MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 12/1982

Finite Geometries

21.3. bis 27.3.1982

This year's Finite Geometries conference was held under the direction of F. Buekenhout (Bruxelles), D.R. Hughes (London) and H. Lüneburg (Kaiserslautern). One of the mainstreams of the conference was the theory of buildings in the sense of J. Tits (e.g. generalized quadrangles or hexagons, polar spaces) and the axiomatisation of these geometries. Many lectures treated relationships to combinatorics, design theory, various aspects of group theory, algebraic geometry, coding theory, graph theory and number theory. Prof. Hering reported on the discovery of an interesting new class of finite projective planes, and Prof. Ott studied finite geometries with the help of concepts inspired by group representation theory.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Vortragsauszüge

E. Bannai, On spherical t-designs which have transitive automorphism groups.

The following question is discussed: Question. For large t, can one find a spherical t-design X in S^d on which a finite subgroup G of O(d+1) (real orthogonal group in R^{d+1}) acts transitively, ($d \geq 2$)? Theorem. Let $d \geq 2$. If $G \subseteq O(d+1)$ acts transitively on a spherical t-design S, then p_i/G remains irreducible for all $i=0,1,\ldots,s=[(1/2)[t/2]]$. Remark. No example of G such that the above conclusion of theorem is true is known for $s \geq 6$. Also, by assuming the classification of finite simple groups, it can be shown that there are no such G for large s. So the answer to the above question is negative.

T. Beth, Application of geometries.

Apart from the well-known connections between coding theory and geometry, two recent applications of geometric structures to practical problems of digital signal-processing are discussed: (a) Finite Radon-transformations, arising from incidence matrices of suitable geometries D, are used to provide a fast and stable inversion algorithm for computer tomography. The decoding procedure can be performed fast by parallel computation in the group-ring GF(p)[G] of an automorphism group G of D. (b) Geometric configurations in PG(n,2) are used for the design of non-linear pseudonoise generators of large complexity. The generatores are sucessfully implemented as key-stream cipher generatores for cryptographic purposes.

A. Beutelspacher, Finite h-semiaffine planes.

An h-semiaffine plane is a linear space S with the property that through any point outside a line L there are h or h-1 lines disjoint to L. By the work of Dembowski (1962) and Oehler (1975) all finite 1- and 2-semiplanes are known. One can prove the following: Theorem 1. Let S be a finite 3-semiaffine plane. Suppose that there are two disjoint parallel classes of "long" lines. Then S is a projective plane from which a triangle is removed. Theorem 2. For any h 4 there are at most finitely many finite h-semiaffine planes. (This work was done together with J. Meinhardt.)

A. Bichara, <u>On the independence of the axioms in Grassmann spaces.</u> We prove that the axioms characterizing the Grassmann spaces are independent.

A.E. Brouwer, <u>Near polygons.</u>
For the definition of a near polygon see Shult and Yanushka (Geom., Dedicata 9 (1980) 1-72). Theorem. A regular near polygon with thick lines and quads (i.e., with s>1 and t_2>0) does contain sub-near polygons: any two points at distance i determine a unique geodetically closed sub-near 2i-gon. Theorem 2. A regular near octagon satisfies (at least) one of the following: (i) s=1; (ii) t_2=0; (iii) t_3=1; (iv) it is a dual polar space. The known near polygons are usually unique, the unknown one's usually do not exist.

F. Buekenhout, <u>Some geometries for the Harada-Norton Group.</u>
We discuss a geometry for the prime p=2, having the intersection property and the diagram

$$\underset{3}{\bullet}\!\!-\!\!-\!\!-\!\!\underset{5}{\bullet}\overset{(4,5,6)^*}{-\!\!-\!\!-\!\!-\!\!-}\underset{4}{\bullet}$$

$$2^{1+8}(A_5 \times A_5)^2 \qquad\qquad 2^6.O_6^-(2)$$

The points are the central involutions of HN. Lines have 3 points and planes-quads have 27 points. The residues of type $(4,5,6)^*$ have 72 points each on 4 lines and 60 lines each on 5 points. In the incidence graph of the latter rank 2 geometries, the girth is 2x4, the greatest distance from a point (resp. lines) is 6 (resp. 5). Geometries for the primes 3 and 5 are also discussed.

P.J. Cameron, <u>Some questions and results about permutation groups.</u> The talk discussed some recent results and open problem in asymptotic permutation group theory, some of which assume the classification of finite simple groups. (i) For almost all n, the only primitive groups of degree n are the symmetric and alternating groups (Cameron, Neumann, Teague). (ii) With known exception, a primitive group of degree n has small order. If no large alternating groups or classical groups of large rank occur as composition factors, the order is polynomially bounded (Babai, Cameron, Palfy). (iii) There are only finitely many distance-transitive graphs of given valency greater than 2. (iv) There are inequalities connecting minimal degree or minimal base results asserting that if a graph, Steiner system, etc. has sufficiently many automorphisms, then it is "known" in some sense.

A.M. Cohen, Some point-line geometries of Tits buildings.
Let (P,B) be an incidence system. The following result both strenghtens and generalizes a theorem of Cooperstein:
Theorem. Let (P,B) be an incidence system where lines have length >2 satisfying: (1) $X \in P$, $y \in B$, $|X^{\perp} \cap y| > 1 \Rightarrow y \subseteq X^{\perp}$, (2) the collinearity graph of (P,B) is connected but not complete, (3) $X, Y \in P$, $d(X,Y) = 2 \Rightarrow \{X,Y\}^{\perp}$ is a non-degenerate generalized quadrangle, (4) $X \in P$, $y \in B$, $X^{\perp} \cap y = \emptyset$, $|X^{\perp} \cap y^{\perp}| > 0 \Rightarrow X^{\perp} \cap y^{\perp} \in B$. Suppose furthermore that all complete subspaces have finite rank. Then one of the following holds: (i) (P,B) is a non-degenerate polar space of rank 3, (ii) there are $n \geq 4$, $d \leq \frac{n+1}{2}$ and a division ring F such that $(P,B) \cong A_{n,d}(F)$, the space of (d-1)-subspaces of PG(n,F), (iii) there are $d \geq 5$, an infinite division ring F and an involutory automorphism σ of $A_{2d-1,d}(F)$ induced by a polarity of PG(2d-1,F) of Witt index $\leq d-5$, such that $(P,B) \cong A_{2d-1,d}/\langle\sigma\rangle$.
Also it is shown how this theorem fits into a program of classifying distance-transitive and root groups geometries of groups of exceptional Lie type. For Cooperstein's original theorem and notation, see Geometriae Dedicata 6 (1977), 205-258.

B.N. Cooperstein, A geometric characterization of a graph related to $_{2n}^{+}(q)$, m > 4.
Let V be an orthogonal space over a field F with dimension $2m \geq 8$

and assume V has maximal Witt index. Let m be the maximal (totally) singular subspaces of V. Define a graph $\hat{\Gamma}$ on m as follows: for M, N ∈ m, {M,N} ∈ $\hat{\Gamma}$ if and only if $\dim_F M/M\cap N = 2$. Under this relation m has two connected components. Let P be such a component and denote by Γ the restriction of $\hat{\Gamma}$ to P. Using the usual construction we get a gamma space with thick lines from the graph (P,Γ). We obtain a characterization of this gamma space when F is finite in terms of axioms on points and lines. This extends part of theorem B in A characterization of some incidence-structures. Progress on extending the result to infinite F and an application will be discussed.

Frank de Clerck, Embedding of triangular copolar spaces.
A(O,α)-geometry (α > 1) is a connected incidence structure S = (P,B,I) satisfying (a) two distinct points are incident with at most one line, (b) if a point X and a line are not incident, then there are O or α (α > 1) points which are collinear with X and incident with the line, (c) each line is incident with at least two points, and each point is incidentwith at least two lines. We determine all (O,α)-geometries with q+1 points on a line which are embedded in PG(n,q), n > 3, and q > 2. As a particular case all semi-partial geometries with parameters s = q,t,α(> 1) and μ, embeddable in PG(n,q), q ≠ 2, are obtained. If q = 2, then the triangular geometries (associated with the triangular graphs) are examples. We give some unusual embeddings of these geometries, in particular T(7) in PG(4,2) and T(9) in PG(5,2). The triangular geometries are the only (O,2)-geometries which can have unusual embeddings.

Anne Delandtsheer, Finite linear spaces with metrically regular incidence graphs.
We consider incidence graphs of finite linear spaces in which for a certain 6-tuple (i,j,k,l,m,n) of distances, there exists an integer p^{ijk}_{lmn} such that for any triple (x,y,z) of verices with d(x,y) = i, d(y,z) = j, d(z,x) = k, the number of vertices at distance 1 (resp. m,n) from x (resp. y,z) is equal to p^{ijk}_{lmn}. We characterize the finite linear spaces satisfying some of these conditions.

O. Domenico, A characteristic property of the Grassmann manifold representing the lines of a projective space.
Given any projective space P, dim P $\geq$ 3, the partial line space G(P) = (S,R) can be considered. The points of G(P) are the pencils of lines of P. G(P) is called the Grassmann-space of P. A star of lines (i.e. the set of all lines of P through a point) is a maximal subspace of G(P). The family Σ of stars is a covering of S with the following property (α):
$\forall T \in \Sigma$ and $\forall p \in S-T$ each element of Σ through p meets T in a single point. The points are on a line of T, locus of points in T collinear with p. In a joint work with Melaue, I proved the following: Theorem. Let (S,R) be a proper partial line space whose lines are not maximal subspaces. If (S,R) has a covering Σ of maximal subspaces verifying (α), then there exists a projective space P such that (S,R) is isomorphic to G(P).

Jean Doyen, A characterization of conference matrices.
A finite graph G is said to have property $P^{\geq t}_{m,n}$ (resp. $P^{t}_{m,n}$) if for every sequence of m+n vertices of G, there are at least t (resp. exactly t) other vertices adjacent to the first m vertices and non adjacent to the last n vertices. It is known that, for any given m,n and t, almost all graphs have property $P^{\geq t}_{m,n}$. G has property $P^{1}_{2,0}$ iff G is a friendship graph (Erdös, Renyi and Sos, 1967). G has property $P^{t}_{2,0}$ ($t \geq 2$) iff G is a strongly regular graph with $\lambda = \mu = t$ (Bose and Shrikhande, 1970). G has property $P^{t}_{m,0}$ ($m \geq 3$) iff $G = K_{m+t}$ (Carstens and Kruse, 1977). Theorem: There is a graph having property $P^{t}_{m,n}$ ($m,n \geq 1$) iff m = n = 1. A graph G has property $P^{t}_{1,1}$ iff G is a conference graph, that is a (4t+1, 2t, t-1, t) strongly regular graph.

D.A. Foulser, Do irreducible SL(3,q)-modules support spreads?
A report on joint work in progress with G. Mason and M. Walker. If $q = p^n$ and p = 2, then Mason proved the answer is "no" (Pullman Conference, 1981). If p > 2, our answer is incomplete although much geometrical and group theoretical structure is

available. For example, there is an elementary abelian group Q of order q^2 whose fixed-point space F(Q) is a subplane such that dim V = $(\dim F(Q))p^{fd}$, where f ≥ 1 and d ≤ p-1 (where V is an irreducible SL(3,q)-module supporting a spread whose kernel is GF(q)). Let V = $V_1^\theta \times V_2^\theta \times \ldots \times V_n^\theta$ where V_i is a basic module and $< \theta >$ = Aut(GF(q)), then at least one V_i is the basic Steinberg module St. As a partial result, one obtains: Proposition. If 3 ∤ p-1 and 2|n, then V does not support a spread. Proof. A certain element P, P = 2(p-1), has 2(p-1) distinct eigenvalues on St and hence on V. Moreover on V, the eigenvalues are subplanes of equal dimension. This implies the number of eigenvalues, 2(p-1), divides p^{fd}, a contradiction.

M.J. Ganley, Weak nucleus semifields.
A weak nucleus semifield (w.n.s.) is a semifield S of order q^2 having a subfield F = GF(q) s.t. x(yz) = (xy)z, whenever 2 of x, y, z ∈ F. This idea was first introduced by Knuth in 1965. Such semifields give rise to projective planes of Lenz-Barloti class V.1. We begin by considering commutative examples having F as the middle nucleus of S, and show that there exist exactly two infinite families when q is odd (where "infinite family" is defined in a special way) and no proper example at all when q is even. These examples (with q odd) can be combined to produce other w.n.s. and we describe all infinite families having F central in S. Finally, we explain how to construct any w.n.s. and give some further examples. All of the examples given give rise to new families of projective planes. We also show how the original examples of Knuth arise in a very natural way.

B. Ganter, Some new perfect codes.
We give examples of perfect binary (15,11)-codes which are not equivalent to previously known codes with these parameters. The main tool is the Jacobian matrix of a code, which generalizes the parity check matrix of a linear code. (Joint work with H. Bauer and F. Hergert.)

Dina Ghinelli Smit, Nonexistence Theorems for automorphism group of divisible designs.

Using the Hasse-Minowski theory, we have proved a nonexistence result for standard automorphism groups of point-divisible designs (see [2] and [3]). Here we show how, if the design is also block-divisible (or simply divisible) the Bose-Connor theorem (see [1]) and the above mentioned result can be improved. If the automorphism group has odd order, yet another improvement is given, using techniques, due to E. Lander (see [4]), derived from coding theory and modular representation theory. References.

[1] Bose and Connor, Combinatorial properties of group divisible incomplete block designs, Ann. Math. Statist., 23 (1952), 367-383. [2] D. Ghinelli Smit, Automorphisms and generalized incidence matrices of point-divisible designs, Proc. of the Intern. Conf. on Combinatorial Geometry and its applications, Annals of Discr. Math., 1982). [3] D. Ghinelli Smit, Nonexistence theorems for automorphism groups of divisible square designs (Ph. D. Thesis, University of London, to be submitted). [4] E.S. Lander, Topics in algebraic coding theory (Ph.D. Thesis, University of Oxford, October 1980).

T. Grundhöfer, The groups of projectivities of finite planes.

Assuming the classification of finite simple groups to be complete, one can prove the following: Theorem. Let Π be the group of projectivities of a finite non-desarguesian projective plane of order n. Then $\Pi = A_{n+1}$ or $\Pi = S_{n+1}$, or $n = 23$ and $\Pi = M_{24}$. The last exceptional case probably does not occur. Work of Prof. Hering gives together with the classification of simple groups a complete list of all finite linear groups acting transitively on the non-zero vectors of a vector space. Applying this we obtain: Theorem. Let Π^{aff} be the group of affine projectivities (= products of parallel projections) of a translation plane of order q^n with kernel GF(q). Then $ASL(n,q) \leq \Pi^{aff} \leq AGL(n,q)$. Apart from a finite number of groups, one has to exclude the possibility that Π^{aff} is contained in a symplectic group.

C. Hering, <u>On the new projective plane of Figueroa.</u>
We define a proper projective plane to be a projective plane whose automorphism group does not fix any point or line. Up until recently only 2 types of finite proper projective planes were known: the classical planes and the Hughes planes discovered by Hughes in 1957. R. Figueroa has constructed a third class. A geometric existence proof for these new planes was presented.

J.W.P. Hirschfeld, <u>Finite semi-linear groups.</u>
A comprehensive table of orders of finite semi-linear groups DX(n,q) is considered for D = I, S, S', G, Γ, ΓS, P, PS, PS', PG, PΓ, PΓS and X = L, O, O_+, O_-, U, Sp, Ps, Ps^*. The invariants of the respective X are PG(n-1,q), the parabolic quadric $\mathcal{P}_{n-1}$, the hyperbolic quadric $\mathcal{H}_{n-1}$, the elliptic quadric $\mathcal{E}_{n-1}$, the Hermitian variety $\mathcal{U}_{n-1}$, the symplectic polarity and in the last two cases the pseudo polarity. In the first six cases of D, the groups comprise linear transformations of the vector space and, in the latter six, the groups comprise projectivities of the projective space.

D.R. Hughes, <u>Semi-symmetric 3-designs: Hadamard case.</u>
A semi-symmetric 3-design (ss3D) for $\lambda + 1$, $\lambda > 0$, is a connected structure in which any 3 points are on 0 or $\lambda + 1$ common blocks and any 3 blocks are on 0 or $\lambda + 1$ points. If s is an ss3D for $\lambda + 1$, then s_y, for any block y, is an extended symmetric design. The case when s_y is a 3-(22,6,1) has been studied. If s_y is a Hadamard 3-design, we show that: (1) if s_y is not a PG(n,2), there exists exactly one ss3D associated with each Hadamard 3-design of the appropriate parameters; (2) if s_y is a PG(n,2), then there exist exactly two ss3D's s. (This classifies, among other things, the geometries belonging to $\underset{1}{\circ}\overset{\subset}{—}\underset{2}{\circ}—\circ \cdots \circ—\underset{n-1}{\circ}\overset{\supset}{—}\underset{n}{\circ}$, $n \geq 4$, except for a finite number of possibilities associated with a projective plane of order 10.)

Th. Ihringer, On linear congruence class geometries.

According to Wille a congruence class geometry arises from a (universal) algebra essentially by taking the elements of the base set of the algebra as points and the congruence classes as subspaces of the geometry. Such a geometry is called linear if every of its lines is specified uniquely by any two of its points.

Using results of Pasini and Wille on linear congruence class geometries and of André and Biliotti on translation structures, all nonsimple finite algebras (with at least one binary admissible operation) having a linear congruence class geometry can be determined. In particular, it can be shown that the congruence class geometry of each such algebra is affine and desarguesian. Much less is known about infinite linear congruence class geometries. But it can be proved that every nonsimple groupoid with neutral element and linear congruence class geometry is an elementary abelian group. Thus one is led to suspect that linearity has very strong consequences in the infinite case as well.

V. Jha, Groups of Baer collineations.

A collineation group G acting on a projective plane Π of order n, is a B-group if every non-trivial element of G is a Baer collinear and $(|G|,n) = 1$. Theorem. Suppose G is a B-group of a translation plane Π of order n. Then G is planar, ie. Fix(G) is a subplane, which we denote by Π_G. Moreover one of the following cases must occur: (i) Π_G is a Baer subplane and G is cyclic, (ii) G is an elementary abelian 2-group and Π_G is a plane of order $n^{1/G}$, (iii) Π_G is a plane of order $n^{1/4^G}$ and G is A_4, A_5, or contains a cyclic normal subgroup T such that $G/T = Z_2$ or Z_2+Z_2. (N.B. It is not clear whether in fact the above possibilities all occur; in particular it is not known if $G = A_5$ is possible.) Part (ii) corresponds to a theorem of Ostrom and we show that it is valid even for cartesian group planes, ie., those which are (P,y)-transitive for some flag.

D. Jungnickel (A joint work with S.S. Sane), On extension of nets.
An (s,r;u)-net is just an affine design S_r $(1,su;s^2u)$. It is well-known that $r\leq(s^2u-1)/(s-1)$ with equality for 2-designs. An (a,su;u)-net with affine dual (i.e. the dual is a net with the same parameters) is called symmetric. We propose the following conjecture: *Every affine 2-design with $s\neq2$ contains a symmetric (s,u)-net. This would imply that the parameters of affine 2-designs with $s\neq2$ are precisely the pairs $(s,u=s^d)$ where s is the order of an affine plane. (Replacing this by "s is a prime power" we obtain a well-known conjecture of S.S. Shrinkhande.) A disproof would be interesting too , as it would require a new construction technique for affine 2-designs. We also construct maximal nets of small deficiency and consider the completion problem. (To appear in Pacific J. Math.)

Vera Matejkova, Bohumil Bydzovsky, Vladimir Mahel, Karel Harlicek, Über einige Konfigurationseigenschaften von Punktbahnen.
Kurz vor dem zweiten Weltkrieg veröffentlichte Prof. Bohumil Bydzovsky eine Arbeit über eine ebene Konfiguration $(12_4,16_3)$. Diese Thematik interessierte ihn ständig und einige seiner Schüler setzten in diesem Gebiet die Arbeit fort. So fand Dozent Vladimir Mahel, daß die Bahn des Punktes der projektiven Ebene über R in einer Gruppe, die durch zwölf quadratische Transformationen und zwölf Kollineationen gebildet wurde, im allgemeinen Fall eine Konfiguration $(24_3,18_4)$ von Punkten und Geraden ist und Prof. Karel Harlicek identifizierte in derselben Ebene die Punktbahn eines allgemein liegenden Punktes in einer tetraedrischen Gruppe von Kollineationen als Konfiguration $(12_2,3_8)$ von Punkten und Kegelschnitten.
In der projektiven Ebene über C führt ähnlich die oktaedrische Gruppe von Kollineationen zu Konfiguration $(24_5,15_8)$ von Punkten und Kegelschnitten, im speziellen Fall dann zu Konfigurationen $(24_2,12_4)$ und $(24_3,18_4)$ von Punkten und Geraden. In der endlichen projektiven Ebene Π_8 über GF(8) ist die Punktbahn in der oktaedrischen Gruppe von Kollineationen im allgemeinen Fall die Konfiguration $(24_3,18_4)$ von Punkten und Geraden, $(24_2,6_8)$ von Punkten

und Ovalen und $(24_5, 15_8)$ von Punkten und Kegelschnitten. Spezielle Fälle führen zur Konfiguration $(12_2, 6_4)$ von Punkten und Geraden und $(12_2, 3_8)$ von Punkten und Kegelschnitten. Die oktaedrische Gruppe von Kollinationen enthält überdies rechtsgliedrige Untergruppen, die es ermöglichen, aus den Punktbahnen Ovale - lauter Kegelschnitte - zusammenzusetzen, und aus diesen Ovalen weitere Ovale, die keine Kegelschnitte sind, zu konstruieren. Ähnliche Konstruktionen lassen sich auch in den Ebenen Π_n, $n < 8$ realisieren.

F. Mazzocca, A characterization of the Dilworth truncations of the combinatorial geometries.

Let R be a non-empty finite set, $G = G(R)$ a combinatorial geometry on R and r the rank function of G. A Dilworth truncation of G is a family S of subsets of R, called blocks, such that: (1) $B_1, B_2 \in S \Rightarrow B_1 \cap B_2 = \emptyset$ and every point of R is contained in at least two different blocks such that: (2) $B \in S$ and $X \subseteq B \Rightarrow$ the closure $\overline{X}$ of X is connected; (3) Let B,B' be different blocks and G a connected flat of G. If the intersection of G with B and B' is not empty, then G contains $B \cap B'$. Let $G' = G'(R)$ be a combinatorial geometry, S' a Dilworth structure of G' and $J: G \to G'$ an isomorphism. The Dilworth structures S and S' will be called equivalent, $S \cong S'$, if $B \in S$ and $B' \in S' \Rightarrow J(B) \in S'$ and $J^{-1}(B') \in S$. Example: Let $\Gamma = \Gamma(P)$ be a combinatorial geometry of rank $r(P) = n + 1 > 1$, R the family of its lines and $T^d(\Gamma) = T^d(\Gamma)[R]$ the Dilworth truncation of Γ. For every point $p \in P$, let B(p) denote the family of lines of Γ through p and let $D(\Gamma)$ be the family of B(p), $p \in P$. We prove that: The family $D(\Gamma)$ is a Dilworth structure of $T^d(\Gamma)$. Now we have the following main result: Theorem. A connected combinatorial geometry $G = G(R)$ is the Dilworth truncation of a geometry if and only if G has a Dilworth structure. However, for every Dilworth structure S of G, there exists a unique geometry Γ such that $G = T^d(\Gamma)$ and $S \cong D(\Gamma)$.

M.S. Montakhab, Embedding theorems.

Let P be a vertex of a multigraph G and a (P),b(P),k(P),1_m(P) and α(P) be given integers, associated to P (for the details see the

Ph. d thesis by me, Westfield college, The university of London, 1982). We prove, if for every vertex P of G, k(P) is large enough in relation to other parameters, then the multigraph has a certain regularity of structure. Claws (P,S) of multiplicity exceeding a(P) do not exist and sufficiently largecliques (called grand cliques) occur in a certain pattern. Then we also prove that if r is large enough, then (i) a $Q_{r,\lambda,c}$-design is embeddable in a unique divisible semi-symmetric design for (c,[λ]); (ii) a non-trivial r-regular linear space with at most r^2-r+1 blocks can be embedded in a unique projective plane of order r-1. The lower bound for r in (ii) can be reduced in some special cases: (iii) A Pseudo-Complement of a Quadrilateral of order n in which n > 17, is embeddable in a unique projective plane of order n; (iv) A non-trivial r-regular linear space with v n^2-e, $o \leq e \leq n$, points and at most $r^2 - r + 1$ blocks is embeddable in a unique projective plane of order n if $n \geq 3e + 1$. (v) If n >, max{e(e+1)/2,3e} and n is not the order of a finite projective plane, then every r-regular linear space with at least $n^2 - e$ points is trivial.

Arnold Neumaier, Pseudo classical distance.

A distance regular graph is pseudo classical if its intersection array is given by

$$b_i = ([{d \atop 1}]-[{i \atop 1}])(\beta-\alpha[{i \atop 1}]),$$

$$c_i = [{i \atop 1}](1+\alpha[{i-1 \atop 1}]),$$

where d is the diameter, and $[{i \atop 1}] = 1+b+...+b^{i-1}$ for a suitable basis b.

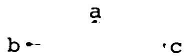
Pseudo classical graphs give rise to P- and Q-polynomial association schemes. Johnson schemes, Haing schemes, and all distance regular graphs belonging to classical groups or their parabolic subgroups belong to the family of pseudo classical graphs. It is conjectured that all pseudo-classical graphs with diameter ≥ 3 are already known.

U. Ott, On the Steinberg module of rank 2 geometries.

Die Definition des Steinberg Moduls einer Geometrie wird gegeben. Für Geometrien vom Rang 2 wird über einige Anwendungen berichtet.

A. Pasini, A Bruck-Ryser type theorem on finite cyclic projective geometries of rank 3.

Let Γ be a finite geometry in the diagram:



(Γ is meant to be pure and we assume (18) on Γ.) Let us assume that every projective plane on the edges of the diagram is non-degenerate. Then all these projective spaces have the same order, say n. We show that there is a positive integer N such that for every type i (i=a,b,c) there are just N i-verieties, and $N \geq f(n)$, where $f(n) = (n(n+1)^2+n(n+1)+1$. The equality $N = f(n)$ holds iff the geometry got from Γ by deleting the i-verieties (for some i = a,b,c) is a projective plane of order n(n+1). It is natural to conjecture that $N > f(n)$ if $n > 1$. But I am not able to prove this conjecture. Nevertheless, I proved that if $n > 1$ and $N = f(n)$, then one of the two following cases holds: 1) $n \equiv 1(8)$ and every odd prime p which divides n to an odd power satisfies $p \equiv 1(4)$ and 2 is a p-adic square, 2) $n \equiv 0,3(4)$ and for every odd prime p which divides n to an odd power, either $p \equiv 1(4)$ and 2 is a p-adic square, or $p \not\equiv 1(4)$ and 2 is not a p-adic square.

S.E. Payne, Subquadrangles of Kantor's Quadrangles in characteristic 2.

Kantor's generalized K(q) of order (q,q^2), $q \equiv 2(\text{mod } 3)$, have many subquadrangles of oder q when q is even. The subquadrangles are all isomorphic to the dual of the usual translation generalized quadrangle associated with the ovoidal permutation $\theta: t \to t^4$ and denoted s_θ. This embedding of the self-polar s_θ in K(q) yields several interesting observations about ovoids and spreads of s_θ, and shows that in some sense certain characterization theorems of Thas are the best possible.

If $x^2 + x + k$ is irreducible over $F = GF(q)$ and if γ is an automorphism of maximal order of F for which θ: $t \to a^2t^{(\gamma-1)/\gamma} + at^{1/2} + kt^{1/\gamma}$ is an ovoidal permutation for each $a \in F$, then there arises a Gθ of order (q,q^2) having subquadrangles of order (q,q) dual to those arising from the ovoidal map θ^{-1}. γ = 2 gives the classical examples. γ = 4 gives Kantor's examples.

N. Percsy, On the grassmannian geometry of polar spaces.
Let T be a polar space of rank $n \leq \infty$ and $d \leq n-1$ an integer. The grassmannian geometry T_d of T is the incidence structure whose points are the d-dimensional singular subspaces of T, and whose lines are the sets of d-dimensional singular subspaces contained in a given (d+1)-dimensional one and intersecting in a (d-1)-dimensional one. We give an axiomatic characterization of these T_d, for $d \leq n-4$, in the spirit of B. Cooperstein, F. Buekenhout and A.M. Cohen's work on building geometry.

D. Ray-Chaudhuri, Geometrical results proved at Ohio State University in recent years.
A finite Möbius plane M of order q after deleting a circle and its points gives rise to a 3-design with parameters $v = q^2-q$, $\lambda_o = q^3+q-1$, $\lambda_1 = q^2+q$, $\lambda_2 = q+1$ and $\lambda_3 = 1$ and set of block sizes {q+1,q,q-1}. Any 3-design with such parameters is called PBRD(q). A block e is said to be r-tangent to a block e' at X if $e \cap e' = \{X\}$ and there exists a block e" which is tangent to e at X and secant to e'. A PBRD(q) is said to satisfy the r-tangency condition if given a circle B, points X and Y not in B there exists at most one circle containing X and Y and r-tangent to B. Theorem. A PBRD(q) can be uniquely embedded into a Möbius plane if and only if it satisfies the r-tangency condition. Let Γ be a finite affine space of dimension n and d be any integer, $1 \leq d < n-1$. Let H_i be the set of i-dimensional flats of Γ and Π be the incidence structure $(W_d, W_{d+1}, \subseteq)$. A theorem is proved giving a geometrical characterization of Π by S axioms. All bit one of the axioms are proved to be essential in a certain sense.

J. Saxl, The sims conjecture.
In a joint work with Cameron, Praeger and Seitz, we use the classification of finite simple groups to settle the conjecture in the title: Theorem. There is an integral function f such that whenever G is a finite primitive permutation group with a subdegree d, then $|G_\alpha| \leq f(d)$. As a corollary we get Theorem. There are only finitely many distance transitive graphs of any given valency $d > 2$.

A. Sprague, Generalized 3-nets and extended dual affine planes.

Geometries admitting diagram $\circ\!\!\frac{\text{net}}{\quad}\!\!\circ\!\!\frac{L^t}{\quad}\!\!\circ$ are examined. Examples of such geometries are plentiful, some like 3-nets, embeddable in affine 3-spaces. It may be shown that any geometry admitting this diagram admits a parallelism on lines, and also on planes A classification or partial classification of several classes is achieved.

Giuseppe Tallini, Partial spreads of Grassmannian manifolds and quadrics in PG(r,q).

We give results on maximal partial spreads of Grassmann manifolds and quadrics in PG(r,q).

Maria Tallini Scafati, Two characters k-sets with respect to a singular space in PG(r,q).

Let $\mathcal{R}$ be a family of lines in PG(r,q). We say that a k-set K is of type (m,n) with respect to $\mathcal{R}$, if every line of $\mathcal{R}$ meets K either in m or in n points and m-secants and n-secants in $\mathcal{R}$ do exist. Two characters k-sets of type (m,n) are studied with respect to the families of all lines in PG(r,q) except those of a hyperplane Π which will be called singular space. Moreover K is studied with respect to all lines in PG(r,q) not passing through a point P.

J.A. Thas, a) Elementary proofs of two fundamental theorems of B. Segre without using the Hasse-Weil theorem. b) Remarks on the classical generalized hexagon H(q) of Tits.

a) If K is a complete k-arc with $k \leq q+1$ in PG(2,q), q even, then B. Segre proved that $k \leq q-\sqrt{q}+1$. For that purpose he showed that the number N of simple points on a plane algebraic curve C_n of order n in PG(2,q) with no regular linear components is less than $n(q+2-n)$ if $\sqrt{q} > n-1$. Here he used the deep theorem of Hasse-Weil on the number of points lying on an algebraic curve of order n and genus g. Here we prove that the number R of real points of a plane algebraic curve C_n of order n with no regular linear components satisfies $R \leq qn-q+n$, which is better than Segre's bound since N ≤ R. Moreover the proof is a few lines just using the classical theorem of Bezout. Further, Segre's result on complete k-arcs for q even readily follows from that new bound on N. b) The classical generalized hexagon H(q) of Tits can be obtained as the "intersection" of 7 linear line complexes of PG(6,q). From thes we deduce a "geometrically" and short proof for the fact that the structure K(q) of Kantor arising from the hexagon H(q), q ≡ 2 (mod 3), is indeed a generalized quadrangle of order (q,q^2).

Patricia Vanden Cruyce, Locally $\overline{T(n)}$ graphs.
For any n ≥ 2, we denote by $\overline{T(n)}$ the graph whose vertices are the unordered pairs of elements of the set E = {1,2,...,n}, two vertices being adjacent if and only if the corresponding pairs of E are disjoint. A graph Γ is locally $\overline{T(n)}$ (for some given n) if for each vertex x of Γ, the graph induced by Γ on the neighbourhood of x is isomorphic to $\overline{T(n)}$. J. Hall has proved that there are (up to isomorphism) exactly three connected locally $\overline{T(5)}$ graphs (note that $\overline{T(5)}$ is isomorphic to Petersen graph). We prove that if Γ is a connected locally $\overline{T(n)}$ graph with n ≥ 7 then $\Gamma \cong \overline{T(n+2)}$.

A.L. Wells, Universal projective embeddings.
Let G = (P,L) be a partial linear incidence structure, and f an injective map from the points of G to the points of a classical projective space P(W). We say that f is a projective embedding (or simply an embedding) when the image of a line of G under f is an entire line of P(W). Let u be an embedding of G in P(u). We say that u is universal when for every embedding f of G in a projective space P(W), there is a similar transformation ψ: u → W, determined by f up to a scalar multiple such that f = ψou. The fundamental theorem of projective geometry shows that any classical projective space is universally embedded. The work of Buekenhout, Dienst, Lefevre-Percsy and Tits shows that a finite polar geometry (P,L) of rank at least 2 of type SP(2n) (characteristic ≠ 2), $O^{\pm}(2n)$, O(2n+1) or u(n) embedded as one of these types is universally embedded. Our main theorem is that

the Exterior power space embeddings of $A_{n,d}$, and the spinor embeddings of $D_{n,n}$ and $B_{n,n}$ (in the last case, provided that the characteristic is odd) are universal projective.

M. Willems, Special Laguerre i-structures and optimal codes.
A special Laguerre i-structure of order n ($i \geq 1$) is an incidence structure $J = (P, B_1 \quad B_2 I)$ for which: (i) Each element of P is incident with one element of B_1. (ii) Each i-residual space of J (with respect to B_1) is a projective plane of order n minus one point. (iii) $B_2 \neq \emptyset$ and each element of B_2 is incident with at least i elements of P which are pairwise not incident with a common element of B_1. We prove some necessary conditions for the existence of special Laguerre i-structures, $i \geq 2$, of order n (resp. optimal (n+i+1,i+2)-codes of order n) and Laguerre i-structures, $i \geq 2$, of even order n (resp. optimal (n+i,i+2)-codes of even order n).

F. Zara, Some graphs related to polar spaces.
We study a class of simple graphs which satisfy the following two axioms: Let G be a graph with vertices set Ω. We let C be the set of cliques of G (complete maximal subgraphs of G).
A1 $\exists r \in N : \forall C \in \mathcal{C}$, then $|C| = r$; A2 $\exists t \in N: \forall C \in \mathcal{C}, \forall X \in \Omega - C$, then $|\Delta(X) \cap C| = t$, where $\Delta(X) = \{Y | Y \in \Omega, X \text{ and } Y \text{ are adjacent}\}$.
Examples: The graphs associated with finite polar spaces. If $\Phi \in \Omega$, we put $\Delta(\Phi) = \bigcap_{X \in \Phi} \Delta(X)$ and G the subgraph induced on $\Delta(\Phi)$.
The study of G for various Φ, enables to obtain some structure theorem on this class of graphs.

Berichterstatter: Th. Grundhöfer

Tagungsteilnehmer

Herrn
Prof. E. Bannai
Dept. of Mathematics
Ohio State University
Columbus/Ohio 43210 U.S.A.

Herrn
Prof. Dr. Th. Beth
Insitut f. Mathe. Maschinen
Martensstraße 3
8520 Erlangen

Herrn
Dr. A. Beutelspacher
FB Mathematik d. Universität
Saarstraße 21
6500 Mainz

Herrn
Prof. D. Bichara
Istituto di Matematica
Università di Roma
Rom/Italien

Herrn
Prof. A. Brouwer
Mathematical Centre
Krumslaan 413
1098 AL Amsterdam/Holland

Herrn
Prof. F. Buekenhout
Dépt. de Mathematique CP 216
Université de Bruxelle
Brüssel 1050/Belgien

Herrn
Dr. P. Cameron
Mathematical Institute
Saint Giles 24
Oxford/England

Herrn
Dr. F. de Clerck
Seminarie voor Hogere Metkunde
Rijksuniversiteit Gent
De Pintelaan - Gebouw 59
9000 Belgien/Gent

Herrn
Prof. A.M. Cohen
Mathematical Centre
Krumslaan 413
1098 AL Amsterdam/Holland

Herrn
Prof. B. Cooperstein
Dept. of Mathematics
University of California
Santa Cruz / Cal. 95060 U.S.A.

Herrn
Dr. P. van den Cruyce
Dépt. de Mathematique CP 216
Université de Bruxelle
Brüssel 1050/Belgien

Herrn
Dr. A. Delandtsheer
Dépt. de Mathematique CP 216
Université de Bruxelle
Brüssel 1050/Belgien

Herrn
Dr. J. Doyen
Dépt. de Mathematique CP 216
Université de Bruxelle
Brüssel 1050/Belgien

Herrn
Prof. D. Foulser
Dept. of Mathematics
UofICC
Chicago/Illinois

Herrn
Dr. M. Ganley
Dept. of Mathematics
The University
Glasgow Cr 12 SQW/Schottland

Herrn
Prof. Dr. B. Ganter
FB Mathematik der TH
Schloßgartenstraße 7
6100 Darmstadt

Herrn
Dr. Theo Grundhöfer
Math. Institut der Universität
Auf der Morgenstelle 10
7400 Tübingen

Herrn
Prof. Dr. C. Hering
Math. Institut der Universität
Auf der Morgenstelle 10
7400 Tübingen

Herrn
Dr. James Hirschfeld
Dept. of Mathematics
University of Sussex
Falmer
Brighton BN 1 9RH/England

Herrn
Prof. Dr. R. Hughes
Dept. of Mathematics
Westfield College
Kidderport Avenue
London NW3 7ST/England

Herrn
Dr. Th. Ihringer
FB Mathematik der TH
Schloßgartenstraße 7
6100 Darmstadt

Herrn
Dr. V. Iha
Dept. of Mathematics
The University
Glasgow Cr 12 SQW/Schottland

Herrn
Prof. Dr. D. Jungnickel
Math. Institut
Arndtstraße 2
6300 Gießen

Herrn
Prof. Dr. E. Köhler
Math. Seminar der Universität
Bundesstraße 55
2000 Hamburg 13

Herrn
Dr. C. Lefevre-Percsy

Dépt. de Mathematique CP 216
Université de Bruxelle

Brüssel 1050/Belgien

Herrn
Dr. A. Neumaier

FB Mathematik der Universität

Hebelstraße 27

7800 Freiburg

Herrn
Prof. Dr. D. Livingston

Dept. of Mathematics
University of Birmingham

Birmingham/England

Herrn
D. Olanda

Università di Napoli
Istituto Matematico

Napoli, 80134/Italien

Herrn
Prof. Dr. H. Lüneburg

Fachbereich Mathematik
Universität Kaiserslautern
Postfach 3049

6750 Kaiserslautern

Prof. Dr. U. Ott

Institut für Mathematik

Pockelstraße 14

3300 Braunschweig

Herrn
Dr. V. Matejkova

Elektrotechnicka Fakulta
katedra matematiky

Likova
1600 Prag 6/CSSR

Herrn
A. Pasini

Istituto di Matematica
Università di Siena

Siena/Italien

Herrn
F. Mazzocca

Università di Napoli
Istituto Matematico

Napoli, 80134/Italien

Herrn
Prof. S.E. Payne

Dept. of Mathematics
Miami University

Oxford/Ohio 450 456 U.S.A.

Herrn
M.S. Montakhab

231 West 18th Avenue

Columbus, Ohio 43210/ U.S.A.

Herrn
Dr. N. Percsy

Faculti des Sciences
Universiti de Mons

Avenue Maistrion
7000 Mons/Belgien

Herrn
Prof. D. Ray-Chaudhuri
Dept. of Mathematics
Ohio State University

Columbus/Ohio 43210 U.S.A.

Herrn
Prof. J.A. Thas

Seminarie voor Hogere Metkunde
Rijksuniversiteit Gent

De Pintelaan - Gebouw 59

9000 Belgien/Gent

Herrn
Dr. J. Saxl

Dept. of Mathematics
University of Cambridge

Cambridge/England

Herrn
K. Vedder

Mathematisches Institut

Arndtstraße 2

6300 Gießen

Herrn
Dr. A.P. Sprague

Dept. of Mathematics
Ohio State University

Columbus/Ohio 43210 U.S.A.

Herrn
A.L. Wells

Mathematical Institute

Saint Giles 24

Oxford/England

Herrn
D. Smit-Ghinnelli

Dept. of Mathematics
Westfield College
London NW 3 7 ST/England
Kidderport Avenue

Herrn
Prof. M. Willems

Dept. voor Wiskunde

Rijksuniversitair Centrum

Antwerpen/Belgien

Herrn
Prof. G. Tallini

Istituto di Matematica
Università di Roma

Rom/Italien

Herrn
Prof. F. Zara

U.E.R. de Mathêmatiques

33 rue Saint-Leu

80039 Amiens/Frankreich

Herrn
Prof. M. Tallini-Scafati

Istituto di Matematica
Università di Roma

Rom/Italien