MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 13/1982

Mathematische Stochastik

28.3. bis 3.4.1982

Die Tagung wurde geleitet von den Herren L. Arnold (Bremen) und H. Strasser (Bayreuth).

Die 45 Teilnehmer aus den USA, Indien, Japan und neun europäischen Ländern repräsentierten eine Vielzahl verschiedener Arbeitsrichtungen innerhalb der mathematischen Stochastik und ihrer Grenzgebiete, dies kam auch in der thematischen Vielfalt der 36 Vorträge zum Ausdruck. Besonders erfreulich war die Anwesenheit einer größeren Anzahl international führender Vertreter des Fachgebiets. Bedauert wurde, daß Herr Zabczyk (Warszawa) auf die persönliche Teilnahme an der Tagung verzichten mußte.

Neben Darstellungen speziellerer Resultate wurden einige längere Vorträge zu folgenden Problemkreisen gehalten: Schätzerkonstruktion vermittels Vapnik-Červonenkis-Klassen; Green- und Dirichlet-Räume von Markov-Prozessen und zufälligen Feldern; Martingale auf Mannigfaltigkeiten; Differentialtheoretische Ansätze in der asymptotischen Statistik; Zufällige Medien.

Besonderes Interesse fanden einerseits die asymptotische Entscheidungstheorie, andererseits die Theorie der Wahrscheinlichkeitsmaße auf lokal kompakten Gruppen und schließlich die 'neue' Verbindung zwischen der Differentialgeometrie und der Theorie der stochastischen Prozesse, dies spiegelte sich sowohl in der Anzahl der Vorträge dazu als auch in den informellen Diskussionen.

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Vortragsauszüge

M. AKAHIRA

Asymptotic deficiency of estimators under models with nuisance parameters

In many cases of statistical inference, there is often raised the problem of "model selection", that is, to specify the appropriate model for observed data. In typical situations we have observations which are assumed to be independently and identically distributed with a parameter θ to be estimated and also "shape" parameters. If we choose a "large" model, that is, with many shape parameters, the model will be more accurate, or it will include a distribution which is close to the "true" distribution. On the other hand, however, the presence of many nuisance parameters would increase the error of estimation of θ due to the errors of those estimated nuisance parameters. This problem can not be approached when we only consider the first order asymptotic efficiency, since the presence of nuisance parameters will not affect the asymptotic variance of estimators of θ,provided that the parameters are orthogonal. Hence we have to consider the second (or the third) order asymptotic efficiency and discuss the problem in terms of "asymptotic deficiency". And in this term we may consider the trade-off between "aacuracy" and "simplicity" of the model.

R. BERAN

Estimated sampling distributions: the bootstrap and competitors

Let $X_1, X_2, ..., X_n$ be i.i.d random variables with cdf F. Suppose the $\{\hat{T}_n = \hat{T}_n(X_1, X_2, ..., X_n);\ n \geq 1\}$ are real-valued statistics and the $\{T_n(F);\ n \geq 1\}$ are centering functionals such that the asymptotic distribution of $\{n^{1/2}(\hat{T}_n - T_n(F));\ n \geq 1\}$ is normal with mean zero. Let $H_n(x,F)$ be the exact cdf of $n^{1/2}(\hat{T}_n - T_n(F))$. The problem is to estimate $H_n(x,F)$ or functionals of $H_n(x,F)$ from the sample. Under regularity assumptions, it is shown that the bootstrap estimate $H_n(x,\hat{F}_n)$, where $\hat{F}_n$ is the sample cdf, is asymptotically minimax. On the other hand, the commonly used normal approximation with estimated variance ist not asymptotically minimax for $H_n(x,F)$ (except in special cases) because of bias. However, the estimated first-order Edgeworth expansion of $H_n(x,F)$ is, again, asymptotically minimax and is equivalent to the bootstrap estimate $H_n(x,\hat{F}_n)$ up to terms of order $n^{-1/2}$. The results for estimating functionals of $H_n(x,F)$ are similar.

L. BIRGÉ

Stability and instability of minimax risk for i.i.d. variables under perturbations

Consider the problem of estimating ϑ in Θ from n i.i.d. variables of law P. Suppose we define the minimax risk corresponding to this experiment by

$$R_n(\Theta) = \inf_T \sup_\vartheta E_\vartheta[nh^2(P_T, P_\vartheta)],$$

T being any estimate of ϑ and h Hellinger distance. If we enlarge the parameter space or make some small perturbations of it we get a new parameter space Θ with the property that for any ϑ' in $\bar{\Theta}$, there exists ϑ in Θ and $h(\vartheta,\vartheta') \leq \varepsilon$. How much will $R_n(\bar{\Theta})$ differ from $R_n(\Theta)$? We search for results of the type

$$R_n(\bar{\Theta}) \leq K\, R_n(\Theta)$$

with K not depending on n or $R_n(\Theta)$ but only on ε. We consider the cases of $\varepsilon = \frac{c}{n}$ and prove a stability result (K being a function of c only).

For larger perturbations $\varepsilon = cn^{-\frac{1}{2}} R_n(\Theta)$ the result is false in the general case, but it holds for a large subclass of models which are regular in the sense that their risk is determined by the dimensional properties of the parameter space (Θ,h) considered as a metric space.

E. B. DYNKIN

Dirichlet spaces and Green's spaces for Markov processes and Gaussian random fields

For symmetric Markov processes, a dual pair of Hilbert spaces: the Dirichlet space H and Green's space K are the most natural domain for the corresponding semi-group of operators T_t. The infinitesimal operator A is an isometry from H onto K and Green's operator $G=(-A)^{-1}$ is the inverse isometry of K onto H. Elements of H can be represented by fine functions i.e. by functions which are right continuous along almost all paths, and elements of K can be represented as measures or generalized measures. To every positive element $k \in K$, there corresponds a measure on the space of paths and an additive functional of the Markov process. Using these tools a general Dirichlet problem can be solved. The results can be extended to families of symmetric Markov processes and can be used for the investigation of symmetric fields associated with symmetric Markov processes (in particular, the so called free field considered in quantum fields theory).

K. D. ELWORTHY

Flows of stochastic dynamical systems: the functional analytic approach

Let $F_t(u)$ be a solution of the stochastic differential equation $dx_t = Y(x_t)\circ dB_t + A(x_t)dt$ on R^n, with B an m-dimensional Brownian motion, and initial point $x_o = u$. A 'flow' for the equation is a version of $F.(\cdot)$ which is jointly continuous in t and u. Following work by Malliavin there has been recent interest in the existence of flows and their properties. By lifting our equation to an equation on a suitable Hilbert space of functions we will show that many of the long technical details of the proofs can be subsumed once and for all in known results about Sobolev spaces.
After giving a simple example to clear away some misconceptions: in particular to show that it is not enough to simply add a coffin state to deal with explosions, we will prove the main results when the coefficients have compact support: existence, differentiability, equation for the inverse, backward equations. The general case (with smooth coefficients) is then deduced from these, as is Kunita's criterion for surjectivity of F_t.
The discussion is based on an expository article written with A. P. Carverhill.

B. FEREBEE

Asymptotic expansion of Brownian exit densities

Let p(t) be the density of the first-exit time of a Brownian motion over the moving boundary given by $x = f(t)$. Let $\lambda(t):=f(t)-tf'(t)$ be the intercept on the vertical axis of the tangent to the boundary at t. If, for a given t, the quantity $\Delta:=\frac{1}{2}(\frac{f(s)}{s} - \frac{f(t)}{t})/(\frac{1}{s} - \frac{1}{t})$ is "large enough" for $s\in(0,t)$, then p(t) is mainly determined by the behavior of f near t; in fact,

$$p(t) \sim \varphi(\frac{f(t)}{\sqrt{t}}) \{\frac{\lambda(t)}{t^{3/2}} - \sum_{n=1}^{\infty} c_n t^{n/2} m_n(\frac{\lambda(t)}{\sqrt{t}})\}.$$

Here $m_n(x):=(-1)^n \frac{d^n}{dx^n} \frac{1-\Phi(x)}{\varphi(x)}$ and the coefficients c_n depend only on the derivatives of f at t .

W. HAZOD

On stable measures

M. Sharpe (Trans. Amer. Math. Soc. 136 (1969) 51-65) introduced the concept of operator stable laws on R^d: Let $(\mu_t)_{t\geq o}$ be a continuous convolution semigroup of probability measures (c.c.s.) on R^d, let further $(\tau_t)_{t>o} \subseteq Aut(R^d)$ be of the form $\tau_t = t^B$, $B \in End(R^d)$, $Spect(B) \subseteq \{\lambda : Re\ \lambda > 0\}$, such that

(*) $\tau_t(\mu_s) = \mu_{t^\alpha s} * \varepsilon_{b(t)}$ for some fixed $\alpha > 0$,

$b(t) \in R^d$, $s,t > 0$. Then (μ_t) is operator stable.

We need the following fact: Let $(\mu_t)_{t\geq o}$ be a c.c.s. on a locally compact group, then for $\varphi \in \mathcal{D}(G)$ - the Schwartz-Bruhat-space - the generating functional $\langle A,\varphi\rangle := \frac{d}{dt} \langle\mu_t,\varphi\rangle$ is defined, and the correspondence $(\mu_t) \to A$ is 1-1.

(*) is equivalent to

(**) $\tau_t(A) = t^\alpha A + P(t)$, where $P(t)$ is a generating functional of a group of Point measures.

Defintion Let G be a locally compact group. Let (μ_t) be a c.c.s. with generating functional A and let $(\tau_t)_{t>o} \subseteq Aut(G)$ be a group, such that $\tau_t\tau_s = \tau_{ts}$, $t,s>0$ and $\tau_t x \xrightarrow[t\downarrow 0]{} e$, $x \in G$.

A resp. (μ_t) is stable if (**) holds.

For a class of connected, simply connected nilpotent Lie groups G (- which admit such automorphisms -) the generating functionals of stable measures can be completely described:

Theorem: Let G be as above, $\mathfrak{g}$ the tangent space at e (i.e. the Lie Algebra). Via the exponential map there is a 1-1-correspondence between generating functionals on G and on the vectorgroup $\mathfrak{g}$. To stable generating functionals on G there correspond operator-stable generating functionals on $\mathfrak{g}$ in the sense of M. Sharpe.

H. HEYER

Embedding of invariant probability measures

The theory of probability measures on the hyperbolic plane vieved as the homogeneous space $X = G/_K$ with $G := SL(2,R)$ and $K := SO(2,R)$ is based on a detailed analysis of the group G, the semigroup $M^1(K,G,K)$ of K-biinvariant probabilities on G and the theory of K-spherical functions on G. The fact that X is a Riemannian symmetric space of noncompact type makes it possible to solve the following central problems.

(1) The embedding of infinitely divisible (K-biinvariant) measures into continuous convolution semigroups.

(2) The central limit theorem for infinitesimal triangular arrays of measures.

(3) The transience property of nondegenerate continuous convolution semigroups.

This talk deals only with problem (1), but in a more general framework. The results obtained extend "classical" contributions to the embedding problem for infinitely divisible measures on an arbitrary weakly compact group (as discussed in chapter III of the author's monograph) to large classes of homogeneous spaces (including the euclidean spaces, the spheres and the real hyperbolic spaces).

A. JANSSEN

Infinitely divisible experiments and exponential families

We refer to three points:

1. Limit theorems for statistical experiments, infinitely divisible experiments, stable exp. ...,
2. Exponential families,
3. Asymptotically normal families.

Infinitely divisible experiments are weak limits of infinitesimal triangle systems of statistical experiments. Especially we consider weak limit points of

$$\left(X^n,\ \bigotimes_{i=1}^{n} \mathfrak{A},\ (P^{\otimes n}_{\delta_n \vartheta})_{\vartheta \in T}\right),\quad \delta_n \downarrow 0,$$ which we call stable (semistable).

A k-parametric exponential family on R^k is stable (infinitely divisible) if and only if the measure P_o is stable (infinitely divisible).

Finally, we give necessary and sufficient conditions for a limit experiment to be a Gauß-experiment.

These results can be applied to prove that under certain conditions the loglikelihood ratio process is asymptotically normal.

P. JEGANATHAN

Some remarks on Hâjek's local asymptotic minimax and admissibility results

In recent times there occur several estimation problems, especially in stochastic processes, e.g. in Galton-Watson super-critical branching processes, where the limit of the loglikelihood ratios is mixed normal. It was shown by Swenson (1980, Ph. D. thesis, Univ. Calif., Berkeley) and also by the present author that J. Hâjek's (1972) local asymptotic minimax and admissibility results can be extended to the fore-mentioned case also. The present purpose is to give an elementary proof of these and other related results in asymptotic theory.

W. KIRSCH

Selfadjointness of Schrödinger operators with stochastic potentials

We consider Schrödinger operators on $L^2(R^d)$ of the form $H_\omega = H_o + V_\omega$ where $H_o = -\Delta$ is the d dimensional Laplacian and V_ω is a stochastic potential; i.e. $(\omega,x) \rightarrow V_\omega(x)$ is a jointly measurable function on $\Omega \times R^d$, where (Ω,F,P) is a probability space. Such operators arise naturally in the quantum theory of disordered systems. We are interested in criteria which ensure that the operator H_ω is essentially selfadjoint on the domain $C_o^\infty(R^d)$, the space of infinitely differentiable functions with compact support. Physically speaking the essential selfadjointness means that the time evolution of the system described by H_ω is uniquely given by the action of H_ω on $C_o^\infty(R^d)$. Our main result is: Let C_o be an open set and denote by $\{x_i\}_{i\in I}$ an indexed set in R^d such that the sets $\overline{C}_o + x_i$ $(i \in I)$ cover R^d, then H_ω is essentially selfadjoint, if for some $p > \max(2,d/2)$ the moment of order k of the random variables $\int_{C_o+x_i} |V_\omega(x)|^p dx$ exists and is bounded by a constant C independent of i. Here k is a number depending on p and the dimension d. (joint paper with F. Martinelli)

A. KOZEK

Minimum Lévy distance estimation of a translation parameter

Let x_i, $1 \leq i \leq n$, be independent identically distributed random variables with a common distribution function F and let G be a smooth distribution function. We derive the limit distribution of $\sqrt{n}(\rho_L(F_n,G)-\rho_L(F,G))$, where F_n is the empirical distribution function based on $X_1,\dots,X_n$, and ρ_L is the Levy distance between distribution functions. If, moreover, F is continuous and $\{G_\Theta = G(\cdot-\Theta): \Theta \in R^1\}$ is a family of probability measures on R^1 with a translation parameter Θ, we obtain the limit distribution of $\sqrt{n}(\rho_L(F_n,G_{\Theta_n}) - \rho_L(F,G_{\bar{\Theta}}))$, $\sqrt{n}(\Theta_n-\bar{\Theta})$ and $\sqrt{n}\rho_L(G_{\Theta_n},G_{\bar{\Theta}})$, where Θ_n and $\bar{\Theta}$ denote the minimum Levy distance translation parameters for F_n and F, respectively. These results are compared with the corresponding ones for the Kolmogorov metric.

L. LECAM

Construction of estimates through Vapnik - Červonenkis classes

Let $\mathcal{E} = \{P_\theta:\theta\in\Theta\}$ be an experiment where each P_θ is a product measure $P_\theta = \underset{j}{X}\, p_{\theta_j}$. Metrize Θ through the sum H^2 of the square Hellinger distances on the components. The problem is to find estimates $\hat{\theta}_n$ such that $E_\theta\, H^2(\hat{\theta}_n,\theta)$ remains bounded by a function of the dimension of Θ for H. This has been done (by LeCam and then Birgé) using tests between Hellinger tubes. One can also attempt to do it by a minimum distance method using as a norm $||\lambda||_{\mathcal{S}} = \sup\{|\lambda(s)|: s\in\mathcal{S}\}$ where $\mathcal{S}$ is a class of measurable sets in the space that carries the p_{θ_j}. Properties of classes $\mathcal{S}$ leading to acceptable estimates are described as well as inequalities on quantities such as $\sqrt{n}\sup_{s\in\mathcal{S}}|\mu_n(s) - \bar{p}(s)|$ where μ_n is the empirical measure and where $\bar{p} = \frac{1}{n}\sum_j p_{\theta_j}$.

P. MAJOR

Dyson's hierarchical models and limit theorems in statistical physics

Our aim is to give a general overlook about Dyson's hierarchical model. It is investigated in order to understand the general behaviour of equilibrium states in statistical physics. We are interested in the behaviour of the model at critical temperature. We get limit theorems for sums of random variables with unorthodox norming. The limit measure must be the fixed point of a complicated transformation. There is always a Gaussian solution, but it is in certain cases unstable. In such cases a stable non-Gaussian solution can be found be means of perturbation theory. This is the interesting solution from physical point of view.

P. MALLIAVIN

Elliptic estimates in infinite dimension

Stochastic calculus of variation, stochastic flow, weak processes, estimates of laws. Application to non linear filtering and models of classical statistical mechanics.
References. Journal of Functional Analysis year 1981 and 1982.

E. MAMMEN

The information (measured by the deficiency distance) of additional observations

In the case of asymptotically Gaussian experiments we give an asymptotic lower bound for the information contained in additional observations as measured by the deficiency distance of LeCam. For exponential families we use Edgeworth expansions of the Bayes risks to study the asymptotic gain of information.

P. A. MEYER

Martingale convergence theorem in Riemannian manifolds

Given a continuous semimartingale X_t with values in a Riemannian manifold V one can define intrinsically 1) the property of X_t of being a martingale 2) the scalar increasing process $d\langle x,x\rangle_t = g_{ij} d\langle x^i,x^j\rangle_t$. Some propaganda is made for the following results:

W. DARLING: on the set $\{\langle x,x\rangle_\infty < \infty\}$, X_∞ a.s. exists in the one point compactification of V

W. A. ZHENG: on the set $\{X_\infty$ exists in V$\}$, $\langle x,x\rangle_\infty$ a.s. is finite.

D. W. MÜLLER

Factorizing the information contained in an experiment, conditionally on the observed value of a statistic

No matter which value t of a statistic T_n has been observed the loss of information, in comparison with the original data, will asymptotically $(n\to\infty)$ always be the same. This statement is interpreted and proved in the framework of "comparison of experiments". The loss of information is described by the conditional experiments $\{ \mathcal{L}_\theta(\text{data}|T_n=t) : \theta \in \Theta \}$. Under assumptions commonly accepted in asymptotic statistics these conditional experiments are shown to be all of the same "type", as $n \to \infty$. Joint work with Werner EHM, University of Frankfurt, W-Germany.

S. OREY

Strong laws for shrinking Brownian Tubes

Let X be R^d-valued Brownian motion, I_T $(I_T(\varphi) = \int_0^T |\dot\varphi(s)|^2 ds)$ the associated action functional and define the functional

$$\alpha_T(\varepsilon) = \inf \{I_T(\varphi) : ||\varphi - X||_T \le \varepsilon\}, \text{ where } ||\psi||_T = \sup_{0\le s\le T} |\psi(s)|.$$

Next let X' be a second R^d-valued Brownian motion, independent of X; say (X,X') are defined on a product space $(\Omega\times\Omega', \mathcal{B}\times\mathcal{B}', P\times P')$. Let $q_T(\varepsilon,\varphi) = P'[||X'-\varphi||_T \le \varepsilon]$. Interest will focus on the random functional $q_T(\varepsilon,X)$.

Theorem 1: ∃ positive constant m_1 such that $\varepsilon^2 \alpha_T(\varepsilon)/T \longrightarrow m_1$ as $\varepsilon \downarrow o$.

Theorem 2: ∃ positive constant m_2 such that $-\varepsilon^2_{\overline{T}} \log q_T(\varepsilon,x) \longrightarrow m_2$ as $\varepsilon \downarrow o$.

G. PALM

Stochastic identification of nonlinear systems

A system is a mapping $S: I \to R$, where I is a set of functions of time T. Usually $T = R, Z$, or $[o,\ell]$, and $I = C(T)$, $L^2(T)$, $\mathcal{D}(T)$, ... Identification of a system S requires its representation, for example as $Sx = k_o + \int k_1(t_1)x(t_1)dt_1 + \int\int k_2(t_1,t_2)x(t_1)x(t_2)dt_1dt_2 + \ldots$ (this is called a Volterra series). Then one can identify the system by experimentally determining the 'kernels' $k_1(t), k_2(t_1,t_2), \ldots$. This is done by testing the system by appropriate test inputs $x(t)$. Norbert Wiener had the idea to use stochastic test inputs, i.e. a process (I,Σ,p) and to assume $S \in L^2(I,p)$. S is then identified by expansion with respect to a specific CONS $H_{nk} \subseteq L^2(I,p)$. $S = \Sigma \langle S,H_{nk}\rangle H_{nk}$. Here $I = C[o,1]$, p Brownian Motion, and H_{nk} are Hermite-Laguerre polynomials, which are of the form (A) $H_{nk} = \sum_{i=o}^{n} G_{nki}$, where (B) $G_{nki}\, x = \int\ldots\int g_{nki}(t_1,\ldots,t_i)dx(t_1)\ldots dx(t_i)$. This result can be generalized to quite arbitrary stochastic processes (I,Σ,p).

Theorem: For every stochastic process (I,Σ,p) (where Σ is generated by cylindersets, $x(o) = o$ a.s. and something like $E(x(t) - x(t_o))^2 \to o\ (t \to t_o)$) with polynomials dense in $L^p(\mathbb{R},P_t)$ for every $p \geq 1$, $t \in T$ there exists a CONS in $L^2(I,p)$ of functionals H_{nk} of the form (A), (B).

G. PAPANICOLAOU

Characterization of set of values of conductivity in random media

We give a suitable definition of conductivity in a random medium. Then we give a representation formula for the conductivity and discuss its implications. Multicomponent random media will be discussed also.

J. PFANZAGL

A differential approach to asymptotic statistical theory

Let $\mathcal{P}$ be a family of probability measures with densities $h(\cdot,P)$. The tangent space $T(P,\mathcal{P})$ is defined as the set of all functions $g \in \mathcal{L}_2(P)$ with $\int g(x)P(dx) = 0$ which are limits of sequences $t^{-1}(h(\cdot,P_t)/h(\cdot,P) - 1)$, $t \downarrow 0$. The gradient $\varkappa^{\cdot}(\cdot,P)$ of a functional $\varkappa\colon \mathcal{P} \to R$ is defined by the property that, for P_t near P, $\varkappa(P_t)$ can be approximated by $\varkappa(P) + \int \varkappa^{\cdot}(x,P)P_t(dx)$. Then $\int \varkappa^{\cdot}(x,P)^2 P(dx)$ is a bound for the asymptotic variance of asymptotically median unbiased estimator-sequences for $\varkappa$, if $\varkappa^{\cdot}(\cdot,P) \in T(P,\mathcal{P})$. As an example, estimation of quantites of symmetric distributions is discussed.

G. Ch. PFLUG

Recursive estimation in nonregular cases

Für nicht reguläre Dichtefamilien (das sind solche, bei denen der euklidische Abstand im Parameterraum und die Hellingerdistanzen der Dichten eine Relation vom Grade ρ erfüllen) werden, aufbauend auf dem Konzept der Pitmanschätzer, rekursive Schätzer definiert. Schätzt man auch die a-posteriori Varianz rekursiv mit, so erreichen so definierte Schätzer jene maximale Konvergenzgeschwindigkeit, welche von LeCam resp. Strasser als "quick-consistency" bezeichnet wird. Weiter ist es möglich, asymptotische Verteilungen (die in der Regel nicht normal sind) anzugeben.

P. RÉVÉSZ

How big are the increments of the local time of a Wiener process?

Let $\{\eta(x,t);\ 0<t<\infty,\ -\infty<x<\infty\}$ be the local time of the Wiener process $\{W(t);\ 0<t<\infty\}$ i.e. for any Borel set A let

$$\int_A \eta(x,t)dx = \lambda\{s\colon s\leq t,\ W(s)\in A\}$$

where λ is the Lebesgue measure. Further let $0<a(t)<t$ $(t>0)$ be a non-decreasing function. Our aim is to study the properties of the processes

$$y_1(t) = \sup_{0 \leqq s \leqq t-a(t)} (\eta(0,s+a(t))-\eta(0,s)),$$

$$y_2(t) = \sup_{0 \leqq s \leqq t-a(t)} \max_{-\infty<x<\infty} (\eta(x,s+a(t))-\eta(x,s)).$$

A new continuity modulus of the local time will be also given.

M. DENKER und U. RÖSLER

On Chernoff-Savage theorems

The subject of this talk is a new proof and an extension of the two-sample linear rank statistic of the Chernoff-Savage type. Let X_i, Y_j, $i=1,\dots,n$, $j=1,\dots,m$ be independent r.v. with distribution function F,G and consider

$$\sqrt{N} \int_0^1 h(\tfrac{N}{N+1} H_N(t))dF_n(t) - \sqrt{N} \int_0^1 h(\hat{H}_N(t))dF(t)$$

where F_n,G_n are the empirical distributions and $H_N = \frac{n}{N} F_n + \frac{m}{N} G_m$, $\hat{H}_N = \frac{n}{N} F + \frac{m}{N} G$. The problem is to find a large class of functions h, such that the above converges in distribution to a normal distribution.

Our basic assumption on the two processes X_i,Y_j is the independence of the processes and the condition, $0 \leq \eta \leq 1/2$

$$E(\sqrt{n}(F_n(z)-F(z))^2) = O([F(z)(1-F(z))]^{1-2\eta})$$

for all $z \in [0,1]$ and similar for G_m,G. This enables us to treat processes with some dependence structure, like uniformly mixing, absolutely regular or strongly mixing.

Chernoff, H. / I. R. Savage: Asymptotic normality and effciency of certain nonparametric test statistics, Ann. Math. Stat. 29 (1958), 972-994.

Pyke, R. / G. R. Shorak: Weak convergence of a two-sample empirical process and a new approach to Chernoff-Savage theorems. Ann. Math. Stat. 39 (1968), 755-771.

H. ROST

Probabilistic modelling of hydrodynamic behaviour

If one is interested in the macroscopic evolution law of a many particle system (concerning mass density) and expects a non-equilibrium behaviour of the form

$$\dot{f} = \frac{1}{2} \operatorname{div}(k(f) \operatorname{grad} f)$$

one way of trying to identify the density dependent diffusion coefficient $k(\cdot)$ is - following an old idea of Onsager - to look at fluctuations around the equilibrium state of the system characterized by the fixed density ρ. It is shown here that for a zero-range jump process with arbitrary jump rates in dimension $d = 1$ or 2 k can be identified and a relation of the form (for $\varphi, \psi \in \mathcal{S}$)

$$\lim_{\varepsilon \to 0} \varepsilon N^{\varepsilon}(\varphi, 0) N^{\varepsilon}(\psi, t) = \chi \cdot \iint \varphi(x) \psi(y) g\left(\frac{x-y}{\sqrt{kt}}\right) dx\, dy$$

can be proven. Here $g(\cdot)$ is standard Gaussian density, the other quantities are defined as follows:

$X(i,t)$ = number of particles at site i at time t,

$$N^{\varepsilon}(\varphi, t) = \varepsilon^{d/2} \cdot \sum_{i \in Z^d} (X(i, t\varepsilon^{-2}) - \rho) \varphi(i\varepsilon),$$

χ = "susceptibility" = $\varepsilon(X(o,o)-\rho)^2$ in that particular model

ρ = density = $\varepsilon X(o,o)$.

M. SCHEUTZOW

Stationary solutions of stochastic differential equations with bounded memory

We study solutions of stochastic delay equations of the form $dX(t) = F(X_t)dt + dW(t)$, where W is a one-dimensional Wiener process, $F: C[0,1] \to R$ and $X_t(s) := X(t+s)$, $s \in [0,1]$. Such equations have a bounded memory because the drift function F only depends on the past of X during the last unit time interval. In case there exists a unique weak solution, $(X_t)_{t \geq o}$ is a Markov-process. If X_t has an invariant probability measure, it is unique and every initial p.m. converges to it. Sufficient conditions for this case are formulated via Lyapunov functionals. We give two examples one of which is the well-known logistic equation of population growth. Finally we state a convergence result of invariant p.ms of the discretized equation towards the invariant p.m. of the original equation.

E. SIEBERT

Holomorphic convolution semigroups

A continuous convolution semigroup $(\mu_t)_{t>0}$ of probability measures on a locally compact group G is said to be (strongly) holomorphic if for every representation π of G by isometries on a Hilbert space (resp. Banach space) the operator semigroup $(\pi(\mu_t))_{t>0}$ is holomorphic. For example compound Poisson semigroups are strongly holomorphic; and symmetric convolution semigroups are holomorphic.

Holomorphic semigroups possess interesting properties with regard to support, densities, etc.

Further important examples of strongly holomorphic semigroups are given by (strictly) stable convolution semigroups; by certain (absolutely continuous) Gaussian semigroups; and by subordination by means of strongly holomorphic semigroups on $[0,\infty[$ (as one-sided stable and Gamma semigroups). For operator-stable probability measures on R^d (in the sense of Sharpe) more precise results can be derived.

J. STOYANOV

One approach to estimation problems for continuous time stochastic processes

We shall consider continuous time stochastic processes depending on some unknown parameters. The finding of good estimators for these parameters is a natural and important problem. Often the following assumption is made: the process is observed continuously during some time interval. In this case and if the process is of a diffusion type the estimators contain integrals in Lebesgue and Ito's sense which is not so convenient from the practice point of view.

We shall present another approach. Suppose the process is observed only at the moments of some discrete window, deterministic or random. The aim is to find estimators for the unknown parameters on the basis of the obtained discrete data. The solution of this general problem will be given for some specific classes of continuous time processes and discrete windows.

H. STRASSER

Experiments with independent increments

The starting point are those non-regular situations in asymptotic statistics where the densities may have jumps. Such situations have been considered under a couple of conditions by Ibragimov & Has'minskiĭ 1972-1981, and by Pflug, 1981. We treat the problem by LeCam's theory of infinitely divisible experiments. The limit experiments occurring with densities with jumps are particular cases of what we call experiments with independent increments. This class of experiments can be described by the associated semigroups of binary experiments. We give a simple set of conditions which implies the situation considered by Ibragimov & Has'minskiĭ.

E. N. TORGERSEN

Statistical information obtainable by sampling plans in survey sampling

Consider a finite population I and a characteristic of interest which, with varying amount (value, degree,...) is possessed by all individuals in I. Let $\theta(i)$ be the amount of this characteristic for individual i.

It is known that θ belongs to some set Θ of functions on I. Let α be a sampling plan, i.e. a probability distribution on the set of finite sequences of elements from I. If this sampling plan is used and if the characteristics of sampled individuals are determined without error, then the outcome

$$x = ((i_1,\theta(i_1)),(i_2,\theta(i_2)),\ldots,(i_n,\theta(i_n))$$

is obtained with probability $\alpha(i_1,i_2,\ldots,i_n)$.

We shall here discuss how the statistical information in x depends on the chosen sampling plan α .

S. R. S. VARADHAN

Transport processes in random media

We consider a transport process in R^d with a finite set $\{v_1,...,v_N\}$ of possible velocities. The infinitesimal generator takes the form

$$G = V\cdot\nabla_x F + \frac{\rho(x)}{N}\sum_{v'}\{F(x,v') - F(x,v)\} .$$

$\rho(x)$ controls the Poisson rate for transitions in velocities and new velocity is chosen randomly with equal probabilities at each turn. $\rho(x)$ is assumed to be random and forms a stationary stochastic process in x with very general ergodicity properties. Under the assumption that the set of velocities is balanced (i.e. v is a velocity implies that -v is so) we prove a central limit theorem for the position $x(t)$ of the process at time t as $t \to \infty$.

H. v. WEIZSÄCKER

What is a perfect experiment?

Let $(\Omega,\mathcal{B},\{p_\vartheta\}_{\vartheta\in\Theta})$ be a statistical experiment, Ω and Θ being Polish spaces and $\vartheta \to p_\vartheta(B)$ Borel for each $B \in \mathcal{B}$. Each of the following conditions is strictly weaker than the next. Each may be viewed as a possible definition of 'perfectness' of the experiment.

a) $p_\vartheta \perp p_{\vartheta'}$ if $\vartheta \neq \vartheta'$.

b) There is a family $(B_\vartheta)_{\vartheta\in\Theta}$ of subsets of Ω satisfying

i) $\{(\omega,\vartheta): \omega \in B_\vartheta\} \in \text{Borel}(\Omega\times\Theta)$

ii) $p_\vartheta(B_{\vartheta'}) = \begin{cases}1 & \text{if } \vartheta = \vartheta' \\ 0 & \text{if } \vartheta = \vartheta'.\end{cases}$

c) If $\mu \perp \nu$ on Θ then $\int p_\vartheta d\mu \perp \int p_\vartheta d\nu$ on Ω

d) There is a Borel map $\varphi: \Omega \to \Theta$ such that $p_\vartheta\{\omega: \varphi(\omega) = \vartheta\} = 1$ for all $\vartheta \in \Theta$

The main points here are $c \Rightarrow b$ and $c \nRightarrow d$. For Gaussian shift experiments $a \Rightarrow c$ is known and $c \Rightarrow d$ is an unsolved problem. c has many interesting reformulations.

V. WIHSTUTZ

Stabilization of linear systems by noise

It is proved that the biggest Lyapunov number λ_{max} of the system $\dot{x} = (A+F(t))x$, where A is a fixed $d \times d$-matrix and $F(t)$ is a zero-mean strictly stationary matrix-valued stochastic process, satisfies $\frac{1}{d}$ trace $A \le \lambda_{max}$. On the other hand, for each $\varepsilon > 0$ there is a process $F(t)$ for which $\lambda_{max} \le \frac{1}{\alpha}$ trace $A + \varepsilon$. In particular, the system $\dot{x} = Ax$ can be stabilized by zero mean stationary parameter noise if and only if trace(A) < 0. The stabilization can be accomplished by a one-dimensional noise source. The results carry over to the case where A is a stationary process.

(Joint paper of L. Arnold, H. Crauel, V. Wihstutz)

H. WITTING

On the convergence rate of linear rank statistics

The convergence rate of signed linear rank statistics is proved to be not worse than $O(n^{-1/2}(\log n)^2)$ if the score generating function b satisfies the Chernoff-Savage type condition $|b''(t)| \le C\,[t(1-t)]^{-2}$ $\forall t \in (0,1)$. The main idea is to approximate the statistic by a U-statistic and to apply a result of Helmers - van Zwet (1981) about the convergence rate of U-statistics. By an appropriate generalization of U-statistics and the Helmers - van Zwet result the convergence rate of general linear rank statistics can be shown to be not worse than $O(\max|c_{nj}|(\log n)^{-2})$, if the regression coefficients satisfy the Noether condition $\max|c_{nj}| \to 0$, $\Sigma c_{nj}^2 \to 1$. (Joint with U. Müller-Funk and K. O. Friedrich).

J. ZABCZYK

Structural properties of stochastic linear systems in Hilbert spaces

The object of the talk is to discuss structural properties of a stochastic linear system $(x_t)_{t \ge 0}$ of the form

$dx_t = (Ax_t+Bu_t)dt + C\,dw_t$, $x_0 = x \in H$, where A denotes infinitesimal generator of a C_0-semigroup defined on a Hilbert space H; B and C are bounded linear operators and $(w_t)_{t \geq 0}$ is a Wiener process. In particular conditions will be given under which the controlled system is non-degenerate, recurrent or positive recurrent.

W. R. van ZWET

Contiguity relative to the randomness hypothesis

For $N = 1,2,\ldots,$ consider product probability measures $Q_N^{(N)} = \prod_{i=1}^{N} Q_{Ni}$ and let P_N^N denote the product of N identical probabilities P_N. We investigate the following statements:

(i) The sequence $\{Q_N^{(N)}\}$ is contiguous to $\{P_N^N\}$ for some choice of $\{P_N\}$;

(ii) The sequences $\{Q_N^{(N)}\}$ and $\{P_N^N\}$ are mutually contiguous for some choice of $\{P_N\}$;

(iii) The log-likelihood ratio $\Sigma \log \frac{dQ_{Ni}}{dP_N}$ is asymptotically $N(-\frac{1}{2}\sigma_N^2, \sigma_N^2)$ under $\{P_N^N\}$ and its summands are asymptotically negligible.

Each of these statements is equivalent to the same statement with $\bar{Q}_N = \frac{1}{N} \Sigma Q_{Ni}$ instead of P_N. Necessary and sufficient conditions in terms of the marginals Q_{Ni} are given for each statement.

Berichterstatter: Ludwig Arnold

Tagungsteilnehmer

Dr. M. A k a h i r a
Department of Mathematics
The University of Electro-Communications
1-5-1, Chofugaoka,
Chofu-Shi
T o k y o / JAPAN

Prof. L. Arnold
Fachbereich Mathematik
Universität, Postfach
D-2800 Bremen 33

C. B e c k e r
Universität Bayreuth
Mathematik VII
Postfach 3008
8580 B a y r e u t h

Prof. R. B e r a n
University of California
Department of Statistics
Berkeley, California 94720
U S A

Dr. L. B i r g é
Université Paris VII
U.E.R. de Mathématiques
Tour 45/55, 5. Etage
2, Place Jussieu
75251-Paris Cedex 05 / FRANCE

H. C r a u e l
Fachbereich Mathematik
Universität
Postfach 330 440
2800 Bremen 33

Prof. E. B. D y n k i n
Department of Mathematics
Cornell University
Ithaca, New York 14853
U S A

Prof. K. D. Elworthy
Mathematics Institute
University of Warwick
Coventry CV4 7AL
GREAT BRITAIN

Prof. B. Ferebee
Institut für Angewandte Mathem.
Universität Heidelberg
Im Neuenheimer Feld 294
6900 H e i d e l b e r g

Prof. W. H a z o d
Abteilung Mathematik
Universität
Postfach 500 500
4600 Dortmund 50

Prof. H. H e y e r
Mathematisches Institut
der Universität Tübingen
Auf der Morgenstelle 10
7400 Tübingen 1

Dr. A. J a n s s e n
Abteilung Mathematik
Universität
Postfach 500 500
4600 Dortmund 50

Dr. P. Jeganathan
Indian Statistical Institute
Stat-Math Division
203, Barrackpore Trunk Road

Calcutta 700 035 / INDIA

Herrn
E. M a m m e n
Universität Heidelberg
SFB "Stoch.Mathem.Modelle"

Im Neuenheimer Feld 294

6900 - Heidelberg

Dr. W. K i r s c h
Institut für Mathematik
Ruhr-Universität
Universitätsstraße 150 NA

4630 B o c h u m

Prof. M. Metivier
Centre de Mathématiques
Appliquées
Ecole Polytechnique

F-91128 Palaiseau Cedex

F R A N C E

Dr. A. K o z e k
Mathematisches Institut
der Universität zu Köln
Weyertal 86-90

D-5000 K ö l n 41

Prof. P. A. Meyer
Département de Mathématiques
Université de Strasbourg
7, rue René Descartes

F-67084 Strasbourg Cédex
F R A N C E

Prof. L. L e C a m
University of California
Department of Statistics

Berkeley, California 94720

U S A

Dr. H. M i l b r o d t
Universität Bayreuth
Mathematik VII
Postfach 3008

8580 B a y r e u t h

Prof. P. M a j o r
Mathematical Institute
Hungarian Academy of Sciences
Réaltanado-U.

H-1053 Budapest / HUNGARY

Prof. D. W. Müller
Universität Frankfurt
Fachbereich Mathematik
Robert-Mayer-Straße 10

6000 Frankfurt 1

Prof. P. Malliavin
10 Rue Saint Louis en l'Isle

F-75004 P a r i s

F R A N C E

Prof. G. N e u h a u s
Institut für Mathematische
Stochastik der Universität Hbg.
Bundesstraße 55

2000 H a m b u r g

Prof. S. O r e y
Department of Mathematics
University of Minnesota

Minneapolis, MN 55455
U S A

Prof. H. R i e d e r
Universität Bayreuth
Mathematisches Institut
Postfach 3008

D-8580 B a y r e u t h

Dr. G. P a l m
Max-Planck-Institut für
Biologische Kybernetik
Spemannstraße 38

7400 Tübingen 1

Dr. U. R ö s l e r
Institut für Mathematische
Statistik
Lotzestraße 13

3400 G ö t t i n g e n

Prof. G. Papanicolaou
Courant Institute
New York University
251 Mercer Str.

New York, NY 10012
U S A

Prof. H. R o s t
Institut für Angewandte Mathematik
Universität Heidelberg
Im Neuenheimer Feld 294

D-6900 H e i d e l b e r g

Prof. J. Pfanzagl
Mathematisches Institut der
Universität zu Köln
Weyertal 86-90

D-5000 K ö l n 41

Dr. M. Scheutzow
Fachbereich Mathematik
Universität
Erwin-Schrödinger-Straße

6750 Kaiserslautern

Herrn
Prof. Dr. G. P f l u g
Institut für Statistik der
Universität W i e n
Rathausstraße 19

A 1010 - W i e n / AUSTRIA

Prof. E. S i e b e r t
Mathematisches Institut der
Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen 1

Prof. P. Révész
Mathematical Institute
Hungarian Academy of Sciences

H-1053 B u d a p e s t
Reáltanodo - U. 13-15
HUNGARY

Prof. J. M. Stoyanov
Institute of Mathematics
Bulgarian Academy of Sciences

S o f i a 1000 / BULGARIA

Prof. H. Strasser
Universität Bayreuth
Mathematisches Institut
Postfach 3008

8580 B a y r e u t h

Prof. V. Wihstutz
Universität Bremen
Fachbereich Mathematik
Kufsteinerstraße/Postfach

2800 Bremen 33

Prof. E. N. Torgersen
Institute of Mathematics
University of Oslo
P. O. Box 1053, Blindern

O s l o 3 / NORWAY

Prof. H. Witting
Universität Freiburg
Institut für Mathematische
Stochastik
Hermann-Herder-Straße 10

7800 F r e i b u r g

Prof. S. R. S. Varadhan
Courant Institute
New York University
251 Mercer St.

New York, NY 10012
U S A

Prof. J. Z a b c z y k
Institute of Mathematics
Polish Academy of Sciences

W a r s z a w a / POLAND

Dr. W. Wefelmeyer
Mathematisches Institut
der Universität zu Köln
Weyertal 86-90

5000 K ö l n 41

Prof. W. R. van Zwet
Centraal Reken-Institut
Rijksuniversiteit te Leiden

Wassenaarseweg 80

NL - L e i d e n
THE NETHERLANDS

Prof. H. von Weizsäcker
Fachbereich Mathematik
Universität
Erwin-Schrödinger-Straße

6750 Kaiserslautern