MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

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Die Tagung fand unter der Leitung von Herrn W. Felscher (Tübingen) und Herrn H. Schwichtenberg (München) statt. Es wurden 33 Vorträge über verschiedene Gebiete der Mathematischen Logik gehalten.

Vortragsauszüge

P. ACZEL: Choice Principles and Inductive Definitions in Constructive Set Theory

In the proceedings of the 1977 Wroclaw logic symposium I gave a model of a theory $CZF + DC$ of constructive set theory. This model was found within a version of Martin-Löf's intuitionistic theory of types. By working in a stronger but still constructive version of type theory it is possible to show that the model satisfies some additional axioms. The presentation axiom (PA) is a choice principal which I had introduced in the Wrocław paper but had then been unable to prove true. Another true choice principal implies that all sets built up from ω using exponentiation are bases, i.e. sets on which choice functions can always be found. A final axiom can be formulated in terms of inductive definitions and implies that for any set Φ of rules there is a smallest set $I(\Phi)$ closed under the rules.

K. AMBOS-SPIES: Paare von rekursiv aufzählbaren Turing-Graden

Lachlan und Yates haben gezeigt, daß manche Paare inkomparabler r.a. Grade ein Infimum besitzen, andere jedoch nicht. Wir betrachten Fragen, die sich aus dieser Situation ergeben haben. Im Mittelpunkt stehen r.a. Grade, die mit keinem inkomparablen r.a. Grad ein Paar von Graden bilden, die ein Infimum besitzen.

H.P. BARENDREGT: Survey Lambda Calculus

There are four main subjects in the lambda calculus: conversion, reduction, theories and models. A flavour of each of these will be presented.

B. BENNINGHOFEN: Monaden-Familien

Ist $(\mathcal{F}_i)_{i\in I}$ eine Standard-Familie von Filtern, $X_i := \cup \mathcal{F}_i$, so ist durch

$$\underset{i}{\pi}|\mu(\mathcal{F}_i)[j] := {}^{E}\{x\in X_j \,|\, \forall^{st}\tilde{F}\in \prod_{i\in I}\mathcal{F}_i \quad x\in\tilde{F}(j)\}$$

die Monade von $\mathcal{F}_j$ auch für nichtstandard $j\in I$ erklärt. Ist für $i\in I$ standard $\mu(\mathcal{F}_i) = {}^{E}\{x\in X_i\,|\,\varphi_i(x)\}$, wobei $\varphi_i(x)$ eine "einfach reduzierbare" Formel ist, so läßt sich daraus eine neue Formel $\hat{\Phi}(i,x)$ konstruieren, so daß für alle $j\in I$ gilt:

$$\underset{i}{\pi}|\mu(\mathcal{F}_i)[j] = {}^{E}\{x\in X_i\,|\,\hat{\Phi}(j,x)\}\ .$$

Ganz allgemein gilt dann für diese Konstruktion

SATZ (Transfer-Theorem)

$$(\forall^{st} i\in I\varphi_i) \Leftrightarrow (\forall i\in I\hat{\Phi}(i))\ .$$

Anwendungen sind

1) Induktive Limites von lokalkonvexen Räumen
2) Vereinfachung von einigen Nichtstandard-Beweisen
3) Charakterisierung von lokalzusammenhängenden Räumen
4) Vereinigung von unendlich vielen Monaden; Berechnung von Hüllen und Kernen in der S-Topologie.

E. BÖRGER: The new proof of James P. Jones and Yuri Matijasevich for the Davis-Putnam-Robinson theorem that all r.e. sets are exponential diophantine

I reported the above mentioned proof which will appear in the JSL 47 (1982) or 48 (1983).

D.S. BRIDGES: Compact Linear Mappings with Complete Range - A Constructive Discussion

A well known result in the elementary theory of compact linear mappings says that:

(*) If a compact linear mapping between normed linear spaces has complete range, then its range is finite-dimensional.

This can be proved classically by an application of the Baire Category Theorem. Within Bishop's constructive framework, the Baire Category Theorem is the contrapositive of the version applied to (*), and we have to work much harder to prove (*). In this talk, I outline the constructive proof of (*), paying particular attention to the proofs of two lemmas, each of which illustrates a technique which occurs repeatedly in the context of a complete metric space.

Reference: F. Richman, D. Bridges, A. Calder, W. Julian and R. Mines, "Compactly Generated Banach Spaces", Archiv der Math., Vol. 36, 1981, 240 - 243.

W. BUCHHOLZ: On Homogeneous Proof Trees

Schütte's proof of the completeness theorem for 1st order predicate logic (given in his book "Proof Theory") is extended to an infinitary cutfree system of inductive definitions. It is shown that the formula trees which are constructed in this proof are homogeneous. This fact is then used to give new proofs for the following theorems:

Theorem (Girard)

If $\forall x \in O \exists y \in O \psi(x,y)$ (ψ arithmetical and positive in O) is true, then there exists a recursive dilator D such that, for each $\alpha > \omega$, the formula $\forall x \in O^{<\alpha} \exists y \in O^{<D(\alpha)} \psi(x,y)$ is true.

Corollary $\omega_1^{CK} = \sup\{D(\omega) : D \text{ a rec. dilator}\}$.

Theorem (van de Wiele)
If the function $f : V \to V$ is uniformly Σ_1-definable on all admissibles, then there exists a recursive dilator D such that $rk(F(x)) < D(rk(x))$ for all sets $x \in V$.

W. FELSCHER: On the Philosophy of Dialogues

P. Lorenzen, in 1958, proposed the foundation of logic upon the notions of a dialogue and of a strategy for dialogues; only recently the author succeeded in a proof establishing the equivalence between intuitionistic provability and provability by strategies for certain dialogues. The rules governing these dialogues were, essentially, proposed already by Lorenzen and Lorenz, but this was done employing mainly concepts referring to games, and no foundational discussion had as yet been developed. In order to obtain such a foundation, the author starts from the observation that provable formulas should be those which, for purely formal reasons, can be defended by a strategy; he then stresses the fact that it is necessary to clarify simultaneously the notions of (i) rules for dialogues, (ii) winning a dialogue, (iii) employing strategies for dialogues. He distinguishes between contentions and hypotheses and points out that, in general, not every hypothesis may be relevant for every contention. Rather than developing an additional notion of relevance, the author shows how dialogue rules can be used in order to avoid distinctions of relevance; thus in a dialogue governed by suitable rules all hypotheses become relevant for all contentions. The principal notions, permitting an analysis of rules, are those of delayed answers and of relevance-creating ramifications.

J.Y. GIRARD: Proof-Theoretical Analysis of Π^1_2 - CA

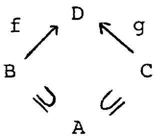
I develop a proof-theoretical analysis of the Π^1_2 - comprehension axiom, by extension of methods of "Π^1_2 - logic" to the Π^1_3 case. The treatment is very similar to my treatment of the Π^1_1 - comprehension axiom by means of Π^1_2 - logic.

R.J. GRAYSON: Formal Spaces

Infinitary geometric propositional theories are those expressed in terms of sequents $\varphi \Rightarrow \bigvee\Gamma$, where φ is a formula built up by conjunction only, and Γ is any set of such. Completeness for such theories corresponds to various kinds of induction principle, for example, bar induction is equivalent to completeness for the theory of an infinite sequence of natural numbers (Fourman). These principles are mainly of interest from a constructive viewpoint. The "formal space" of a theory may be used to construct a "generic" model (over a complete Heyting algebra) for a theory which classically has no models, for example, that of a partial function from $\mathbb{N}$ onto $\mathbb{N}^{\mathbb{N}}$.

W. HODGES: Forking (without uncountable cardinals)

Forking is a concept used in analysing analgans



of parts of models. Intuitively, the analgan is non-forking with respect to a formula $\varphi(\bar{x};\bar{y})$ iff for each $\bar{b}$ in B, there is no $\bar{c}$ in C such that $\models\varphi(f\bar{b};g\bar{c})$ unless there has to be one (given $B \supseteq A \subseteq C$). This can be made precise in several equivalent ways. The notion works well (e.g. is left-right symmetric) iff $\varphi(\bar{x};\bar{y})$ is stable. Examples are given, e.g. in a skew field every formula of form $p_1(\bar{x})q_1(\bar{x}) + \ldots + p_n(\bar{x})q_n(\bar{x}) = 0$ is stable. Shelah's 'non-forking' is 'non-forking with respect to all formulas of the language'. But the theory can be strengthened by being developed one formula at a time, and without the transfinite cardinal arithmetic which has usually been used.

S. KOPPELBERG: Maximal Chains in Boolean Algebras

Let, for a Boolean algebra B, $m(B)$ be the cardinality of the set of maximal chains in B.

Proposition 1 If B is atomless, then $|B| \leq m(B) \leq 2^{|B|}$.

<u>Proposition 2</u> If B is complete and atomless, then $m(B)^{\omega} = m(B)$.

<u>Problem</u> Does Proposition 2 generalize to arbitrary atomless Boolean algebras?

<u>Proposition 3</u> Let $(X,\leqslant)$ be a complete dense linear order and C a maximal chain in the interval algebra B of X. Then C contains 0_B and 1_B , C is dense and isomorphic to an F_σ - subset of a complete linear order.

<u>Theorem</u> If $F \subset [0,1]$ (the real unit interval) is an F_σ - set, dense in $[0,1]$ and contains 0 and 1 , then F is isomorphic to a maximal chain in the interval algebra of $\mathbb{R}$.

H. KOTLARSKI: <u>A Survey of Non-Standard Satisfaction Classes</u>

Let $M \models PA$ (= Peano Arithmetic). $S \subset M$ is a full satisfaction class iff S contains sentences (in the sense of M , i.e. need not be standard) and satisfies Tarski's condition on truth, i.e.

$$a + b = c \in S \quad \text{iff} \quad \text{it is true,}$$
$$(\neg\varphi) \in S \quad \text{iff} \quad \varphi \notin S \text{ , } \quad \text{etc.}$$

This is an axiomatic framework for studying the notion of truth for non-standard sentences.

S is inductive (substitutable) iff $(M,S) \models$ induction.

THEOREM 1 (Krajewski). There exists a countable M which has 2^{ω} many distinct inductive full satisfaction classes.

Observations: (i) If M has a full inductive satisfaction class and M is non-standard then M is recursively saturated.

(ii) If M has a full satisfaction class which is inductive then $M \models$ "Peano is consistent".

THEOREM 2 (Lachlan). If M has a full satisfaction class and is non-standard then M is recursively saturated (i.e. induction is not needed in (i) above).

THEOREM 3 (Lachlan, Krajewski and myself). Every countable recursively saturated M has a full satisfaction class. Moreover define in M : A_0 is $0 = 0$, A_{i+1} is $A_i \vee A_i$. If i is non-standard then we can choose a satisfaction class on M which makes false A_i .

THEOREM 4 (Kotlarski). If M is countable recursively saturated then M has a satisfaction class S such that $(M,S) \models \Delta_0$ - induction. Moreover define in M A_k to be

$\forall x_1 \forall x_2 \ldots \forall x_k \bigwedge_{i=1}^{k} x_i = x_i'$. If k is non-standard then we can choose S to make A_k false.

Ratajczyk has given an explicit recursive axiomatisation of the arithmetical part of the theory PA + S is an inductive satisfaction class.

K. KOYMANS: Models of the λ - Calculus

In this lecture a category theoretic framework will be presented to interpret λ - calculus theory. This framework incorporates the well-known λ - models as constructed by D.S. Scott and others, but extends these by including the so called λ - algebras, a notion of model not satisfying extensionality of functions. In fact this setup is equivalent to the λ - algebras, but provides a far more natural theory. As an intermediate notion the so called cartesion closed monoids will be considered.

P.H. KRAUSS: Topological Quasi Varieties

We consider topological structures of similarity type $\underset{\sim}{Op} \cup \underset{\sim}{POp} \cup \underset{\sim}{RI}$, where $\underset{\sim}{Op}$, $\underset{\sim}{POp}$ and $\underset{\sim}{RI}$ are sets of operation symbols, partial operation symbols and relation symbols. The corresponding language has a proper class Vb of variables, a class Tm of terms as usual, and atomic formulas $\sigma \approx \rho$, $r\sigma_o \ldots \sigma_{n-1}$, $\tau_d \overset{d \in D}{\rightarrow} \sigma$, where $\langle D, \leq \rangle$ is a directed set, $\tau : D \rightarrow Tm$ is a net in Tm , and $\sigma, \rho, \sigma_o, \ldots, \sigma_{n-1} \in Tm$. $\tau_d \overset{d \in D}{\rightarrow} \sigma$ is interpreted as net convergence. A topological quasi atomic formula is a formula of the form $\bigwedge\{\Phi_\xi | \xi \in \Delta\} \Rightarrow \Psi$, where Δ is a set and Φ_ξ , Ψ are atomic formulas. A (compact) topological quasi variety is a class of (compact) models of a class of topological quasi atomic formulas.

Theorem (Clarc and Krauss)

$$\underset{\sim}{Y} \models Th_{tqa} \mathcal{K} \Leftrightarrow \underset{\sim}{Y} \in \mathbb{I}\,\mathbb{S}\,\mathbb{P}\,\mathcal{K} .$$

Corollary $\mathcal{K}$ is a (compact) topological quasi variety iff $\mathcal{K} = \mathbb{I}\mathcal{K} = \mathbb{S}_{(c)}\mathcal{K} = \mathbb{P}\mathcal{K}$.

In algebraic duality theory the category of topological spaces is a compact topological quasi variety. An important problem of duality theory is to find an axiomatisation of this compact topological quasi variety (D.M. Clark and Peter H. Krauss, Topological quasi varieties, Preprint 1981).

J.D. MONK: Cardinality of Homomorphs of Products of Boolean Algebras

(Joint work with B. KOPPELBERG, R. McKENZIE)

Theorem 1. If there is no measurable cardinal $\leqslant |I|$, $\langle A_i : i \in I\rangle$ is a system of Boolean algebras, B is a homomorph of $\Pi_{i\in I} A_i$, and $\omega \leqslant |B| > |A_i|$ for each $i \in I$, then $|B|^{\omega} = |B|$.

Theorem 2. Suppose $\lambda \geqslant \omega$ and there is a countably complete $(\lambda,\lambda^{+\omega})$-regular ultrafilter on some set I such that $|I| > \lambda^{+\omega}$. Then there is a system $\langle A_i : i \in I\rangle$ of Boolean algebras and an infinite homomorph B of $\Pi_{i\in I} A_i$ such that $|A_i| < |B|$ for all $i \in I$ and $|B|^{\omega} > |B|$.

A.W. MOSTOWSKI: Determinancy of Sinking Automata over Infinite Trees and Inequalities between Various Rabin's Pair Indices

An automaton $\mathfrak{A} = \langle S,M,F,\Omega\rangle$ over an alphabet V is climbing/sinking according to $M : S \times V \to 2^{S^k}$ / $M : S^k \times V \to 2^S$. The I in $\Omega = \{(U_i,L_i)_{o\leq i<I}\}$ is called a Rabin's pair index.

Theorem. For any $\mathfrak{A}$ an equivalent deterministic sinking automaton $\tilde{\mathfrak{A}}$ can be effectively constructed. For k-ary trees $k \geqslant 2$ for constant: I and the cardinality of V the construction is square, space and number of the states and cubic time. Evidently no analogon for climbing automata exists. For ω-sequences i.e. $k = 1$, the index of $\tilde{\mathfrak{A}}$ is 1-empty $(I=1, L_o=\emptyset)$ and the number of states is of the order 2^n, (not 2^{n^2} as in McNaughton's construction for climbing automata). For a set of trees the following indices can be defined: NI-nondeterministic climbing, DI-deterministic climbing and SI-sinking. Always $NI = SI \leq DI$. There exists a set with $NI = DI = 2$. For a set such that $DI < \infty$ or NI = 1-full (i.e. $I = 1$, $U_0 = S$) the non-deterministic index of the complement is 1-empty.

G.H. MÜLLER: Choice and Partitions

The main problem considered was to find axioms of set theory ZF, violating V = HOD but keeping AC alive. (In passing the question to find reasonable borderlines (in terms of length of the von Neumann hierarchy) of the validity of the axiom of choice was mentioned.) - To attack these problems a survey of newer results was given (i) concerning the structure of HOD (e.g. by S. Roguski and K. Gloede) and (ii) concerning typical violations of AC through elementary embeddings from V into V , determinacy and infinitary partitions. Finally the following type of such partitions were considered: Let $DEF[\kappa]^\omega = \{S \mid S \in [\kappa]^\omega$ and "definable" $(S)\}$; then $\forall f \in DEF$ from $DEF[\kappa]^\omega$ into $\{0,1\}$ $\exists C \subseteq \kappa$, $card(C) = \kappa$ such that $[C]^\omega$ is homogeneous for the partition f . Various definability concepts to interpret DEF are considered, e.g. her. ordinal definability; in this case $[C]^\omega$ does not contain any ordinal definable element hence $V \neq HOD$. Research is going on to adapt Silver's thesis to these new type of axioms.

D. NORMANN: An Introduction to the Continuous Functionals

The continuous functionals are defined, several characterizations described and the naturaleness of the structure discussed. We then give the basic structural properties and discuss the connection between Kleene-computations (internal algorithms) and countable recursion (external algorithms). Finally we define the continuous r.e. degrees and discuss some of the properties of this degree-structure.

H. OSSWALD: Hyperendliche Maßräume

Sei Ω eine hyperendliche interne Menge, $\mathfrak{A}$ eine Algebra von Teilmengen von Ω und sei μ_p ein internes endlich additives $\mathbb{R}^*$-wertiges Wahrscheinlichkeitsmaß auf $\mathfrak{A}$, das von einer internen Zählfunktion $p : \Omega \to \{x \in \mathbb{R}^* \mid 0 \leq x \approx 0\}$ mit $\sum_{\omega\in\Omega} p(\omega) \aleph 1$ induziert wird.

$$\mu_p(A) := \sum_{\omega\in A} p(\omega) .$$

P.Loeb zeigte in den T.A.M.S. 1975, wie dieses Maß zu einem reellwertigen Maß übersetzt werden kann. Man definiert

$$\hat{\mu}_p(A) = {}^{o}\sum_{\omega\in\Omega} p(\omega) \quad (= \text{Standardteil von } \mu_p(A)).$$

Das Maß $\hat{\mu}_p$ ist σ-additiv, weil man das Nonstandardmodell als ω_1-saturiert voraussetzt. Deshalb kann man dieses Maß eindeutig zu einem σ-additiven Maß auf der kleinsten σ-Algebra, die $\mathfrak{A}$ umfaßt, fortsetzen. Sei $L(\mathfrak{A})$ die Vervollständigung dieses Maßes, das wir auch mit $\hat{\mu}_p$ bezeichnen.

$$(\Omega, L(\mathfrak{A}), \hat{\mu}_p) \quad \text{heißt Loebscher Maßraum.}$$

Im folgenden werden einige Sätze angegeben, die die Integrationstheorie in diesen Räumen betreffen, und es wird gezeigt, wie man diese Ergebnisse auf gewisse stochastische Integralgleichungen anwenden kann. Die angegebenen Ergebnisse stammen von P. Loeb, Anderson, Keisler, Fisher, Hoover, Perkins, Lindström u.a.

P. PÄPPINGHAUS: Extensible Algorithms on Graphs

In two areas work is done about algorithms on graphs for problems like coloring, embedding, matching, etc. The results obtained in recursive combinatorics about the existence of such algorithms for infinite graphs have no obvious relationship to the results obtained in complexity theory about the complexity of such algorithms for finite graphs. Such a relationship is established to another property of well-behaviour, called extensibility. Theorem: For a good set of data (Y,Υ,P,Y^*,P^*) there is a P^*-solving uniform algorithm for Y^* iff there is a P-solving extensible algorithm for (Y,Υ). Moreover for some general notion of a trap, one can prove that the existence of a trap implies that there is no P^*-solving uniform algorithm for Y^*, and provided the trap satisfies some additional conditions, one can even exhibit a particular highly recursive graph in Y^* for which no P^*-solving algorithm exists.

H. PFEIFFER: Ein Beitrag zur Zeitlogik

Es wird ein zeitlogisches System innerhalb eines zweisortigen Prädikatenkalküls angegeben, der eine Sorte Variablen für Individuen und eine für Zeitpunkte enthält. Die Sprache L dieses Systems besteht aus Formeln mit genau einer Zeitstelle und eventuell mehreren Individuenstellen, einem Existenzprädikat E und zwei Zeitprädikaten P_1 und P_2 als Hilfsprädikaten. Die Semantik wird mit sogenannten L-Sturkturen gebildet, die die Eigenschaft besitzen, daß ein Individuum, das zu einem Zeitpunkt existiert, auch zu jedem bzgl. P_1 späteren und zu jedem bzgl. P_2 früheren Zeitpunkt existiert. Die Formeln haben Analoga in der Sprache des Systems WK_t (vgl. D. Gabbay, Ann. Math. Log. 8, 1975). φ aus L ist L-gültig genau dann, wenn das Analogon von φ in WK_t gilt. Der Beweis des Vollständigkeitssatzes des den L-Formeln angepaßten Prädikatenkalküls bzgl. der L-Semantik motiviert die Vorgehensweise beim Vollständigkeitsbeweis für WK_t. Auch ein zeitlogisches Teilsystem des zweisortigen Prädikatenkalküls, das Quantifikation über die Zeit zwischen zwei Zeitpunkten erlaubt, wird vorgestellt. Der Ableitungskalkül ist bezüglich einer Semantik vollständig, die mit Hilfe von sogenannten B-Strukturen geeignet defininiert wird.

W. RAUTENBERG: Simplifications in Applying Modal Logic to Metamathematics

A simple tableau calculus for G (Gödel - Löb's modal logic) is presented who yields at the same time all basic properties of G : finite model property, and various refinements, decidability, interpolation, a simplified proof of the fixed point theorem, cut free axiomatizability etc. Similar calculi exist for Gr (Grzegorczyk's system) and related systems.

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G.R. RENARDEL DE LAVALETTE: Axiomatisation of the Arithmetical Fragment of an Intuitionistic Theory with Extended Bar Induction

Extended bar induction (EBI) is the schema

EBI $\quad \forall a \in A^{N} \exists n P\bar{a}n \wedge \forall x \in A^{<\omega}(\forall y \in A P\hat{x}*y \leftrightarrow Px) \rightarrow P\langle\rangle$.

We add EBI to (an extension of) APP, an intuitionistic theory based on type-free (partial) application, which can be considered as a conservative extension of HA and EL (intuitionistic arithmetic and elementary analysis). The arithmetical fragment of the theory thus formed is reduced to a theory with inductively defined sets $K = K_A$ satisfying

'K is the least set s.t. $\forall x \lambda a.x \in K \wedge \forall f(\forall x \in A \lambda a.f(\hat{x}*a) \in K \rightarrow f \in K)$'

The reduction is carried out in several steps, involving elimination of choice sequences, realizability and forcing.

B. SCARPELLINI: Bemerkungen über die Quantorenelimination bei der Presburger Arithmetik

In a paper (1968) Hodes - Specker investigate the effectiveness of quantifier elimination methods for certain elementary theories such as ordered dense Abelian groups. They show that for every k there is a polynomial deterministic machine T_k which tranforms every formula $F = (Q_1 y_1 \dots Q_k y_k) L(x_1 \dots x_s y_1 \dots y_k)$ (L quantifierfree) into an equivalent quantifierfree formula F^* . Here it is shown that for Presburger arithmetic the opposite is true. By using in part variants of the Rabin - Fischer construction, in part the $P \neq NP$ hypothesis we show that polynomially bounded quantifier elimination methods do not exist for Presburger arithmetic.

U.R. SCHMERL: On Unprovability of Diophantine Equations and Inequations in Weak Systems of Arithmetic

Let Z_1 be a formal system of arithmetic in the language of O,N (successor), + , and $\cdot$ with induction restricted to quantifier-free formulae. By proof-theoretic means we establish a characterization deciding the provability of diophantine equations

$t_1 = t_2$ or inequations $t_1 \neq t_2$ in this system. When slightly extended this characterization contains the independence results by Shepherdson 1965 for his system T_5 .

J. SCHULTE MÖNTING: The Notion of Codimension for Heyting Algebras

The codimension of a Heyting algebra H is a pair $\langle d,c\rangle$, $c,d \in \omega \cup \{\infty\}$, where c is the number of minimal prime filters on H , and d is the number of those minimal prime filters which do not contain the filter D of dense elements of H .
In a Heyting algebra of codimension $\langle d,c\rangle$, there exists an orthogonal antichain $\langle a_\mu | \mu < c\rangle$ satisfying $a_\mu \vee a_\nu = 1$ $(\mu \neq \nu)$, $a'_\mu = 0$ for $\mu < d$, a_μ strictly regular (i.e. not covered by a dense element $\neq 1$) for $d \leq \mu < c$. The elements of such an antichain generate a subalgebra of the form $\underline{2}^d \times \underline{2}^{c-d}$, "the" prime algebra of codimension $\langle d,c\rangle$. Codimensions are partially ordered by the product order.
This concept seems to be useful for the structure theory of Heyting algebras. As an example, on can prove the
Theorem A Heyting algebra H can be embedded into every algebraically closed Heyting algebra $\hat{H}$ of finite codimension $\langle\hat{d},\hat{c}\rangle$ if and only if it is countable and locally finite and has a codimension not greater than $\langle\hat{d},\hat{c}\rangle$.
A similar theorem holds for the infinite case.

H. SCHWICHTENBERG: On Martin-Löf's Theory of Types

Für eine Version von Martin-Löf's Typentheorie wird gezeigt, daß sich jede Herleitung in Normalform bringen läßt. Als Folgerung ergibt sich die Konservativität über der Heyting-Arithmetik bezüglich arithmetischer Formeln, die weder $\vee$ noch $\exists$ in Prämissen von Implikationen enthalten.

E. SIBILLE: Combinatorial Meaning of an Ordinal Assignment for HERBRAND - GÖDEL - STYLE PRIMITIVE RECURSIVE FUNCTIONALS

Besides λ-abstraction, the only initial equations (in a system as described by Kleene, 1962, Proc. Symp. pure Math., vol.5, A.M.S., Providence R.I.) here are for recursion.

A s.c. ("simple combination") of the terms $s_1^{\sigma_1},\dots,s_p^{\sigma_p}$ is a term of the form $u^{(\tau_1,\dots,\tau_q\to o)}(u_1^{\tau_1},\dots,u_q^{\tau_q})$, where u is some s_i $(1 \leq i \leq p)$ and each u_j $(1 \leq j \leq q)$ is either some s_i , or - if τ_j is $\neq$ from all σ_i - a variable, or - if τ_i is the type O - also a s.c. of $s_1,\dots,s_p$. The ordinal $\| t \|$ for a normal term t of the form $x^{(\underline{\sigma}\to o)}(\underline{s}^{\underline{\sigma}})$ is $1+\sup\{\| u \|$; u ranges over all s.c. of $\underline{s}\}$. For recursion:
$\| f^{(o,\underline{\sigma}\to o)}(x^o,\underline{s}^{\underline{\sigma}})\| = \sup\{\| f(\overline{n},\underline{s}^{\underline{\sigma}})\| \; ; n \in \mathbb{N}\}$.

The proof of $\| t \| < \varepsilon_o$ leads to a direct syntactical ordinal assignment which has the lowering ordinal property for <u>all</u> reductions.
Instead of using "evaluation rules" (cf. Kleene), we interpret type$\neq$ O-variables by terms built by means of Hilbert's ε operator (applied to simple formulas) and λ-abstraction: $\| . \|$ is in concordance with the arithmetical hierarchy. If we interpret these variables by prim. rec. functionals, we correlate the constants of height 1 with $< \varepsilon_o$-prim.-rec. functions.

J. SMITH: <u>Programming in Intuitionistic Type Theory: An Example</u>

The strong development of computers during the last years has necessiated new programming languages. Per Martin-Löf has suggested that his formalization of constructive mathematics, intuitionistic type theory, may be used as a programming language.

In type theory only primitive recursion is allowed. Since general recursion is extensively used in programming, new methods of consructing programs must be developed for type theory. It is shown how to reduce a useful scheme of course-of-values recursion on lists to primitive recursion, using a higher-order function. As an example, the method will be used to construct a program in type theory for the sorting algorithm quicksort.

E. SPECKER: <u>Strukturzahlen von monadischen Theorien zweiter Ordnung</u>

Für einen Satz S der monadischen Sprache zweiter Ordnung in binären Relationen sei $a^S(n)$ die Anzahl der Strukturen auf der Menge $\{1,2,\ldots,n\}$, welche S erfüllen. Satz: a^S ist nach jedem Modul periodisch von einer gewissen Stelle an, d.h.

$$(\forall m)_{>0}(\exists p)_{>0}(\exists n_0)(\forall n)\; n_0 \leqslant n \to a^S(n+p) \underset{(m)}{\equiv} a^S(n) \;.$$

(Resultat einer gemeinsamen Arbeit mit Chr. Blatter)

B.G. SUNDHOLM: Constructions, Proofs and the Meaning of the Logical Constants

A critical examination of the use of "theories of constructions" as "theories of meaning" for the constructive logical constants, with particular reference to the problem of the second clauses.

W. THOMAS: On Definability of Sets of $\mathbb{Z}$ - Sequences

A nonempty word w over $\Sigma = \{0,1\}$ is identified with the finite model $M_w = (\{1,\ldots,|w|\},<,\mathbb{O}_w,\mathbb{1}_w)$ where $\mathbb{O}_w$, $\mathbb{1}_w$ are unary predicates which hold for the positions in w carrying the letter 0 , 1 respectively. Similarly, a $\mathbb{Z}$-sequence α over Σ determines the model $M_\alpha = (\mathbb{Z},<,\mathbb{O}_\alpha,\mathbb{1}_\alpha)$. Call a set $W \subset \Sigma^*$ (resp. a set $S \subset \Sigma^{\mathbb{Z}}$) first-order definable if there is a sentence φ in the first-order language with symbols $<,\mathbb{O},\mathbb{1}$ such that: $w \in W \Leftrightarrow M_w \models \varphi$ (resp. $\alpha \in S \Leftrightarrow M_\alpha \models \varphi$) . Similarly for monadic second-order definability. Let

$$\overleftrightarrow{W} = \{\alpha \in \Sigma^{\mathbb{Z}} \mid \text{there are } \ldots < k_2 < k_1 < k_0 < l_0 < l_1 < l_2 < \ldots \text{ such that } \alpha(k_i)\alpha(k_i+1)\ldots\alpha(l_i) \in W \text{ for } i \geqslant 0\} \,.$$

Theorem $S \subset S^{\mathbb{Z}}$ is first-order definable (resp. monadic second-order definable) iff S is a Boolean combination of sets $\overleftrightarrow{W}$ with W first-order definable (resp. monadic second-order definable).

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