MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 18/1982

FLÄCHEN IN DER GEOMETRISCHEN DATENVERARBEITUNG

26.4. bis 30.4.1982

Die erste Tagung über "Flächen in der geometrischen Datenverarbeitung" im Mathematischen Forschungsinstitut Oberwolfach stand unter der Leitung von W. Böhm (Braunschweig) und J. Hoschek (Darmstadt). Unter den 27 Teilnehmern (darunter 4 aus dem europäischen Ausland, 5 aus den USA) befanden sich nicht nur Wissenschaftler, die an Universitäten und Hochschulen überwiegend theoretische Aspekte des "Computer Aided Geometric Design (CAGD)" untersuchen, sondern auch in der Industrie (Automobil-, Flugzeug-, Schiffsbau) und in Forschungsinstituten tätige Mathematiker, die sich mehr mit praktischen Anwendungen des CAGD befassen.

Im Mittelpunkt der Vorträge über neuere Forschungsergebnisse standen Verfahren zur Approximation bzw. Interpolation von Flächenstücken, die gewisse Vorgaben (z.B. vorgegebene Randkurven, Punktmengen oder Stetigkeitsforderungen) erfüllen. In diesem Zusammenhang wurden als neuartige Ansätze mehrparametrige Spline-Funktionen und die "Dualisierung" bisheriger Verfahren vorgestellt. Daneben wurde auch auf Fragen der Berechnung und graphischen Darstellung von Flächen eingegangen. In den anwendungsorientierten Vorträgen wurde aufgezeigt, welche Verfahren man zur Computer-unterstützten Behandlung von Flächen bzw. Flächendaten momentan in der Praxis anwendet. Die dort auftretenden Probleme waren u.a. ein Anlaß für interessante Diskussionen im Anschluß an die Vorträge und auch später im privaten Gespräch.

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Vortragsauszüge

R. E. Barnhill:

Computer Aided Surface Representation

We discuss some of our recent Surface research. This includes C^2 triangular interpolants for 3D surfaces and C^1 tetrahedral interpolants for 4D surfaces. We also mention a new adaptive (bivariate) cubature method. The various algorithms are illustrated by means of 16 mm movies.

J. A. Gregory:

Non-Rectangular Surface Patches

A surface in computer-aided geometric design is usually represented by piecewise defined vector valued surface patches, where the individual patches have a rectangular domain of definition. However, non-standard surface patches can occur within a rectangular patch framework and this talk will consider a method of constructing such patches, for example, those having a triangular or pentagonal domain of definition. The removal of the twist conditions from the definition of rectangular and non-rectangular surface patches will also be considered.

H.-J. Hochfeld:

Oberflächen-Beschreibungen in der Anwendung bei Volkswagen

Methods and applications of the mathematical curve and surface description are discussed that are operational or in the pilot phase in the passenger car development respectively in the production environment at Volkswagen. The main accent lies on the Bézier surface model.

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G. Farin:

Smooth Interpolation to Scattered 3D data

An outline of triangular Bernstein-Bézier methods is given. This includes the de Casteljau algorithm for triangular patches, their subdivision and the conditions for arbitrary smoothness between adjacent triangular patches. As an application of the theory, an improved version of the standard 9 parameter cubic is presented (with quadratic instead of linear precision) and a modification of the Clough-Tocher scheme is described, both interpolants being formulated in Bernstein-Bézier form.
As the major application, the following problem is considered:

given: a triangulated set of 3D points plus a tangent plane at each point,

find: a smooth, piecewise polynomial surface through these points and tangent planes.

Since smoothness cannot be defined in terms of differentiability, a more general concept of tangent plane continuity is formulated in Bernstein-Bézier form. This wider concept allows to construct a generalized Clough-Tocher surface, consisting of quartic polynomials.
The presented algorithm also allows the treatment of (otherwise untenable) closed surfaces.

J. Kahmann:

Continuity of Curvature between adjacent biparametric Bézier-Polynomials

In some applications of CAD it is required to join surface patches - most convenient defined by biparametric vectorvalued Bézier-polynomials - with coincident tangent planes and identical normal curvature in every direction

at every point of their common boundary curve, i.e. the Dupin indicatrices have to coincide. This is trivially fullfilled if the iso-parametric curves have continuous first and second derivatives at junction points. More generally second order smoothness between both patches can be ensured if the relations

$$\vec{b}_x = \alpha\, \vec{a}_u + l\, \vec{a}_v \quad ; \quad \alpha > 0,\ l = \beta(1 - v) + \gamma v$$

$$\vec{b}_{xx} = \alpha^2\, \vec{a}_{uu} + 2\alpha l \vec{a}_{uv} + l^2 \vec{a}_{vv} + \rho \vec{a}_u + [\sigma(1-v) + \tau v]\vec{a}_v$$

hold along the common boundary curve $\vec{c}(v)$, where $\vec{b}(x,y)$ means the patch, which is to be constructed and $\vec{a}(u,v)$ is the given patch. By comparing coefficients one gets some constructions of the Bézier-points of $\vec{b}(x,y)$.

W. Böhm:

<u>Generating the Bézier Points of Triangular Splines</u>

The Bézier points of splines over regular partitions (i.e. one-dimensional splines, their tensor products, splines over regular triangulations) can be obtained from the following general <u>algorithm</u>:

- Take a polygon or net that corresponds to the knots or knotlines of the desired spline
- Subdivide the sides of the polygon or net with respect to the degree of the spline
- Determine the Bézier points of the spline as centres of gravity of certain neighbouring subdivision points, selected by a fixed mask
- Superpose different surfaces, if they exist.

This construction is an almost immediate consequence of the following <u>lemma</u>:

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- Consider a surface that has been generated by Sabin-type convolutions from truncated Bernstein polynomials. Its Bézier points are the subdivision points of a hexagonal pyramid of base length 1. (An analogous lemma holds for spline curves).

In the case of quartic splines over triangles the mentioned mask consists of a simple hexagon, in the case of bicubic splines over squares it is a simple quadrilateral.

W.J. Gordon:

Piecewise polynomial blended surface interpolation techniques

The most commonly used surfacing methods in Computer-Aided Geometric Design are based upon bivariate interpolation schemes which use piecewise polynomials. This paper presents a survey of the theoretical foundations of these techniques and their practical application to surface definition in Euclidean 3-space.

G. Farin:

CAGD at Daimler-Benz

The talk focusses on three main aspects of surface generation for car bodies:

1. smoothing boundary lines

   an interactive algorithm is employed that manipulates the derivative curves instead of the original curve

2. generating surfaces

   an algorithm is described that is more faithful to the input geometry than the standard Boolean sum approach is.

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3. reflection lines

the use and basic algorithms of reflection lines for surface interrogation is described.

F. Little:

Convex Combination Surfaces

Shepard's surface is a convex combination of Taylor interpolants. It has been generalized to have arbitrary continuity and interpolation properties on a domain of arbitrary dimension. Recently, interpolants which match data all along the boundaries of squares and triangles were formulated as convex combinations. Terms from Shepard's surface can be included into these interpolants to extend their interpolation properties to a few additional interior points. Polynomial interpolants of low degree can be combined using convex weights to union their interpolation and local shape-preserving properties. The differentiation of this surface produces a local high precision estimate for derivative data. This surface also extrapolates in a desirable manner. Implicit variations of this surface are immediate and produce an implicit surface with implicit precision and shape producing properties which interpolates.

S. Turk:

A Taxonomic Approach to the Comparison of some Interpolation Algorithmi

Several interpolation algorithmi are compared in some of their features and properties. The basis for the comparison is the algorithmus definition: The compared features are procedure principles, tautness, boundary conditions,

local and global property, parameter dependance. The results of the analysis procedure principles are presented in the treelike way. Beside this, a table is composed in which the calculation methods are summarized. Some attention is also given to the nonlinear splines. As the next, the algorithmi are compared with the respect on the required number of operations. Results for hypothetical computers with and without the floating point processor are presented graphically.

Keywords: computer graphics, interpolation, approximation, splines.

W. Böhm:

The Algorithm of Cox - de Boor for Triangular Splines

Suppose one can express a basis spline $N_{i,k}(u..)$ of order k over a partition as a linear combination of basis splines of lower order, where the coefficients in the linear combination are linear in u.. . Then every spline of order k can be written as a linear combination of B-splines of lower order, the coefficients being polynomials in u.. .

A geometric analogue of this approach is described for splines over regular triangulations; an unsolved problem is the relationship between splines of order 5 and 6. In the discussion W. Dahmen pointed out possible connections with the box splines of de Boor and Höllig (to be published).

H. Nowacki:

Definition and Fairing of Ship Surfaces

Procedures for developing a ship surface definition and for improving its fairness are reviewed. In initial design of ship lines a set of characteristic

control curves is developed from form parameter requirements. This step is supported by curve design techniques accounting for offset point, derivative, and integral property constraints. Exploiting the local properties of B-spline curves leads to important reductions in the size of the resulting equation systems. The skeleton mesh is extended to surface representation, e.g., using B-spline surfaces as Coons patches. Local fairness properties can be discussed in terms of maps of principal curvature and Gaussian curvature isolines. Fairness criteria based on modified strain energy concepts can be used to evaluate and automically improve the quality of the surface. The problem should be further investigated with geometric and non-geometric constraints in mind.

R. Franke:

Surface Approximation with Imposed Conditions

Several general purpose methods for interpolation of scattered data have been recently developed. We will describe and compare some methods which have the capability to adapt to special features of the data. For example, while we assume a smooth surface is usually desirable, the user may wish to impose ridges or even discontinuities on the surface and we discuss ways to model such surfaces.

J. Hoschek:

Dual Bézier Curves and Surfaces

The usual approximation techniques like Bézier curves, Bézier surfaces, B-spline curves etc. interpret the curve or the surface as locus of points, i.e. these methods are working in the point space. It is also possible to

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interpret a curve or a surface as an envelope of lines or of planes, i.e. we can describe a curve or a surface in the line space or the plane space too. The transformation of the Bêzier curves, etc., from the pointspace into the line space or the plane space by the principle of duality leads to a new concept for the approximation of free form curves or surfaces. The properties of the dual Bêzier curves, etc. can be deduced from the well-known properties of the Bêzier curves etc. by a polarity. The polarity shows that the dual Bêzier curves etc. are related to the rational Bêzier curves etc. .

F. Elsässer:

Surfaces and their Applications at Opel

The intention of this presentation is to give an impression of existing mathematical surfaces and their applications at Opel, especially in the area of body design. Two groups of surfaces are mainly used:

parametric surfaces for body design and

bicubic splines for different kinds of representation of 2-dimensional functions especially for generation of contour map lines.

Additionally two pseudo surfaces and a special windshield surface will be presented.

G.M. Nielson:

Surface Construction based upon triangulations of nonconvex, multiply connected regions.

This presentation will consist of a discussion of a general class of methods for constructing surface approximations based upon a triangulation. We will survey the topic of triangulation and present a new method which covers the

case of nonconvex domains. Several methods of estimating local shape characteristics of the surface will be reviewed and compared. A discussion of various triangular patch methods will also be included.

H. Seybold:

Construction of Functions for the Representation of Surfaces

Some algorithms of computational geometry are based on representations of surfaces Φ by equations $F(P) = 0$, where

$$F : \mathbb{R}^3 \to \mathbb{R}, \qquad F(P) = 0 \iff P \in \Phi ,$$
$$(F(P_1) \cdot F(P_2) < 0 \wedge |\overline{P_1P_2}| < s) \Rightarrow \overline{P_1P_2} \cap \Phi \neq \emptyset ,$$

(s is some boundary dependent upon the actual problem).
The construction of $F : \mathbb{R}^3 \to \mathbb{R}$ can be reduced to the analogous 2- and 1-dimensional problem, if Φ can be generated by a sufficiently simple motion (and deformation) M of a sufficiently simple curve g: We "project" the points P of the s-neighbourhood N of Φ via their M-orbits into an appropriate 2-dimensional region G containing the "generator" g. The intersections of appropriate curves in G, passing through the projections P' of P, with the generator g give informations about the position of P with respect to Φ which we use for the construction of F.
Tested examples: Generalized helical surfaces.

R. Schmidt:

Flächenapproximation durch Tensor-Produkt-B-Splines

Es wird ein einstufiges Verfahren zur Konstruktion von Flächen durch Tensor-Produkt-B-Splines vorgestellt. Die Stützstellen der Daten dürfen

beliebig unregelmäßig verteilt sein. Die approximierende Fläche besteht aus Polynomstücken eines beliebig zu wählenden Produktgrades über einem äquidistanten Rechteckgitter, die über die Gitterlinien hinweg stetig differenzierbar verhaftet sind. Die Glattheit der Fläche wird durch den Polynomgrad im Sinne der Splinetheorie bestimmt, die Daten nach der Methode der kleinsten Abweichungsquadratsumme approximiert. Datenlücken werden durch stetiges Fortsetzen der Flächenstücke kompensiert.

W. Dahmen:

Multivariate Splines - A New Constructive Approach

The purpose of this talk is to survey some recent developments in multivariate spline theory centering upon the notion of the multivariate B-spline as the basic ingredient of a purely knot dependent concept. In particular, practical recurrence relations for the evaluation of B-splines as well as their derivatives and inner products are discussed. As a further central issue a combinatorial recipe for constructing certain flexible configurations of knot sets is described. Ultimately exploiting the geometrical interpretation of the B-spline one can show that the corresponding collections of B-splines form 'stable' basis for globally smooth spline spaces admitting the realization of optimal local and global approximation rates.

H. Eckert:

Use of Biparametric Surfaces in the NC Process

MBB is using the APT 4 processor with CAM-1 SS enhancements (Sculptured Surface) since 4 years to manufacture moulds and models. Routines developed and enhanced by MBB enable the connection of APT 4/SS with all surface

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modellers by means of a simple interface. The method of "non native" use of geometry with APT 4 and a production related topology scheme will be presented.

J. Novák:

Rechnerunterstützter Entwurf von kanalartigen Flächen

Es wird über den rechnerunterstützten Entwurf von kanalartigen Flächen berichtet, die durch vorgegebene Lagen von geschlossenen Stützquerschnitten und durch weitere wählbare Bedingungen bestimmt werden. Die Aufgabe gliedert sich in eine Reihe von Teilaufgaben mit Interpolationsproblematik

- Verarbeitung der in graphischer Form vorgegebenen Stützquerschnitte als Interpolationskurven (Schwerpunkt, Transformation),
- Konstruktion der räumlichen Kanalachse, welche durch die Stützquerschnittschwerpunkte hindurchgeht und deren Normalebenen in diesen Punkten mit den entsprechenden Stützquerschnittebenen zusammenfallen,
- Konstruktion der Flächenzwischenquerschnitte und ihre Abbildung,
- Konstruktion von Schichtenlinien der Fläche, welche zur Erzeugung des Flächenmodells dienen.

Schließlich werden Anwendungsmöglichkeiten besprochen.

Berichterstatter: L. Hering

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