MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 19/1982

Gruppentheorie

2.5. bis 8.5.1982

Die Tagung fand unter der Leitung der Herren K. W. Gruenberg, London, und O. H. Kegel, Freiburg, statt. Es wurden Fragen sowohl aus der Theorie der endlichen Gruppen als auch der unendlichen Gruppen behandelt. Bei den endlichen Gruppen standen Ergebnisse zur Theorie der Fitting-Klassen endlicher auflösbarer Gruppen und zur Darstellungstheorie im Vordergrund. Gebäude-Geometrien für endliche einfache Gruppen, deren Struktur und Darstellungstheorie sowie die Auswirkungen des Klassifikationssatzes bildeten einen weiteren Schwerpunkt.
Bei den unendlichen Gruppen wurden Ergebnisse zur Struktur linearer Gruppen über Schiefkörpern und zur Struktur endlich präsentierter Gruppen vorgestellt. Mit Ergebnissen zur Entscheidbarkeit (bzw. Unentscheidbarkeit) des Wortproblems und des Isomorphieproblems wurden algorithmische Fragen behandelt; hier wurden auch Beziehungen zur Theorie der formalen Sprachen beleuchtet.
In den 38 Vorträgen des offiziellen Tagungsprogramms wurden große Teile der Gruppentheorie anregend behandelt. Trotz dieser Fülle kam auch die informelle Seite nicht zu kurz, macht sie doch eine wesentliche Eigenheit der Oberwolfacher Tagungen aus: In oft langen persönlichen Gesprächen wurden Erfahrungen und Ergebnisse ausgetauscht und gruppentheoretische Fragestellungen diskutiert.
Bei der großen Zahl der Interessenten ist leider nur festzustellen, daß bei weitem nicht alle eingeladen werden konnten.
(auch wenn Herr Merzljakov schließlich nicht an der Tagung teilnehmen konnte, erscheint sein Vortragsauszug nachstehend.)

Vortragsauszüge

S. I. Adian   Some recent results on periodic groups

A survey of some new applications the Noviko-Adian method of classification of periodic words relatively to a given odd exponent n. Here is one of them: Let $n \geq 665$ be an odd integer and $m \geq 66$. We consider words in the alphabet $a_1, a_2, \ldots, a_m, a_1^{-1}, a_2^{-1}, \ldots, a_m^{-1}$. Put $A_1 = a_1 a_2 a_1^{-1}$, and define inductively: $A_{i+1} = A_i a_2 A_i^{-1}$. For any word X denote by X' the result of replacing each letter a_j^{σ} in X by $a_{j+_m 1}^{\sigma}$, where $+_m$ is addition modulo m.

THEOREM : The set of relations $R = \{A_i A_i' A_i \ldots A_i = 1;\ i=1,2,\ldots\}$ is an independent set of relations in the group

$$C(m,n) = \langle a_1, a_2, \ldots, a_m;\ x^n = 1,\ R \rangle .$$

Corollary : There exists a recursively presented group with m generators that has unsolvable word problem and satisfies the identity $x^n = 1$.

J. L. Alperin   Trees, determinants and blocks

A treatment of the theory of blocks with cyclic defect groups, which proceeds by establishing the Janusz-Kupisch structure theorem by purely module-theoretic means in characteristic p, will be discussed after an exposition of the main results of the theory. The standard character theorems fall out easily from the results in characteristic p. A combinatorial result on trees and a lemma on determinants of matrices of overlapping submatrices are used at one point.

J. C. Beidleman   Fitting functors of finite solvable groups

Fitting functors are introduced and such functors are used to construct new conjugate classes of subgroups of finite solvable groups. Further, some new Fitting classes are studied. Specialized Fitting functors include Fischer functors, normally embedded functors, and Lockett functors. These specialized functors are used to study various special types of Fitting classes and their properties.

R. Bieri   G. M. Bergman's conjecture on the logarithmic limit set of an algebraic variety

This is joint work with John Groves (Melbourne): Let G be a free abelian group of rank $n < \infty$, I an ideal in the group algebra $\mathbb{C}G$, and $V \subseteq \mathbb{C}^n$ the corresponding algebraic variety. In order to investigate automorphisms of G stabilizing I, G. M. Bergman introduced the logarithmic limit set V_∞ of V which is the subset of the unit sphere

S^{n-1} given by $V_\infty = \left\{ \frac{(\log|x_1|,\ldots,\log|x_n|)}{\sqrt{1+\sum(\log|x_i|)^2}} ; x \in V \right\} \cap S^{n-1}$.

He proved that V_∞ is contained in a finite union of great subspheres of S^{n-1}, and conjectured that V_∞ is, in fact, a rational spherical polyhedron (= finite union of finite intersections of closed hemispheres given by inequalities with integer coefficients). We prove this.

R. A. Bryce   Subgroup closed Fitting classes of finite soluble groups

A class of finite soluble groups is a Fitting class if it contains
every normal subgroup of each of its groups, and
every group which is aproduct of two normal subgroup, each from the class.

Examples of Fitting classes are the class of all finite soluble groups of (at most) prescribed length, and the class $\underline{S}_\pi$ of all finite soluble groups the prime divisors of whose orders come from a given set of primes π. Moreover, intersections of Fitting classes and products of Fitting classes are again Fitting classes. A Fitting class is called primitive if it is an intersection of products of the form $\underline{S}_{\pi(1)}\underline{S}_{\pi(2)}\cdots\underline{S}_{\pi(r)}$ where the $\pi(i)$ are sets of primes. Of course, primitive Fitting classes have a number of closure properties not enjoyed by Fitting classes generally, for example, they are subgroup closed. Recently, John Cossey and I succeeded in establishing that subgroup closure characterizes primitive Fitting classes.

THEOREM : A Fitting class is primitive if and only if it is subgroup closed.

M. J. Collins   Projective modules, filtrations and Cartan invariants

The following joint work with J.L. Alperin and D. A. Sibley was reported: Let G be a finite group, N a normal subgroup, $\tilde{G} = G/N$ and k an algebraically closed field of characteristic p. Any (simple) $k\tilde{G}$-module is a (simple) kG-module: if S is such a simple module, we ask what is the relationship between its projective covers as a kG-module and as a $k\tilde{G}$-module.

THEOREM : Let $\tilde{Q}_1,\ldots,\tilde{Q}_s$ be the projective indecomposable kG-modules, and let $Q_1,\ldots,Q_s$ be the projective covers as kG-modules of the corresponding simple modules. Then there exists a kG-module M such that, for each i, the modules Q_i and $M \otimes \tilde{Q}_i$ have the same composition factors.

If N is a p-group, we may take $M = kN$ (with the action of G given by conjugation). As an application, we give aproof of Brauer's theorem that the determinant of the Cartan matrix is $\pm$ a power of p, independent of the characterisation of characters.

P. Hauck   Subnormal subgroups in direct products

A group is called normally (subnormally) detectable if the following holds : Whenever G is normal (subnormal) in $G_1 \times \ldots \times G_n$ with $G_j \simeq G$ for all j, then $G = G_i$ for some i.

THEOREM 1 : If the group G satisfies min-sn then the following are equivalent : (a) G is subnormally detectable, (b) G is directly indecomposable and does not admit non-trivial homomorphisms into Fit(G).

The situation for normally detectable groups is more complicated. A full classification is not yet known. Some partial answers are given by

THEOREM 2 : Let the group G satisfy min-sn and max-sn.
(a) If G is not of the form $G = NS$, $1 \neq N \trianglelefteq G$, $S \trianglelefteq\trianglelefteq G$, S not nilpotent, $N \cap S = 1$, $S^G/\mathrm{Core}_G S$ nilpotent, and if $\mathrm{Hom}(G/G', Z(G)) = 0$ then G is normally detectable.
(b) If G is directly indecomposable and if $\mathrm{Hom}(\bar{G}/\bar{G}', Z(\bar{G})) = 0$ for all essential factor groups $\bar{G}$ of G (i.e. $\bar{G} = G$ or $\bar{G}$ is not nilpotent, directly decomposable, and isomorphic to a subnormal subgroup of G) then G is normally detectable.

G. Higman   Automorphism groups of pairs of trees

Let the groups NW, NE, SE, SW be disjoint products of their subgroups N, S, E, W. The amalgam $NW \cup NE \cup SE \cup SW$ is embeddable in a group, and its free product G is the disjoint product of ordinary free products $N * S$ and $E * W$. We are particularly interested in the case in which the groups N, S, E, W are finite, of orders n,2,2,m respectively. Then G has a flag transitive action on the tree Γ_m of valency m, in which $(N * S)W$ is a vertex stabiliser, and $(N * S)E$ an edge stabiliser and a similar action on the tree Γ_n of valency n. The fact that G is the disjoint product of $N * S$ and $E * W$ implies that it acts faithfully and regularly on the set $Fl(\Gamma_m) \times Fl(\Gamma_n)$, where $Fl(\Delta)$ denotes the set of flags of Δ, and so is an automorphism group of the pair of trees (Γ_m, Γ_n). – A lemma of Djokovic states that if G is transitive on the arcs of length s of Γ_m for all s, then it is faithful on Γ_n, and, of course, vice versa. We study in particular the case when NW is A_5, NE is S_4, and SW is D_{10}, and show that then G is indeed transitive on s-arcs for all s, and so faithful on both Γ_5 and Γ_{12}.

P. Hilton A contribution to nilpotent group theory

Let $\varphi : G \to H$ be a homomorphism of nilpotent groups. Then φ completes to a surjection if and only if $\varphi^*: (H,F) \to (G,F)$ is injective for any finite group F, where $(-,-)$ is the set of momomorphisms. Now let P be any family of primes. We may then generalize the statement above to P-completion, restricting F to be a finite P-group. We show that, in fact, it suffices to check φ^* for $F = Z/p$, p P; and that this remains true (in an obvious sense) if we remove the condition that G be nilpotent.

We will also discuss a relativization of this result, obtained by replacing "groups" by "surjections of groups". In this relativization there is no real loss of generality in restricting attention to "groups over a fixed group Q". The results have applications to the study of fibre spaces. The main result is due to Vidhyanath Rao.

D. F. Holt Machine computation of Schur multipliers of finite groups

We will give a brief description of a series of computer programmes which together calculate the Schur multiplier $M(G)$ of a finite group G defined by generating permutations on a set Ω . The idea is to find $P \in Syl_p(G)$, and then to calculate $M(P)$ using the well-known nilpotent quotient algorithm, originally due to I. D. Macdonald. $M(G)_p$ can then be calculated as a factor group of $M(P)$, using double coset representatives of P in G, and some cohomology theory.

L. G. Kovács Simply generated indecomposable modules for SL_2

Let p be a prime, F the algebraic closure of the field of p elements, and G the group $SL(2,F)$ acting naturally on a 2-dimensional F-space V. The composition factors of the tensor powers of V (as FG-modules) are well-known. We determine the indecomposable direct summands of the tensor products of these irreducible modules. We find that two direct sums of directs summands of tensor powers of V are isomorphic if they have the same composition factors with the same multiplicities. This was first conjectured by Schooneveldt, and enables one to determine the module structure of the homogeneous components of degree prime to p in the free Lie algebra of rank 2. -- The methods are those of finite group representations. In particular, one obtains on the way the $(2p - 1)^n$ isomorphism types of indecomposable direct summands of tensor products of irreducible $SL(2,p^n)$-modules (over any field containing the field of p^n elements). For p = 2, this was done by Alperin some years ago.

P. Landrock — Experimental group theory

We are interested in the Adams operators on finite groups, i.e. the maps $x \to x^n$ for n arbitrary but fixed, or similarly, an operation on the character ring of a finite groups, given by

$$\tilde{\psi}(x) = \psi(x^n) := \psi^{[n]}(x).$$

Our aim is to discuss the corresponding generalized Frobenius-Schur values, $\varepsilon_n(\psi) := (\psi^{[n]},1)_G$. We will state some remarkable observations, as well as applications in modular representation theory.

H. Lausch — Kategorientheoretische Begriffe in der Theorie der Fittingklassen

Die Blessenohl-Gaschütz Konstruktion normaler Fittingklassen stellt nichts anderes dar als die Konstruktion eines Kokegels über einem gewissen Funktor, wobei der universale Kokegel die kleinste normale Fittingklasse liefert. Ähnlich lassen sich allgemeiner ganze Lockett-Abschnitte beschreiben. Auch Fittingklassen können durch kategorische Konstruktionen beschrieben werden, sowie das Lockett-Problem. Es ist eine offene Frage, inwieweit jedoch kategorientheoretische Methoden zur Lösung von Problemen über Fittingklassen beitragen können.

A. I. Lichtman — On linear groups over a field of fractions of a polycyclic group ring

Let D be a skew field with center Z and let D^* be its multiplicative group. Let G be a subgroup of D^* and T a subfield of Z. We denote the subfield generated by T and G by T(G).

THEOREM 1 : Let D = T(G) where G is polycyclic-by-finite. Then any non-central normal subgroup of D contains a non-cyclic free subgroup.

COROLLARY : Let D be a field of fractions of a group algebra of a torsion free polycyclic-by-finite group. Then every non-central normal subgroup of D contains a non-cyclic free subgroup.

THEOREM 2 : Let D = T(G) be a field generated by a polycyclic-by-finite group G. Then every periodic subgroup of $D_n = GL(n,D)$ is locally finite. (This answers a question of D. Farkas in Comm. in Algebra 8 (1980), 585 - 602.)

V. D. Mazurov **Über maximale Untergruppen endlicher einfacher Gruppen**

Wenn alle Kompositionsfaktoren maximaler Untergruppen der bekannten endlichen einfachen Gruppen bekannt sind, dann sind auch alle endlichen einfachen Gruppen bekannt. Diese Bemerkung zeigt einen anderen Weg zur Klassifizierung der endlichen einfachen Gruppen.
Sei M eine maximale Untergruppe der endlichen einfachen Gruppe G; dann gibt es einen nicht-trivialen irreduziblen G-Modul V und einen Vektor $v \in V$ derart, daß $M = \{m \in G;\ vm = v\}$. Zum Beispiel ist die Gruppe M_{12} von Mathieu gleich dem Stabilisator eines Vektors im A_{12}-Modul des Grades 132, der dem Typ (6,6) in der Jungschen Tabelle entspricht. Dies ergibt neue kombinatorische Eigenschaften von M_{12}.

Ju. I. Merzljakov **Locally polycyclic groups with abelian factors of finite but non-bounded ranks**

In the paper "On locally soluble groups of finite rank" Algebra i Logika 8_6(1969), 686 - 690. Zbl. 244.20021 (1973) the author constructed some examples of locally polycyclic torsion free groups whose abelian subgroups are of finite but non-bounded ranks. The aim of the present report is to show that many of the locally polycyclic groups constructed by means of the technique of $\varkappa$-deformations developed in this paper possess, indeed, the following much stronger property: not only the abelian subgroups but also all abelian sections of these groups have finite ranks while the ranks of the abelian subgroups are non-bounded.

G. Michler **Some applications of the classification theorem to modular representation theory**

THEOREM 1 : Let G be a finite group with a cyclic Sylow p-subgroup D ($\neq 1$). Then the principal block B_0 is the only p-block of G if and only if $D \triangleleft G$ and $O_p(G) = 1$.
COROLLARY : If G is a non-solvable transitive permutation group of prime degree p 5, then G has a p-block of defect zero.
THEOREM 2 (jointly with P. Brockhaus) : If G is a finite simple group and $p \neq 2$, then G has at least two p-blocks.
Using this result, P. Brockhaus could solve D. A. R. Wallace's problem by showing that the group algebra FG of a finite group G over a field F of characteristic $p > 0$ has a Jacobson radical J(FG) of maxiaml possible vector space dimension if and only if G has a normal Sylow p-subgroup.

P. M. Neumann Subnormal subgroups of infinite soluble groups

What are the soluble groups that have at most countably many 2-step subnormal subgroups ? That question arose in work with Gerhard Behrendt, a former student of mine at Oxford. - Such groups are minimax groups (that is, they have a finite subnormal series in which each factor is either cyclic or C_{p^∞} for some prime p); further, for each prime p, C_{p^∞} can occur at most twice as a factor in such a series; and, finally, these groups are nilpotent-by-abelian-by-finite and the third term of the lower-central series of the nilpotent piece is finite. From this structure theorem it follows that these groups have only countably many subgroups.

J. B. Olsson Character correspondences in GL(n,q)

(Joint work with G.O. Michler) Recently, P. Fong and B. Srinivasan determined the r-blocks of GL(n,q) and U(n,q) for $r \neq 2$, $r \nmid q$. Consider an r-block $B = B_{s,\lambda}$ of GL(n,q) with defect group R, and let Ch(B) (resp. $Ch_0(B)$) be the set of irreducible characters (of height 0) in B.
Proposition 1 : There exists a canonical heightpreserving bijection between Ch(B) and $Ch(\tilde{B}_0)$, where $\tilde{B}_0$ is the principal r-block of a subgroup $\tilde{G}$ of G depending on (s,λ).
Proposition 2 : The Brauer correspondents b and $\tilde{b}_0$ of B and $\tilde{B}_0$ are Morita equivalent; so the above statement is also true for Ch(b) and $Ch(\tilde{b}_0)$.
Proposition 3 : There is a canonical bijection between $Ch_0(\tilde{B}_0)$ and $Ch_0(\tilde{b}_0)$. - As a corollary we obtain Alperin's height 0-conjecture for GL(n,q). - Similar results should hold for the unitary groups. In the course of the proof a height 0-correspondence is also given for the symmetric groups. The correspondence in Prop. 3 is established using Green's correspondence and some combinatorics.

W. Plesken Irreducible lattices of some finite groups

Let $K/\mathbb{Q}_p$ be a finite field extension of the p-adic field $\mathbb{Q}_p$, R the valuation ring in K. Group rings RG are investigated with the aim to describe the epimorphic images eRG of RG and hence the irreducible RG-lattices, where e runs through the central primitive idempotents of KG. In case the p-decomposition numbers of G are all 0 and 1 this problem is reduced to the determination of some well-defined parameters. A generalization of Jacobinski's conductor formula is proved in order to have a method to find these parameters. Some examples are discussed, e.g. $SL_2(q)$ for q odd at the prime 2; M_{11} at all primes; S_{10} at prime 5, blocks of multiplicity 1 or of cyclic defect.

L. Puig   <u>On block source algebras</u>   Let P be a finite p-group, p a prime number, and B an interior OP-algebra, that is, an O-algebra, O-free of finite rank, endowed with a group homomorphism $P \rightarrow B^*$, O being a complete discrete valuation ring with quotient field of characteristic zero and with algebraically closed residue class field of characteristic p. Call B a block source OP-algebra if there are a finite group G and a block b of G such that P is a defect group of b (up to isomorphism) and B is a source algebra of b (that is, there is a primitive idempotent j in $(OGb)^P$ such that $j \notin (OGb)^P_Q$ for any proper subgroup Q of P, and such that B jOGj as interior OP-algebras, where the homomorphism from P to $(jOGj)^*$ maps u to uj). Then, on the one hand the O-algebras OG and B are Morita equivalent, and vertices, sources, and generalized decomposition numbers of any OGb-module can be computed from the corresponding B-module. On the other hand, it seems reasonable to conjecture that the set of isomorphism classes of block source OP-algebras is finite indeed, (i) for any n, the set of isomorphism classes of block source OP-algebras of O-rank n is finite, and (ii) Brauer's conjecture on the existence of a function of the defect bounding the Cartan numbers and Feit's conjecture on the finiteness of the set of isomorphism classes of irreducible source modules over a given vertex imply the existence of a function of P bounding the O-rank of block source OP-algebras.

Derek Robinson   <u>Soluble groups with polycyclic quotients</u>   If a finitely generated soluble group is not polycyclic, it has a non-polycyclic quotient group the proper quotient groups of which are all polycyclic. Soluble groups with the latter property are called just-non-polycyclic. (JNP). Every JNP-group is finitely generated and abelian-by-polycyclic. Thus the theory of finitely generated modules over polycyclic group rings is relavant, and theorems of P. Hall and J. E. Roseblade can be applied to yield structural information about JNP-groups.

K. W. Roggenkamp   <u>Units in group rings of finite groups</u> (Report on joint work with L. Scott) Let G be a finite group of order $|G|$ and $Z_{(G)} =: R$ the semilocalisation of Z at all rational prime divisors of $|G|$. Denote by V(RG) the group of normalized units in RG. The following two proplems are considered :

P. I.: If $U \leq V(RG)$, $U \simeq G$, when is U conjugate to G in QG^*, the units in QG ?

P.II.: If $U \leq V(RG)$, $U \simeq G$, when is U conjugate to G in V(RG) ?

It is stated that a large class of simple groups admit an unconditional positive answer to P.I.; among them are A_n, PSL(n,q), M_{11}, M_{12}, the Suzuki- and Janko groups.

P. II. admits a positive answer for p-subgroups of

$G_o = \{ e^{2\pi i/p^r} \} \rtimes \mathrm{Gal}\, Z[\sqrt[p^r]{1}\,]$ for p-odd, for dihedral groups, and for metacyclic subgroups $C_{2^m} \rtimes C_{2^n}$, $n > 1$ of G_o for $p = 2$. If G is one of these groups, and if A is a G-module in characteristic p, then the isomopphism problem has a positive solution for any group in $H^2(G,A)$.

M. A. Ronan <u>Buildings and sporadic group geometries</u> The purpose of the talk was to give a dèscription of buildings, and generalizations thereof pertaining to the sporadic simple groups. We discussed the development of some of these ideas in roughly chronological order, beginning with buildings, then describing the work of Buekenhout on geometric amalgams using not just generalized n-gons as for buildings, but also complete graphs. Then we discussed the recent theorem of Tits showing that (almost) all geometric amalgams of generalized n-gons are covered by buildings. In conclusion we discussed geometries built from p-local subgroups of the sporadic groups which in some cases are, and in some cases come very close to being, amalgams of generalized polygons.

P. Rowley <u>Automorphisms of trees</u> Suppose G is a group with subgroups P_1 and P_2 satisfying

i) $\langle P_1, P_2 \rangle = G$; ii) $P_1 \cap P_2$ does not contain any normal $\neq 1$ of G

iii) $P_i/O_2(P_i)$ is a dihedral group of order $2p_i$, p_i an odd prime, $i=1,2$;

iv) $P_1 \cap P_2$ is a Sylow 2-subgroup of P_i, $i = 1,2$.

If, under these assumptions, $\{p_1, p_2\} \neq \{3\}$, then either P_1 is p_1-closed or P_2 is p_2-closed.

Peter Schmid <u>Lifting irreducible modules of p-soluble groups</u>

Let p be aprime and let G be a (finite) p-soluble group. The celebrated Fong-Swan theorem states that every (absolutely) irreducible p-modular character of G can be lifted to characteristic 0. It is unnecessary to assume spiitting fields : Theorem : Let (K,R,k) be a p-modular system (with R complete). Let W be an irreducible kG-module. Then there exists a KG-module V lifting W. Moreover, V is uniquely determined (up to isomorphism) by W provided p is odd and K is unramified over $\mathbb{Q}_p$. This should be compared with the surjectivity of the decomposition map (for arbitrary finite groups) which likewise holds without assuming that the fields are large enough (Chevalley, Dress). The proof is based on recent work of Isaacs and the (simple) observation that the p-rational characters are precisely those which can be realized over unramified extensions of $\mathbb{Q}_p$.

P. Schupp Group theory and formal language theory

Given a finitely generated group presentation $G = \langle X; R \rangle$, define the word problem $W(G)$ to be the set of all words in $X^{\pm 1}$ which represent the identity of G. Anisimov raised the question " If $W(G)$ is a regular or context-free language in the usual sense of formal language theory, what can one say about G algebraically ?" This question has turned out to be very fruitful. It is easy to see that G has regular word problem if and only if G is finite. In joint work, Dave Muller and I have shown that a finitely generated group is virtually free if and only if it has context-free word problem and is accessible. (The last condition is surely redundant.) The proof uses the theory of ends. Natural subclasses of the virtually free groups correspond to natural subclasses of context-free languages.

E. Scott Finitely presented infinite simple groups

This is a discussion of work which follows on from R.J. Thompson's paper in "Word Problems II" edited by W.W.Boone and G. Higman and from G. Higman's work in "Finitely presented infinite simple groups" number 8 in the Canberra Notes series. -- The groups $G_{n,1}$ constructed by G. Higman are extended, inside R.J.Thompson's group, by certain subgroups H of the infinite wreath product $S_{n-1}(wrS_{n-1})^{\omega}$ in such a way that if H is finitely presented then so is $\langle G_{n,1}, H \rangle$, and the commutator subgroup of $\langle G_{n,1}, H \rangle$ is simple. When the commutator subgroup has finite index we have a finitely presented simple group. We can choose an integer m such that the split extension $\mathbb{Z}^n \cdot GL(n,\mathbb{Z})$ is a subgroup of $S_{m-1}(wrS_{m-1})^{\omega}$ of the right type and such that the commutator subgroup of $\langle G_{m,1}, \mathbb{Z}^n \cdot GL(n,\mathbb{Z}) \rangle$ is a finitely presented simple group. Thus, any linear group can be embedded in a finitely presented simple group.

D. Segal The isomorphism problem for polycyclic groups

It is known that there is no uniform algorithm for deciding whether an arbitrary pair of finite presentations of groups present isomorphic ones or not. It is therefore of interest to develop algorithms which decide this question for finite presentations of restricted classes of groups. I have constructed such an algorithm (in joint work with N. Maxwell) for the class of polycyclic-by-finite groups. The algorithm is modelled on that which solves the isomorphism problem for nilpotent groups (Grunewald-Segal 1980), but is considerably more involved. The main steps are : a) Effective construction of all inequivalent "semi-simple splittings" for a "splittable" polycyclic group; b) effective location of a "splittable" subgroup of finite index in a polycyclic group; c) A procedure for deciding whether polycyclic subgroups of $GL(n,\mathbb{Z})$

are conjugate or not.
Some of the details may be found in my book "Polycyclic Groups" Cambridge University Press 1983.

R. L. Snider **Division rings of fractions of group rings**

Theorem : If G and H are finitely generated torsion-free nilpotent groups such that the division rings of fractions of their group rings are isomorphic, then G and H are isomorphic.

R. Solomon **Simple groups of 2-rank 2 revisited**

A finite simple group is *large* if by signalizer functor methods it may be shown to have a presentation of Coxeter-Steinberg-Tits type. A finite simple group is *small* if by Thompson-Bender methods combined with Brauer-Suzuki-Feit methods it may be shown to be a split BN-pair of rank 1 or 2. The Classification Theorem asserts that every finite simple group is either large or small (except for approximately 26 errors). The Gorenstein-Lyons Large Reclassification Project is aimed at proving that every simple group of p'-local p-rank ≥ 3 is large. The Small Reclassification Project for $p = 2$ includes the revised treatment of simple groups of 2-rank ≤ 2, triumphantly achieved by Bender and Glauberman in the dihedral case. The Bender approach applied to the semidihedral-wreathed case suggests the dichotomy: $H = C_G(t)$ is maximal in G ($U_3(q)$ and M_{11} cases) or $H \subset M$ with $F^*(M)$ a p-group ($L_3(q)$ case). Aschbacher, Gilman, and Solomon have established the following proposition by "local" arguments : Theorem : Let G be a finite simple group with semidihedral or wreathed Sylow 2-subgroups. Assume that every proper simple section of G is known. Then either $G \simeq L_3(q)$ or $H = C_G(t)$ is maximal in G; $H = (L \times O(H))S$ with S a Sylow 2-subgroup of G, $L \simeq SL(2,q)$ and $O(H)$ an abelian or Frobenius TI-subgroup. Moreover, either $O(H)$ is a cyclic subgroup of $Z(H)$ or H' is a TI-subgroup of G.

U. Stammbach **On the compostion factors of principal indecomposable modules** Let G be a finite group and k the field with p elements, $p \mid |G|$. Denote by J the Jacobson radical of kG. If S is a simple module with projective cover P, let $\mathfrak{Z}(S)$ denote the set of Isomorphism classes of simple modules A appearing as a direct summand in JP/J^2P. Clearly, $A \in \mathfrak{Z}(S)$ if and only if $\mathrm{Ext}^1_{kG}(S,A) \neq 0$. We have

a) (Pahlings, Stammbach) $\bigcap_{A \in \mathfrak{Z}(S)} \mathfrak{C}A \cap \mathfrak{C}S = O_{pp'}\mathfrak{C}S$; here $\mathfrak{C}S = \{x \in G; \; xs = s \text{ for all } s \in S\}$.

b) If G is p-solvable and if $\mathfrak{C}A \neq \mathfrak{C}S$, then $\mathrm{Ext}^1_{kG}(S,A) = \mathrm{Hom}(C/C' \otimes S, A)$ where $C = \mathfrak{C}A \cap \mathfrak{C}S$.

Result a) strengthes a theorem of Mächler and Willems; result b) generalizes a theorem of Gaschütz (S = k).

C. Y. Tang <u>Conjugacy separability of certain one-relator groups</u>

It is well-known that one-relator groups need not be residually finite (R^F). However it is conjectured by G. Baumslag that one-relator groups with torsion are R^F. A number of such groups has been proved to be R^F. In view of Baumslag's conjecture and B. B. Newman's result that one-relator groups with torsion have solvable conjugacy problem it is natural to ask which one-relator groups with torsion are conjugacy separable (c.s.). Using the concept of potency we show that groups with the presentation $\langle a,b;(a^{\ell}b^{m})^{t}\rangle$ are c.s.. It would be interesting to know whether $\langle a,b;(a^{-1}b^{\ell}ab^{m})^{t}\rangle$, $t > 1$, is c.s..

J. G. Thompson <u>Some characters of finite groups via the Riemann-Roch theorem</u> Professors Mennicke and Gaschütz have indicated the relevance of work of Eichler (Einige Anwendungen der Spurformel, 1967) and of Chevalley and Weil (Hamburger Abh. 10, 1934, and 11, 1936) to my subject. – If Γ is commensurable with $PSL(2,\mathbb{Z})$, the normal subgroup A of Γ is said to be anomalous if A is generated by its elements of finite order together with its unipotent elements. If X is of finite index in Γ, G(X) denotes the graded ring of modular forms associated to X; $G_k(X) = \{ f \;;\; f$ is holomorphic on $\mathfrak{H}$ and at all cusps and $f(\frac{a\tau + b}{c\tau + d}) = (c\tau + d)^k f(\tau)$ for all $\binom{a\ b}{c\ d}' \in X \}$.

Then $G_k(X)$ is a module for Γ/X (provided X is normal in Γ), with character $\chi_k = \chi_{k,\Gamma,X}$. – We say that A is good iff whenever $A \subset X \triangleleft \Gamma$ with X finite, the character $\chi_{k,\Gamma,X}$ is rational valued for all k = 2, 4, ... Then with a possible proviso concerning unipotent elements, we get a result :

Theorem : A is good iff for every element $\gamma \in \Gamma$ which is either unipotent or of finite order, $\gamma^2 \in A$.

In any case, if $C = \langle c \rangle$ is the cyclic group of order n, and for each $m | n$, C_m is the subgroup of C of order m, and if χ is a rational valued character of C, then by Möbius inversion, we have

$$\chi(c) = \sum_{m|n} (\chi/C_m, 1_{C_m}) \cdot t_m \quad \text{where} \quad t_m = \sum_{\substack{d|n \\ m|d}} \mu(d)\,\mu(dm/n)/\varphi(d),$$

where μ, φ are the Möbius and Euler functions, respectively.

F. Timmesfeld Tits geometries and parabolic systems in finite groups

A parabolic system $\mathcal{P}$ of the finite group G is a system of subgroups X_i, $i \in I = \{1,\dots,n\}$ and satisfying

1) $G = \langle X_i ;\ i \in I \rangle \neq \langle X_j ;\ j \in J \subsetneq I \rangle$;
2) $\bigcap X_i$ contains a Sylow p-subgroup S of G;
3) $\overline{X}_i = O^{p'}(X_i/O_p(X_i))$ are rank 1 Chevalley groups in characteristic p;
4) $\overline{X}_{i,j} = O^{p'}\langle X_i, X_j\rangle / O_p\langle X_i, X_j\rangle$ is a rank 2 Chevalley group in Characteristic p for $i \neq j$.

The diagram Δ of $\mathcal{P}$ is defined in the usual way. If $p = 2$ and Δ has only "single" bonds a characterisation of $\mathcal{P}$ and G was given. This has lead to an alternative classification of Tits geometries of characteristic 2-type and to a classification of Tits geometries, all the rank 2 residues of which are desarguesian projective planes of even order.

S. Tobin The lower central chain in groups with exponent 4

Let $\gamma_1 G = G$, $\gamma_{i+1} G = [\gamma_i G, G]$ define the lower central chain in a group G. Let G have exponent 4. Then it is known that

i) $(\gamma_2 G)^2 \leq \gamma_4 G$, and hence $(\gamma_n G)^2 \leq \gamma_{n+2} G$ for $n \geq 2$.
ii) $(\gamma_4 G)^2 \leq \gamma_8 G$, and hence $(\gamma_n G)^2 \leq \gamma_{n+4} G$ for $n \geq 4$.
iii) There are groups G in which

$$(\gamma_2 G)^2 \nleq \gamma_5 G$$
$$(\gamma_3 G)^2 \nleq \gamma_6 G .$$

Theorem : If G has exponent 4, $(\gamma_n G)^2 = [\gamma_n G, \gamma_n G]$ for all $n \geq 4$. This result is best possible.

Corollary : Let $B(n)$ be the Burnside group of exponent 4 with n generators, then $\gamma_k B(n)$ is elementary abelian whenever $2k \geq 3n - 1$.

Conjecture : $(\gamma_k B(n))^2 \neq \langle 1 \rangle$ whenever $2k \leq 3n - 2$.

M. R. Vaughan-Lee An aspect of the Nilpotent Quotient Algorithm

Let G be a group with generators $a_1, \dots, a_n$ and relations

$$a_i^p = a_{i+1}^{\alpha_{i,i+1}} a_{i+2}^{\alpha_{i,i+2}} \cdots a_n^{\alpha_{i,n}}, \quad 1 \leq i \leq n,$$
$$a_i a_j = a_j a_i a_{i+1}^{\beta_{i,j,i+1}} a_{i+2}^{\beta_{i,j,i+2}} \cdots a_n^{\beta_{i,j,n}}, \quad 1 \leq j < i \leq n.$$

Then G is a p-group of order at most p^n. A word in the generators is normal if it is of the form $a_1^{\alpha_1} a_2^{\alpha_2} \cdots a_n^{\alpha_n}$ with $0 \leq \alpha_i < p$. Any word in the generators can be reduced to a normal word by the colledtion process. This entails systematically replacing subwords of the form a_i^p and $a_i a_j$ with $i > j$ with the right hand side of the corresponding relation. If the order in which these substitutions are to be made is specified by some algorithm then the collection defines a product on the set of normal words: write one word after the other and collect. It is well known that G has order p^n provided this product satisfies the following associativity conditions :

$$(a_i a_j)a_k = a_i(a_j a_k) \quad \text{for } 1 \leq k < j < i \leq n,$$
$$a_j(a_k^p) = (a_j a_k^{p-1})a_k \quad \text{for } 1 \leq k < j \leq n,$$
$$(a_i^p)a_j = a_i(a_i^{p-1}a_j) \quad \text{for } 1 \leq j \leq i \leq n.$$

We show that if G is a d-generator group then it is sufficient to check the above conditions when k d for the presentations that arise in the nilpotent quotient algorithm. In the case of B(4,4) for example, which has order 2^{422} and class 10, this reduces the number of consistency checks from 168237 to 82062.

P. J. Webb   Character tables of the Green ring

Two character tables are associated to the Green ring A(G) of kG-modules, where k is a field of characteristic p. The first contains the values of the distinct homomorphisms $A(G) \to \mathbb{C}$ on the indecomposable modules, and was considered by Green. The second table satisfies a set of orthogonality relations with respect to the first which extend the usual Brauer orthogonality relations, and this was described by Benson and Parker. Certain coefficients arise in the relations which are analogous to the orders of centralizers of elements of G. We show that these coefficients satisfy certain arithmetic properties, for example of divisibility, and give information which allows them to be computed rather easily. The character tables have a number of uses. For example, they contain complete information about the cohomology of the modules from which they are constructed.

B. A. F. Wehrfritz   Nilpotence in skew linear groups

We discuss the origins and consequences of the following theorem : Let n be a positive integer, D a locally finite-dimensional division algebra of characteristic p ≥ 0, and G any subgroup of GL(n,D). Then $\zeta(G)/\zeta_\omega(G)$ is locally finite, and its p′-part is finite of order dividing n! ,if p = 2 or 4 divides p - 1, and dividing $2^n n!$ otherwise. - These bounds are essentially the best possible for every n and p. - Here $\zeta(G)$ is the hypercentre of G and $\zeta_\omega(G)$ the ω-th term of the upper central series of G.

Berichterstatter : O. H. Kegel

Tagungsteilnehmer

Dr. H. Åberg
Mathematisches Institut
der Universität
Stockholm

Schweden

Dr. R. Bryant
Department of Mathematics
U M I S T
Manchester M60 1QD

England

Prof. Dr. S. I. Adian
Steklov Mathematical Institute
Ul. Vavilova 42
MOSCOW V-333

U.S.S.R

Dr. R. Bryce
Department of Mathematics
Australian National University
Canberra, A. C. T. 2600

Australia

Prof. Dr. J. L. Alperin
Department of Mathematics
University of Chicago
Chicago, Illinois 60637

U S A

Dr. M. J. Collins
Mathematics Institute
24 -29, St. Giles
OXFORD OX1 3LB

England

Prof. Dr. J. C. Beidleman
Department of Mathematics
University of Kentucky
Lexington, Kentucky 40506

U S A

Prof. Dr. K. Doerk
Fachbereich Mathematik
Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. R. Bieri
Mathematisches Seminar
Universität Frankfurt
Robert Mayer Str. 6 - 10

6000 Frankfurt a. M.

Prof. Dr. W. Gaschütz
Mathematisches Seminar
Universität Kiel
Olshausenstr. 40 - 60

2300 Kiel 1

Prof. Dr. N. Blackburn
Department of Mathematics
The University
Manchester M13 9PL

England

Prof. Dr. K. W. Gruenberg
Department of Pure Mathematics
Queen Mary College
Mile End Road
LONDON E1 4WS

England

Prof. Dr. F. Grunewald
Mathematisches Institut
Universität Bonn
Wegelerstr. 10

5300 Bonn 1

Prof. Dr. O. H. Kegel
Mathematisches Institut
Universität Freiburg
Albertstr. 23 b

7800 Freiburg i. Br.

Dr. P. Hauck
Mathematisches Institut
Universität Freiburg
Albertstr. 23 b

7800 Freiburg i. Br.

Prof. Dr. L. G. Kovács
Department of Mathematics
Institute for Advanced Studies
Australian National University
Canberra, A. C. T. 2600

Australia

Prof. Dr. H. Heineken
Mathematisches Institut
Universität Würzburg
Am Hubland

8700 Würzburg

Prof. Dr. P. Landrock
Department of Mathematics
Aarhus University
AARHAUS

Dänemark

Prof. Dr. G. Higman
Mathematics Institute
24 - 29, St. Giles
Oxford OX1 3LB

England

Prof. Dr. H. Lausch
Department of Mathematics
Monash University
Clayton, Victoria 3168

Australia

Prof. P. Hilton
Department of Mathematics
SUNY Centre at Binghamton
Binghamton, N.Y. 13901

U S A

Prof. Dr. A. I. Lichtman
Department of Mathematics
Ben Gurion University of
the Negev
Beer Sheva

Israel

Dr. D. F. Holt
Mathematics Institute
Warwick University
Coventry CV 4 7 AL

England

Prof. Dr. V. D. Mazurov
Institute of Mathematics
Novosibirsk, 90
USSR, 630090

U S S R

Prof. Dr. B. Huppert
Fachbereich Mathematik
Universität Mainz
Saarstr. 21

6500 Mainz

Prof. Dr. Ju. I. Merzljakov
Institute of Mathematics
Novosibirsk, 90
USSR, 630090

U S S R

Prof. Dr. G. O. Michler
Fachbereich 6 - Mathematik
Universität Essen - Gesamthochschule
Universitätsstr. 2

4300 Essen

Prof. Dr. M. A. Ronan
Department of Mathematics
University of Illinois
at Chicago Circle
Chicago, Illinois 60680

U S A

Prof. P. M. Neumann
The Queen's College
Oxford OX1 4AW

England

Dr. P. J. Rowley
Department of Mathematics
U.M.I.S.T.
Manchester M 60 1 QD

England

Prof. J. B. Olsson
Institut für Mathematik
Universität Dortmund

4600 Dortmund 50

Prof. Dr. P. Schmid
Mathematisches Institut
Universität Tübingen
Auf der Morgenstelle 10

7400 Tübingen

Prof. Dr. W. Plesken
Lehrstuhl D für Mathematik
Rheinisch-Westfälische
Technische Hochschule
Templergraben 64

5100 Aachen

Prof. Dr. P. E. Schupp
Department of Mathematics
University of Illinois
Urbana, Illinois 61801

U S A

Prof. Dr. L. Puig
Département de Mathématiques
Université Paris VII
2, Place Jussieu

F - 75221 Paris

Dr. Elizabeth Scott
Mathematics Institute
24 - 29, St. Giles
Oxford OX1 3LB

England

Prof. Dr. D. J. S. Robinson
Department of Mathematics
University of Illinois
Urbana, Illinois 61801

U S A

Dr. D. Segal
Department of Mathematics
U.M.I.S.T.
Manchester M 60 1 QD

England

Prof. Dr. K. W. Roggenkamp
Mathematisches Institut
Universität Stuttgart
Pfaffenwaldring 57

7000 Stuttgart 80

Prof. R. L. Snider
Department of Mathematics
Virginia Polytechnic Institute
and State University
Blacksburg, Virginia 24061

U S A

Prof. R. Solomon
Department of Mathematics
Ohio State University
Columbus, Ohio 43210

U S A

Prof. Dr. S. Tobin
Department of Mathematics
University College of Ireland
Galway

Ireland

Prof. Dr. U. Stammbach
Mathematisches Institut
ETH Zürich

CH - Zürich

Dr. M. R. Vaughan-Lee
Mathematics Institute
24 - 29, St. Giles
Oxford OX1 3LB

England

Prof. Dr. C. Y. Tang
Department of Pure Mathematics
University of Waterloo
Waterloo, Ontario N2L 3G1

Canada

Dr. P. J. Webb
Department of Mathematics
The University
Manchester M13 9PL

England

Prof. Dr. J. G. Thompson
Department of Mathematics
Cambridge University
16 Mill Lane
Cambridge GB2 1SB

England

Prof. B. A. F. Wehrfritz
Department of Pure Mathematics
Queen Mary College
Mile End Road
London E1 4NS

England

Prof. Dr. F. G. Timmesfeld
Mathematisches Institut
Universität Giessen
Arndtstr. 2

6300 Giessen

Dr. J. S. Wilson
Christ's College
Cambridge CB2 3BU

England