig. Ziel der Übersichtsvortrags ist es, an Hand zweier Beispiele diesen umgekehrten Weg aufzuzeigen. Wir charakterisieren die Čebyšev-Polynome $(\check{C}_m)_{m\geq 0}$ zweiter Art als Charaktere der irreduziblen unitären Darstellungen genau derjenigen Sphäre unter den euklidischen n-Sphären S_n in den Vektorräumen $\mathbb{R}^{n+1}$ $(n>1)$, welche eine Liegruppen-Struktur aufweist. Außerdem weisen wir auf eine geometrische Beweisidee für den Existenz- und Eindeutigkeitssatz der kardinalen Spline-Interpolation mit Hilfe der harmonischen Analyse der reellen drei-dimensionalen nilpotenten Heisenberg-Gruppe $\tilde{A}(\mathbb{R})$ hin. Weitere Anwendungen, z.B. auf die Signalübertragung und die Radarortung bewegter Objekte, werden kurz angedeutet.

D. SCHMIDT:

Lipschitz conditions and strong uniqueness for metric projections for almost Chebyshev subspaces of C(X)

When X is an interval in the real line, it is well known that best uniform approximations from a (finite dimensional) Chebyshev subspace of C(X) are strongly unique and that the corresponding metric projection is pointwise Lipschitzian. When X is a multidimensional set, these concepts and their relationship to each other is clouded due to the non-existence of nontrivial Chebyshev subspaces of C(X). In this lecture, the relationship between these concepts is discussed. Among other results, it is found that the equivalence of strong uniqueness of a best approximation and pointwise Lipschitz continuity of the metric projection is a property exclusively possessed by the almost Chebyshev subspaces of C(X).

R. SCHMIDT:

Flächeninterpolation bei unregelmäßig verteilten Daten

Es wird eine Methode zur Konstruktion von Flächen vorgestellt, die in beliebigen Punkten im $\mathbb{R}^2$ vorgegebene Werte annehmen und über einem passenden Gebiet stetig differenzierbar sind. Sie besteht in der Triangulierung der Stützstellenmenge und der stetig differenzierbaren Verheftung von kubischen Flächenstücken über den Dreiecken, die in den Eckpunkten die Interpolations- und auf den Kanten die Glattheitsforderungen erfüllen. Das Verfahren ist global, es erfordert jedoch über die gegebene Datenmenge hinaus keine weiteren Informationen.

L.L. SCHUMAKER:

Dimension of spaces of piecewise polynomials in two variables

In this talk we examine the space of piecewise polynomials of degree d and in $C^{\mu}(\Omega) \cap C^{\rho}(\mathbb{R}^2)$ which vanish outside of a set Ω which has been partitioned into $\{\Omega_i\}_1^n$. Results on the dimension of this space were given at the last conference when $\rho = -1$ - here we consider $\rho \geq 0$. Such spaces are of interest in smooth data fitting, and in the finite element method. For rectangular partition the results are complete and local support bases are constructed for all d, μ, ρ. For certain triangular partitions we have results for $\mu = \rho = 0$, and for $d = 1,2,3$ and $\mu = 1, \rho = 0$. There remain many interesting questions.

H.S. SHAPIRO:

When is a vector sum of closed subspaces closed?

Let H denote a Hilbert space, and $M_1, \ldots, M_n$ closed subspaces. In some connections it is of interest to know whether the vector sum $M_1 + \ldots + M_n$ is closed. For example, in computed tomography this question arises, as well as that of expressing the orthogonal projection P on $M_1+\ldots+M_n$ in terms of the P_i (orthogonal projection on M_i). A simple but effective criterion recently discovered by Lars Swensson will be presented, according to which it suffices if P_iP_j is compact for each pair $i,j,i\neq j$. Application will be made to the "sum of plane waves" problem in tomography.

B. SHEKHTMAN:

Some properties of spline-projections

Every spline projection in a Hilbert space X could be presented as a function of interpolation conditions $\Lambda\subset X$ and operator $T:X\to Y$. Let $P(\Lambda,T)$ be an interpolation projection.

We plan to discuss properties of $P(\Lambda,T)$ as a function of T and Λ. In particular the limits of

$$P(T^n,\Lambda) \text{ as } n\to\infty,$$
$$P(T,\Lambda_n) \text{ as } \underline{\lim}\ \Lambda_n = X,$$
$$P(T_n,\Lambda) \text{ as } T_n\to T,$$
$$P(T^n,\Lambda_n) \text{ as } \lim \Lambda_n = X, n\to\infty.$$

A. VAN DER SLUIS:

Some remarks on cubature

At the meeting on numerical integration, Oberwolfach October 1981, the notion of quadrature rule set was defined, and it was proved that the corresponding repeated quadrature on a simplex gives rise to an error functional which has an asymptotic expansion (see the proceedings in the ISNM series). A generalization of this theorem to arbitrary linear functionals will now be presented. Special quadrature rule sets will be discussed. We will also make some observations on asymptotic expansions for cubature on domains with curved boundaries and functions with severe discontinuities.

J.P. THIRAN:

Minimal H-sets for two-variable rational approximation

H-sets have been introduced by Collatz to yield lower bounds for the minimal error norm in Chebyshev approximation theory. This contribution investigates H-sets relative to ratio of two bivariate polynomials. A modification of the original definition is proposed, which allows a description of extremal point sets in characterization theorem. It is shown that H-sets relative to rational functions in two variables correspond in fact to linear approximation problems with interpolation conditions. These conditions result from a fundamental theorem in algebraic geometry which is due to Noether. In this way, one gets a simple method to construct minimal H-sets for rational approximation from those relative to bivariate polynomials. Some examples are given to illustrate the procedure.

G. WAHBA:

Smoothing splines on the sphere with applications in meteorology

Just, we announce some results concerning smoothing splines on the sphere (SIAM J. Sci. Stat. Comput. 2 (1981)). Some similar results have been obtained independently by W. Freeden (Math. Meth. in the Appl. Sci. 3 (1981)). Some ideas from the theory of smoothing splines on the sphere are being used to estimate the divergence and vorticity of the horizontal wind field, given noisy observations of horizontal wind vectors from the international radio network. The seminorms in the spline smoothing optimization problem are chosen based on some meteorological data which provides information concerning the rate of decay of the energy spectrum of certain meteorological functions with wave number. This rate of decay corresponds to continuity properties of the Green 's function, or reproducing kernel which governs the semi-norm. The smoothing parameter is chosen by generalized cross validation. Numerical results so far are quite promising as far as practical application of the method for the determination of initial conditions in numerical weather prediction are concerned.

G.A. WATSON:

A Lagrangian method for multivariate continuous Chebyshev approximation problems

A method for multivariate continuous Chebyshev approximation problems is developed. It is based on a globally convergent method given by the author for semi-infinite programming problems (B.I.T. 21 (1981), 362-373), in which a sequence of quadratic programming problems is generated whose solutions are descent directions for an exact penalty function. The structure of the present problem is exploited so that an artificial penalty function is not required, but descent is achieved with respect to the norm. The connection with Newton's method applied to the first order necessary conditions, and with the second algorithm of Remes for linear Chebyshev set problems, is demonstrated when the matrix of the quadratic sub-problem is chosen as the Hessian matrix of an appropriate Lagrangian functio

SHEN XIE-CHANG:

A survey of recent results on approximation theory in China

I. Approximation and expansion of a real variable

1. Approximation by linear operators
2. Approximation by some concrete linear operators
3. The exact constant of Jackson type operator
4. Approximation by algebraic polynomials

II. Interpolation and approximation of several variables, harmonic analysis

1. Some results in bivariate interpolation
2. Some results on multiple approximation
3. The Riesz means of the multiple Fourier series

III. Complex approximation

K. ZELLER (mit W. Haußmann und E. Luik):

BOGS procedures in approximation

Numerical approximation is often carried out by ascent methods. But also other methods (like descent, telescoping, preiteration) are useful, especially in the multivariate case. We discuss procedures which are connected with biorthogonal systems (BOGS). In principle the task is to shorten a long expression (about which one has explicit or implicit information). Thereby one uses best or good approximations of single terms or groups of terms. The linear functionals in the BOGS are employed for computing or estimating coefficients and errors. More specifically we consider expansions of Fourier (Chebyshev) type in the univariate and multivariate case.

K. ZELLER (mit W. Haußmann und E. Luik):

BOGS remainder in cubature

Error estimates for cubature formulas are usually given in terms of higher derivatives (Peano-Sard) or in terms of analyticity properties (Davis-Hämmerlin). The approximation method has found little attention. We present the latter method in a generalized and refined form, based on biorthogonal systems (BOGS). The degrees of approximation and the coefficient estimates connected with a BOGS lead to rather good and versatile inequalities for the error. More specifically we treat Clenshaw-Curtis and general product formulas.

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