MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 11/1982

Einhüllende Algebren von Lie-Algebren
14.3. bis 20.3.1982

Nach den Tagungen in den Jahren 1973, 1975, 1978 war dies die vierte Oberwolfacher Tagung über Einhüllende Algebren. Im Mittelpunkt des Interesses stand - wie schon 1978 - auch diesmal die Entwicklung im halbeinfachen Fall, wo eine Reihe spektakulärer Ereignisse in der Zwischenzeit zu einem regelrechten Umbruch geführt haben: Nachdem Duflo, Jantzen, Joseph, Vogan das Studium der primitiven Ideale und andere zentrale Fragen auf die Bestimmung der Multiplizitäten in den Verma-Moduln zurückgeführt hatten, formulierten Kazhdan und Lusztig Anfang 1979 eine Vermutung, derzufolge diese Multiplizitäten durch gewisse topologische Invarianten der Schubert-Varietäten gegeben sind (als lokale Euler-Charakteristiken der Schnitt-Homologie von Deligne - Goresky - MacPherson), die man andererseits durch einen kombinatorischen Algorithmus effektiv berechnen kann (im Prinzip). Diese Vermutung erwies sich als schwer zugänglich mit "konventionellen" Methoden,konnte aber schon Ende 1981 von Beilinson und Bernstein, sowie Brylinski und Kashiwara unter Benutzung der Theorie der holonomen Systeme partieller Differentialgleichungen (D-Moduln) bewiesen werden (im Falle ganzzahligen zentralen Charakters). Der Einsatz neuartiger Methoden aus dem Bereich der Topologie bzw. Analysis, der Schnitt-Homologie bzw. D-Moduln, hat also im Endeffekt wichtige Fragen der Ring- und Darstellungstheorie in Probleme der Kombinatorik übersetzen können. Diese können allerdings derart kniff-lig sein, daß Schlüsse in umgekehrter Richtung, also Anwendungen der Theorie Einhüllender Algebren zur Lösung rein kcmbinatorischer Probleme, schon mehrfach vorgekommen sind. - Diese neuen Methoden haben daneben auch Borho und Brylinski zu einem Beweis der lange vermuteten Zuordnung von primitiven Idealen (mit ganzz.zentr.Charakter) zu nilpotenten Konjugationsklassen (nämlich dichte Orbiten in deren assoziierten Varietäten) geführt, ferner Borho und MacPherson zu einem besseren Verständnis der Springer'schen Korrespondenz von nilpotenten Konjugationsklassen zu irreduziblen Darstellungen der Weyl-Gruppe, was Alvis, Lusztig und Spaltenstein in die Lage versetzte, die Zuordnung explizit zu berechnen. Barbasch und Vogan haben Josephs Korrespondenz von primitiven Idealen zu Weyl-Gruppen-Darstellungen (über Goldie-Rang-Polynome) als Komposition der beiden zuvor erwähnten Zuordnungen identifiziert und dazu benutzt, die Klassifikation der primitiven Ideale zu einem gewissen Abschluß zu bringen.

Auch im Falle allgemeiner Lie-Algebren hat das Studium der primitiven Ideale inzwischen wesentliche Fortschritte gemacht (Arbeiten von Duflo, Moeglin, Rentschler), vor allem durch die Angabe einer Surjektion mit endlichen Fasern auf das primitive Spektrum durch Duflo.
Vom Standpunkt der wissenschaftlichen Organisation dieser Tagung aus betrachtet war es aussichtslos, diese -hier nur grob angedeutete - Entwicklung des Gebietes systematisch oder gar erschöpfend behandeln zu wollen. Dies war auch wegen des unglücklichen Zusammentreffens der bisher lebhaftesten Entwicklungsphase mit dem bisher längsten (vierjährigen) Abstand zwischen den Tagungen über Ein-

hüllende Algebren nicht möglich. Es wurde jedoch mit einer Reihe von Überblicksvorträgen der Versuch gemacht, einige der wichtigsten Ergebnisse, Methoden und Querverbindungen in dieser Entwicklung auf der Tagung wenigstens summarisch zu behandeln.

Insbesondere gab es solche Vorträge zu den folgenden Themen:

1.) D-modules approach to enveloping algebras
2.) Report on Verma modules
3.) On the proof of the Kazhdan-Lusztig conjecture
4.) Report on primitive ideals (semisimple case)
5.) Report on primitive ideals (general case)
6.) Report on geometry of conjugacy classes
7.) Weyl group representations and nilpotent orbits
8.) Topology of Schubert varieties.

Soweit sie nicht in diesen Rahmen gehören, befaßten sich die übrigen Vorträge mit folgenden weiteren Themen: Mit Jantzens Filtrierung der Verma-Moduln (auch in Charakteristik p), die Gabber und Joseph auf eine -bisher noch unbewiesene- Vertiefung der Kazhdan - Lusztig Vermutung führte. Mit der Bernstein - Gelfand - Gelfand´schen Kategorie O. Mit der Krull-Dimension und Lokalisierung von Einhüllenden Algebren und verwandten Ringen. Mit dem von Kostant vorgeschlagenen Konzept eines "verschobenen Cotangentialbündels", einer symplektischen Mannigfaltigkeit mit Hamilton´scher Gruppenaktion, die für das Studium coadjungierter Orbiten interessant ist. Mit der Struktur der Invarianten der äußeren Algebra einer Lie-Algebra. Mit der Homotopie des Koszul-Komplexes unter Verwendung von Lie-Superalgebren. Mit Verma Moduln und Flaggen Varietäten für den Fall der (unendlich - dimensionalen) Kac-Moody Lie-Algebren, sowie einem Zusammenhang mit der Korteweg - de Vries Gleichung.

Die Tagung wurde von W. Borho (Wuppertal), J. Dixmier (Paris) und R. Rentschler (Orsay) geleitet.

TEILNEHMER waren ferner:

Andersen, H.H., Aarhus
Barbasch, D., New Brunswick
Bartels, D., Wuppertal
Berline, N., Rennes
Brown, K., Glasgow
Brylinski, J.-L., Palaiseau
Bürgstein, H., Wuppertal
Carmona, J., Marseille,
Carter, R.W., Coventry
Delorme, P., Palaiseau
Enright, T.J., La Jolla
Hesselink, W., Groningen
Humphreys, J.E., Amherst
Irving, R.S., Seattle
Jantzen, J.C., Bonn
Joseph, A., Paris
Kac, V.G., Cambridge, Ma.
Kempken, G., Basel
Kostant, B., Cambridge, Ma.
Kraft, H., Basel
Lenagan, T.H., Edinburgh
Levasseur, T., Paris
Lorenz, M., Essen
Malliavin, M.-P., Paris
McConnel, J., Leeds
Moeglin, C., Paris
Rocha-Caridi, A.M.P.,New Brunswick
Smith, S.P., Los Angeles
Springer, T.A., Utrecht
Spaltenstein, N., Coventry
Stafford, T., Leeds
Verdier, J.L., Paris
Wigner, D., Paris.

Vortragsauszüge

J.-L. BRYLINSKI: $\mathcal{D}$-MODULES APPROACH TO ENVELOPING ALGEBRAS

The talk was an introduction to sheaves of modules over the sheaf of differential operators on an algebraic variety, with special emphasis on their relations to modules over enveloping algebras. The algebra of the global algebraic differential operators on the flag variety of a semi-simple Lie algebra $\underline{g}$ is isomorphic to the quotient of the enveloping algebra $U(\underline{g})$ by the ideal generated by the "trivial" maximal ideal of the centre. Under some technical condition, Cartan's theorems A and B hold for sheaves of $\mathcal{D}$-modules over the flag variety. This theorem of Beilinson and Bernstein uses a sheaf-theoretical version of the Borho-Jantzen-Zuckerman translation functors. It is crucial in all proofs of the Kazhdan-Lusztig and Vogan conjectures. The talk gave the construction of the functors "integration along fibres" for sheaves of $\mathcal{D}$-modules (especially for the case of a closed immersion) and discussed its relation to induction of modules or ideals in the enveloping algebra context.

A. JOSEPH: REPORT ON VERMA MODULES

Verma modules were reviewed and the following problems proposed.

1) Is Fract $(U(\underline{g})|J): J \in \operatorname{Prim} U(\underline{g})$ a matrix ring over a *Weyl division algebra* (true in type A_n)?

2) Is Kdim $U(\underline{g})|J = \frac{1}{2} d(U(\underline{g})|J): J \in \operatorname{Prim} U(\underline{g})$ (true for J maximal or minimal)?

3) Is $\sum_{k=0}^{\infty} \sum_{w' \in W_\lambda} q^k (-1)^{\ell(w')-\ell(w)-k} \dim \operatorname{Ext}^k(M(w\lambda), M(w'\lambda))\, T_{w'} = T_w$? (The T_w generate the W_λ Hecke algebra).

4) Calculate $z_w := \dfrac{\operatorname{rk} F(L(w\lambda), L(w\lambda))}{\operatorname{rk} (U(\underline{g})/J(w\lambda))} : w \in W_\lambda$ (F denotes "ad $\underline{g}$ finite part" of Hom).

5) Does Fract $F(M,M)$ = Fract $(U(\underline{g})/\operatorname{Ann} M) \Longrightarrow F(M,M) = U(\underline{g})/\operatorname{Ann} M$? (where M is a simple $U(\underline{g})$ module).

6) Set $a(w) = \sum_{w' \in W_\lambda} (L(w\lambda):M(w'\lambda))w'^{-1}$. Show that $F(M,M) = U(g)/\text{Ann } M:M =$

$= L(w\lambda) \iff a(w)$ is cyclic in its left cell.

7) Is $d(L(\sigma\mu)) = \max_{w \in C(\sigma)} \max_{\ell \in \mathbb{N}} \{\ell | H^\ell(n^-, L(w\mu))_{\mu-\rho}\}$? ($\mu$ antidominant, $C(\sigma)$ the left cell containing σ).

8) Is Soc $F(L(w\lambda), L(w'\lambda))$ multiplicity free?

9) The composition factors of $U(g)/J(w\lambda)$ as known. Calculate their multiplicities.

10) Do the functors $M \to C_\alpha M := L(M(s_\alpha\lambda), M) \otimes_{U(g)} M(\lambda) : \alpha \in B_\lambda$ satisfy the Braid relations?

11) The map $C_\alpha(\delta M) \to \delta(C_\alpha M)$ induces a map $F(L(s_\alpha w\lambda), L(s_\alpha w\lambda)) \xrightarrow{\kappa} F(L(w\lambda), L(w\lambda))$ Calculate Ker κ and coker κ.

12) For each $\alpha \in B_\lambda$ does $M(w\lambda)$ admit an orthogonal direct sum of indecomposable $\underline{s}_\alpha \cong \underline{s\ell}(2)$ modules?

13) Is $M^i(w\lambda) = M^{i-1}(s_\alpha w\lambda) + C_\alpha M^i(s_\alpha w\lambda)$? (Superscript denotes Jantzen filtration, $s_\alpha w > w$).

14) Is $V(U(\underline{g})/\text{Ann } M) = \overline{GV(M)}$ (M a finitely generated $U(\underline{g})$ module, $V(M)$ its associated variety).

15) Relate $V(C_\alpha M)$ to $V(M)$.

16) Let $V(w): w \in W$ denote the smallest closed B stable variety containing $\underline{n} \cap \underline{n}^w$ and $I(V(w))$ its ideal of definition in $S(\underline{n}^-)$. Calculate the Poincaré series of $S(\underline{n}^-)/I(V(w))$.

17) Show that for every $w \in W$ there is a simple highest weight module L such that $V(L) = V(w)$.

18) Show that for every nilpotent orbit $\mathcal{O} \subset \underline{g}^* = \underline{g}$ there is a completely prime $J \in$ Prim $U(\underline{g})$ such that $V(U(\underline{g})/J) = \bar{\mathcal{O}}$.

19) Show that every $J \in$ Prim $U(\underline{g})$ is the annihilator of a simple Harish-Chandra module.

20) Has $M \otimes E$ finite length if M is simple and E finite dimensional?

J.-L. BRYLINSKI: ON THE PROOF OF THE KAZHDAN-LUSZTIG CONJECTURE

The proof of the Kazhdan-Lusztig conjecture uses in a crucial way the known strong relations between holonomic $\mathcal{D}$-modules on a complex analytic variety X, and the topology of analytic subspaces of that variety. Kashiwara proved in 1975 that the De Rham complex of a holonomic $\mathcal{D}$-module is a bounded complex of sheaves with constructible cohomology. The condition that such a complex of sheaves is the De Rham complex of a holonomic $\mathcal{D}$-module is explicitly known, and is nowadays called a perversity condition. It is essential here that the Verdier duality on complexes of sheaves with constructible cohomology corresponds to an algebraic duality operation on holonomic $\mathcal{D}$-modules. If one restricts to the holonomic $\mathcal{D}$-modules with regular singularities (a notion which generalizes the old notion in dimension 1, due to Fuchs), one gets a perfect correspondence with the above complexes of sheaves. For instance, for a closed analytic subvariety Y, one may easily characterize a holonomic $\mathcal{D}$-module $L(Y,X)$, which has for De Rham complex the complex $IC^{\cdot}(Y)$ of Goresky-MacPherson-Deligne.

D. BARBASCH: ON THE CLASSIFICATION OF PRIMITIVE IDEALS

Let $\underline{g}$ be a semisimple Lie algebra. As is well known from results of Duflo, any primitive ideal in $U(\underline{g})$ is the annihilator of a Verma module. Several results relating primitive ideals to the characters of the Verma modules were discussed. In particular, techniques for determining the primitive ideal cells in the sense of Joseph were presented.

R. RENTSCHLER: REPORT ON PRIMITIVE IDEALS (GENERAL CASE)

Let $\underline{g}$ be a k-Lie-algebra, k alg. closed field of char. 0 ($\dim \underline{g} < \infty$)

1) A prime ideal P of $U(\underline{g})$ is primitive <=> P is locally closed in Spec $U(\underline{g})$ (Moeglin).

2) Definition and properties of the Dixmier-Duflo map $J : \underline{g}^*_{pr} \to \text{Prim } U(\underline{g})$ (Prim = "primitive ideals", $\underline{g}^*_{pr}$ = {linear forms on $\underline{g}$ having a solvable polarisation}).

3) Duflo splitting map D_δ: Let $\underline{u}$ be a nilpotent ideal of a subalgebra $\underline{b}$ of $\underline{g}$, $\delta \in \underline{u}^*$, $\underline{b} = \underline{h} + \underline{u}$ with $b \subseteq \{x \in b \mid \delta([x,u]) = 0\}$. Let $Z_\delta := \sum_{x \in \underline{h} \cap \underline{u}} U(\underline{h})(x-\delta(x))$. Then $\exists\, D_\delta : {}^{U(\underline{b})}/U(\underline{b})J(\delta) \xrightarrow{\sim} {}^{U(\underline{h})}/U(\underline{h})Z_\delta \otimes {}^{U(\underline{u})}/J(\delta)$, canonical isomorphism.

4) Let $\underline{u}$ be an ideal of $\underline{g}$, $I \in \text{Prim } U(\underline{g})$. Then $\exists\, q \in \text{Prim } U(\underline{u})$ such that $I \cap U(\underline{u}) = \bigcap_{\gamma \in G} \gamma q$ (G:= adjoint algebraic group). Let $\underline{h} := \{x \in \underline{g} \mid [x,q] \subseteq q\}$.

Then $\exists\, J \in \text{Prim } U(\underline{h})$, $I = \text{Ind}_{\underline{h}}^{\underline{g}}(J)$, $J \cap U(\underline{u}) = q$ (Moeglin-Rentschler).

5) Duflo-description of primitive ideals (1981):

Let G be a linear algebraic group, $\underline{g}$: = Lie G. If $\underline{h}$ is a subalgebra of g, denote by ${}^u\underline{h}$ its unipotent part ($\underline{h}$ algebraic).

Let $\Sigma := \{(\underline{g} = \underline{h}_0 \supset \underline{h}_1 \supset \ldots \supset \underline{h}_\ell ; \delta) \mid \delta \in (\Sigma\, {}^u\underline{h}_i)^*, \underline{h}_{i+1} = \{x \in \underline{h}_i \mid \delta([x, {}^u\underline{h}_i]) = 0\}\}$.

For $S = (\underline{h}_0, \underline{h}_1, \ldots, \underline{h}_\ell ; \delta) \in \Sigma$ let $X(S) := \{q \in \text{Prim } U(\underline{h}_\ell) \mid x - \delta(x) \in q$ for $x \in {}^u\underline{h}_\ell\}$. Then $X(S) = \text{Prim } U(\underline{h}_\ell / {}^u\underline{h}_\ell)$ (primitive spectrum, reductive case). There is a naturally defined surjective map $\coprod_{S \in \Sigma} X(S) \twoheadrightarrow \text{Prim } U(\underline{g})$. The factored map $\left(\coprod_{S \in \Sigma} X(S)\right)/G \twoheadrightarrow \text{Prim } U(\underline{g})$ has finite fibres.

S.P. SMITH: KRULL DIMENSION OF FACTOR RINGS OF ENVELOPING ALGEBRAS

A brief review of what is known about the Krull dimension of factor rings of enveloping algebras is given. For such a ring R,d(·) denotes the Gelfand-Kirillov dimension and $|\cdot|$ the Krull dimension.

Proposition ($\underline{g}$ solvable, algebraic). If $U(\underline{g})$ is catenary and R a prime factor ring of $U(\underline{g})$ then $|R| = d(R) - s(R)$ where $s(R) = \min\{d(M) \mid M$ is a non-zero R-module$\}$.

Theorem (g semi-simple) $|U(g)| < \dim g - r(g)$ where $r(g) = \frac{1}{2}$ dimension of a minimal non-zero dimensional G orbit in g^*.

For example, one obtains $|U(s\ell(2))| = 2$ and $5 < |U(S\ell(3))| < 6$. Joseph has pointed out that it follows from Beilinson-Bernstein that if P is a minimal primitive then $|U/P| = \frac{1}{2}d(U/P)$. It is conjectured that this equality holds for all primitives. It also holds for those primitives P with $d(U/P) = 2r(g)$.

T.H. LENAGAN: KRULL DIMENSION OF RINGS OF DIFFERENTIAL OPERATORS

This talk describes joint work with K.R. Goodearl on the problem of determining the Krull dimension (in the sense of Rentschler and Gabriel) of Ore extensions of Noetherian rings. The specific results we report, in the case of $\mathbb{Q}$-algebras, are as follows:

Theorem 1 If R is a right Noetherian ring with finite Krull dimension, and δ is a derivation on R, then $\mathrm{kdim}(R[\theta;\delta]) = \mathrm{kdim}(R) + 1$ if and only if there is a maximal right ideal M and $a \in R$ such that $(\delta + a)M \subseteq M$ and height $(\overset{R}{\tilde{M}}) = \mathrm{kdim}(R)$

Theorem 2 If R is a commutative Noetherian ring with finite Krull dimension and $\delta_1,\ldots,\delta_u$ are commuting derivations on R then $\mathrm{kdim}(R[\theta_1,\ldots,\theta_u;\delta_1,\ldots,\delta_u]) =$

$$\max\{\text{height}(P) + \text{diff.dim}(P) \mid P \in \mathrm{Spec}(R)\}.$$

In the case that R is a finitely generated algebra over a field, the maximum value in the formula of Theorem 2 occurs at a maximal ideal of the ring.

D. WIGNER: A HOMOTOPY OF THE KOSZUL COMPLEX

We employ the formalism of Lie superalgebras of the Poincaré-Birkhoff-Witt theorem to superalgebras to construct a natural homotopy of the Koszul complex for the cohomology of Lie algebras in characteristic zero (joint work with Cartier and Duflo).

T.A. SPRINGER: WEYL GROUP REPRESENTATIONS AND NILPOTENT ORBITS

Let G be a connected semi-simple group on $\mathbb{C}$; denoted by B, T, W a Borel subgroup, a maximal torus contained in it and the corresponding Weyl group, respectively.

Let $\underline{g}$ be the Lie algebra of G and N the nilpotent variety of $\underline{g}$. If $X = G/B$ is the flag variety and T^*X its cotangent bundle, the "moment map" of T^*X induces a morphism of varieties $\pi: T^*X \to N$, which is a resolution of singularities of N. The talk contained a survey of some results which relate representations of W to the geometry of π. The following results were discussed:

(a) The construction, using intersection cohomology, of a W-action on the direct image complex $\mathbb{R}\pi_*\mathbb{Q}$ (see [1]).

(b) If $\xi \in N$ put $X_\xi = \pi^{-1}\xi$. Then X_ξ is of pure dimension $e(\xi) = \frac{1}{2}$ ($\dim Z_G(\xi)$ - rank G) ($Z_G(\ )$ denoting centralizers in G). Let $C(\xi) = Z_G(\xi)/Z_G(\xi)^0$ be the component group of $Z_G(\xi)$. From (a) one sees that $C(\xi) \times W$ operates on the cohomology groups $H^i(X_\xi:\mathbb{Q})$, in particular on $H^{2e(\xi)}(X_\xi;\mathbb{Q})$. For any rational character ϕ of $C(\xi)$, let $V_{(\xi,\phi)}$ be the ϕ-isotypic part of $H^{2e(\xi)}(X_\xi;\mathbb{Q})$. There is a rational representation $\chi_{(\xi,\phi)}$ of W such that $C(\xi) \times W$ acts in $V_{(\xi,\phi)}$ via $\phi \otimes \chi_{(\xi,\phi)}$.

Then:

(i) If $V_{(\xi,\phi)} \neq 0$ the representation $\chi_{(\xi,\phi)}$ is absolutely irreducible.

(ii) Any absolutely irreducible representation of W is equivalent to such a $V_{(\xi,\phi)}$, the pair (ξ,ϕ) is unique up to conjugacy (see [1] , [3]).

(c) Definition of special representations of W after Lusztig [2].

References:

[1] W. Borho, R. MacPherson, Représentations des groupes de Weyl et homologie d'intersection pair les variétés de nilpotents, C.R. Acad. Sc. Paris, 292 (1981), 707-710.

[2] G. Lusztig, A class of irreducible representations of Weyl groups, Proc. Kon. Ak. v. Wet. Amsterdam, 82 (1979), 323-335.

[3] T.A. Springer, Trignometric sums, Green functions of finite groups and representations of Weyl groups, Invent. Math. 36 (1976), 173-207.

N.SPALTENSTEIN: COMPUTATION OF SPRINGER REPRESENTATIONS

Let G be a connected reductive group defined over $\mathbb{C}$, $x \in G$ unipotent, $\mathcal{B}_x$ the variety of all Borel subgroups of G containing x. Springer has defined representations of the Weyl group W of G in the spaces $H^i(\mathcal{B}_x)$. The representations in H^{2e} ($e = \dim \mathcal{B}_x$) have been computed by Shoji (G classical or of type F_4) and Alvis, Lusztig and the author (E_n, $6 \leq n \leq 8$). The techniques used include the relation between truncated induction for Weyl group representations and induction of unipotent classes, the Borho-MacPherson results on Springer representations, Macdonald formula for $\Sigma(-1)^i H^i(\mathcal{B}_x)$ and Springer's formula for the restriction to parabolic subgroups of W. These are applications to Green functions of finite Chevalley groups and to the study of unipotent (or nilpotent) orbits.

H. KRAFT: REPORT ON THE GEOMETRY OF CONJUGACY CLASSES

Consider a semi simple Lie algebra $\underline{g}$ = Lie G and the adjoint action of G on $\underline{g}$. In the study of the geometry of a conjugacy class $C_x = Gx$ we consider the following three problems:

A) C_x *as a homogeneous space:* E.g. it is an open problem, whether C_x is rational or not. Furthermore one gets interesting invariants by looking at the multiplicities in the coordinate ring $\mathcal{O}(C_x) = \mathcal{O}(G)^{G_x}$ and the growth.

B) Degenerations: Here we consider the closure $\bar{C}$, which is a finite union of classes. The inclusion relations between these closures are known (Gerstenhaber-Hesselink, Shoji, Mizu no, Spaltenstein), the normality and singularity problem is settled for classical groups (Procesi-Kraft). Here we should mention the intersection cohomology of $\bar{C}$ and the relations with Springer's Weyl group representations (Borho-MacPherson), which can be used for the study of singularities.

C) Deformations: This leads to the concept of sheets and their parametrization. There is a very nice picture for $s\ell_n$ (Dixmier, Ozeki-Wakimoto, Borho, Kraft, Peterson). The general classification can be obtained via pairs (Levi-subalgebra, original orbit) (Borho; Kempken-Spaltenstein, Elashvili). The sheets in classical groups are smooth and admit an "almost regular" parametrization in general. (However, there exists a non-normal sheet in type G_2.) Another point of view of some of the problems here is obtained via the notion of a moment map; it has been used for the study of primitive ideals and their associated variety (Borho-Brylinski).

B. KOSTANT: ON THE SHIFTED COTANGENT BUNDLE

If L is a line bundle over a manifold M, one defines a symplectic manifold $T^*_L(M)$ referred to as the L-shifted cotangent bundle over M. One has a new symbol calculus for $T^*_L(M)$. If L is a homogeneous G-bundle for a Lie group G then $T^*_L(M)$ has the structure of a Hamiltonian G-space and one has a moment map $\mu_L : T^*_L(M) \to \underline{g}'$ where $\underline{g}'$ is the dual of $\underline{g}$ = Lie G. Let (H,χ) be a pair where H is a closed subgroup and χ is an H-character.

Theorem: (H,χ) *is a polarization of a coadjoint orbit* $\mathcal{O}$ *if and only if there exists an open orbit* $\tilde{\mathcal{O}}$ in $T^*_L(M)$ *for* $M = G/H$ *and* $L = G\times_H \mathbb{C}_\chi$.

Moreover in such a case $\tilde{\mathcal{O}}$ *can be chosen so that* $\mu_L : \tilde{\mathcal{O}} \to \mathcal{O}$ *is a covering of Hamiltonian* G*-spaces.*

One readily obtains a generalisation of a result of Duflo.

Corollary. *If* (H,χ) *is a polarization of a coadjoint orbit and if* $U(\underline{g})$ *is the enveloping algebra of* $\underline{g}$ *one has*

$$\text{Cent } U(\underline{g}) \to \text{scalars}$$

with respect to Ind (H,χ).

Assume $\mathcal{O}$ is closed simply connected and G is algebraic. Let $I \subseteq S(\underline{g})$ be the prime ideal defining $\mathcal{O}$ and let $J \subseteq U(\quad)$ the kernel of Ind (H,χ) for $U(\underline{g})$. Then one has

Theorem: *Let* μ *be the moment map for* $T^*(M)$, *the usual cotangent bundle. Now assume that*

$$\dim \mu(T^*(M)) = \dim \mathcal{O}.$$

Then $\text{Gr } I \subseteq S(\underline{g})$ *is prime if and only if* Gr J *is prime in which case*

$$\text{Gr } I = \text{Gr } J$$

and the corresponding variety is $\overline{\mu(T^*(M))}$.

A.ROCHA-CARIDI: CHARACTERS OF IRREDUCIBLE HIGHEST WEIGHT MODULES OVER INFINITE DIMENSIONAL LIE-ALGEBRAS

We study the irreducible quotients of Verma modules for generalized Cartan matrix (GCM) or Kac-Moody Lie algebras and for the Witt algebra. Our starting point is the resolution of a "standard" module. These modules are the analogs of the finite dimensional irreducible modules over semi-simple Lie algebras. For GCM Lie algebras the resolution, which we obtained in previous work, is a generalization of the Bernstein-Gelfand-Gelfand resolution and a sharpening of the Garland-Lepowsky resolution. For the Witt algebra, we construct a resolution of the trivial module. The second step in our study is the proof of a character sum formula for the quotient of two Verma modules. For GCM Lie algebras this formula is a generalization of J.C. Jantzen's formula for semi-simple Lie algebras.

As an application of the results described above we construct resolutions of irreducible (not necessarily standard) modules over rank 2 GCM Lie algebras and over the Witt algebra. These resolutions imply character formulas as conjectured by D. Kazhdan and G. Lusztig in the GCM case and by V. Kac for the Witt algebra.

R.S. IRVING: PROJECTIVES IN THE CATEGORY $\mathcal{O}$

Let $\underline{g}$ be a semisimple Lie algebra, with Cartan subalgebra $\underline{h}$, Weyl group W. Given $\mu \in \underline{h}^*$, let $P(\mu)$ be the projective cover of $L(\mu)$. Let w_0 be the longest element of W. For a dominant weight λ, a theorem of Humphreys yields that $P(w_0 \cdot \lambda)$ has a p-series in which each $M(\nu)$ occurs once, for $\nu \in W_\lambda \cdot \lambda$.

For any μ in $W_\lambda \cdot \lambda$, the projective $P(\mu)$ can be described in terms of $P(w_0 \cdot \lambda)$, reducing the study of projectives to that of antidominant projectives. For rank 2, and for some non-regular weights, these projectives can be described.

J.E. HUMPHREYS: MODULAR VARIATIONS ON A THEME OF GABBER-JOSEPH

Jantzen has defined filtrations in highest weight modules (such as Verma modules in characteristic 0, Weyl modules in characteristic p) by means of contravariant forms, and has proved a "sum formula" (still conjectural for some small p). Gabber-Joseph have shown how to relate the Verma module filtrations and their behaviour under canonical maps with the Kazhdan-Lusztig polynomials. We conjecture that similar phenomena will occur in characteristic p:(1) Filtrations satisfying sum formulas like Jantzen's should exist in all sheaf cohomology groups $H^i(G/B, \mathcal{L}(\lambda))$, Weyl modules being a special case. Such filtrations would respect Serre duality and be compatible with H.H. Andersen's canonical maps. (2) The conjectured filtrations could also be computed from Lusztig's polynomials [Advances in Math., 1980] in the spirit of Gabber-Joseph. (3) Such filtrations may help to explain and predict "nonstandard vanishing" of cohomology.

J.-L. VERDIER: TOPOLOGY OF SCHUBERT VARIETIES

Report on *Topology of Schubert varieties:* (1) Weyl group of a complex semi-simple group, (2) Varieties of position, Schubert cells, (3) Schubert varieties: Bruhat order, Whitney stratification, Singular locus, Demazure desingularisation; (4) Cohomology of Schubert varieties; (5) Intersection homology; (6) Intersection complex and Kazhdan-Lusztig polynomials.

V.G. KAC: INFINITE-DIMENSIONAL FLAG VARIETIES AND THE KdV EQUATION

In this talk the recent remarkable work of Date-Jimbo-Kashiwara-Miwa on the KdV-type hierachies and affine Lie algebras is related to the geometry of the flag variety of an affine Lie group $\hat{G}$. It is known that the basic representation of $\hat{G}$ can be realized in the space V of polynomials in infinitely many variables x_i (Advances in Math. 42 (1981), 83-112). The projectivization of the orbit of 1 is the (generalized) flag variety P. The group $\hat{T} = T(\mathbb{C}[t,t^{-1}] < \hat{G}$ operates on P, and the translates of 1 form a set of polynomials R_α parametrized by the dual root lattice of G. These are centres of the Schubert cells X_α, and $X_\alpha = \{R_\alpha(x_i + c_i),\ c_i \in \mathbb{C}\}$. The polynomials R_α can be computed explicitly (at least for SL_n) since they are isobaric polynomial τ-functions of the KdV type hierarchies. Relation to the (generalized) Kazhdan-Lusztig conjectures is discussed.

T.J. ENRIGHT: WEIGHTED JANTZEN FILTRATIONS

Let $\underline{g}$ be a semisimple Lie algebra over a discrete valuation ring A with valuation . Let $\underline{b} = \underline{h} \oplus \underline{n}$ be a Borel subalgebra and fix $\lambda \in \mathrm{Hom}_A(\underline{h},A)$. Write $M(\lambda)$ for the Verma module $U(g) \otimes_{U(b)} A_{\lambda-\rho}$, $\rho = \frac{1}{2} \sum_{\alpha>0} \alpha$. Using the contravariant form and the maximal ideal I of A, define $M^i(\lambda) = \{m \in M(\lambda) | \langle m, M(\lambda)\rangle \subseteq I^i\}$. If ϕ denotes the residue map $\phi: A \to A/I = K$, then define filtrations on Verma modules over K by $M^i(\phi\lambda) = \phi\, M^i(\lambda)$.

Jantzen's character sum formula generalizes to:

$$\sum_{i>0} \mathrm{ch}\, M^i(\phi\lambda) = \sum_{\substack{\alpha>0 \\ n\in\mathbb{N}^*}} \nu(\lambda(H_\alpha)-n)\ \mathrm{ch}\, M(s_\alpha\phi\lambda).$$

Based on this character formula and the corresponding one for quotients of two Verma modules we prove:

Theorem: Fix $\alpha > 0$ with $\phi\lambda(H_\alpha) = m \in \mathbb{N}^*$. Assume $\nu(\lambda(H_\alpha)-m) \gg \nu(\lambda(H_\beta)-n)$ for all $n \in \mathbb{Z}$, $\beta > 0$, $\beta \neq \alpha$. Then for all $i \in \mathbb{N}$,

$$M^i(\phi\lambda) \cap M(s_\alpha\phi\lambda) = M^{i-\nu(\lambda(H_\alpha)-m)}(s_\alpha\phi\lambda).$$

K.A. BROWN: LINKS AND LOCALISATION IN ENVELOPING ALGEBRAS

In a Noetherian ring R there are close connections between (i) the structure of certain R-R bimodules, (in particular those of the form I/J where I and J are ideals of R); (ii) the obstructions to localising at a prime ideal of R (in R or an over-ring of R); and (iii) the representation theory of R. We explore and describe these connections in the case where R is the enveloping algebra of a solvable Lie algebra.

J.T. STAFFORD: DIMENSIONS OF DIVISION RINGS

Let D be a division ring. For example, take D = D($\underline{g}$), the quotient division ring of the enveloping algebra of a finite dimensional Lie algebra over a field k or D = D(G), the quotient division ring of the group ring kG of a poly (cyclic-by-finite), torsion-free group G. Then very little is known about the internal structure of D, and there are very few invariants that have been found. One invariant is Resco's matrix transcendence degree which can be calculated by determining the Krull or global dimension of $D \otimes_k k(z_1,\ldots,z_r)$. This suggests that other invariants for D may be obtained by investigating other over-rings of D. We use this approach to provide another possible candidate for the transcendence degree of D by proving

THEOREM (i) $\mathrm{gldim}\, D(\underline{g})^{op} \otimes_k D(\underline{g}) = \dim \underline{g}$

(ii) $\mathrm{Kdim}\, D(\underline{g})^{op} \otimes D(\underline{g}) = \dim \underline{g}$ provided that $\underline{g}$ is either solvable or algebraic and k is an uncountable, algebraically closed field of characteristic zero.

(iii) $\mathrm{gldim}\, D(G)^{op} \otimes D(G) = \mathrm{Kdim}\, D(G)\ op \otimes D(G) = h(G)$, the Hirsch number of G.

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