MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 20/1982

Kombinatorik

9.5. bis 15.5. 1982

Im Mittelpunkt der Tagung, die unter der Leitung von D. Foata (Strasbourg) stattfand, standen Vorträge und Diskussionen über Partitionen, q - Reihen, symmetrische Funktionen, orthogonale Funktionen und spezielle Funktionen. Diese Themen wurden vom Standpunkt der Kombinatorik aus behandelt. Insbesondere kamen die vor allem in den letzten beiden Jahren entwickelten Methoden, klassische Identitäten mit Hilfe von kombinatorischen Überlegungen zu beweisen, öfters zur Sprache.

Für das Gelingen der Tagung waren auch die gute Ausstattung und schöne Lage des Instituts sehr wesentlich. Last not least sei den Mitarbeitern des Instituts für ihre sorgfältige und unbürokratische Betreuung an dieser Stelle herzlich gedankt!

Vortragsauszüge

G. E. ANDREWS:

Frobenius' Representation of Partitions and Related Problems

Frobenius first observed that each ordinary partition of n can be written as $\begin{pmatrix} a_1 & a_2 & \cdots & a_r \\ b_1 & b_2 & \cdots & b_r \end{pmatrix}$ where $a_1 > a_2 > \ldots > a_r \geq 0$; $b_1 > b_2 > \ldots > b_r \geq 0$ and $n = r + \Sigma a + \Sigma b$. If we relax the condition that the a's and b's are respectively distinct to the condition that no integer appears more than k times in each row, we obtain generalized Frobenius partitions. Let $F_k(n)$ denote the number of generalized Frobenius partitions of n of multiplicity k. Numerous interesting results arise for the $F_k(n)$. E. g.

(1) $\sum_{n \geq 0} F_2(n) = \prod_{n=1}^{\infty} (1-q^n)^{-1}(1-q^{12n-10})^{-1}(1-q^{12n-9})^{-1}(1-q^{12n-3})^{-1}(1-q^{12n-2})^{-1}$

(2) $5 \mid F_2(5n+3)$

etc. etc.

G. E. ANDREWS:

What Is (or Should Be) a Simple Combinatorial Proof of the Rogers - Ramanujan Identities

A history of the interplay of bijective and algebraic - analytic proofs of partition identities is presented. Three bijective proofs of Euler's theorem (the partitions of n into distinct parts are equinumerous with the partitions of n into odd parts) are presented along with the refinements of Euler's theorem implied by them. Also treated are the Rogers - Ramanujan Schur identities and Schur's "difference 3" theorem; again the analytic refinements related to combinatorial treatments are presented.

The classical bijections seem to rely on algorithms that terminate in a number of steps related to the number of parts of the considered partitions. Is there such an "efficient" bijection related to the Rogers - Ramanujan identities? The pathbreaking bijection of Garsia - Milne appears to be inefficient by this measure.

M. BARNABEI:

Umbral Methods for Multi - Variate Hermite Polynomials

Multivariate umbral methods can be used to give quick proofs of some basic facts in the theory of multivariate Hermite polynomials, such as recurrence relations, Rodrigues formula and Burchnall - Feldheim - Watson formulas.

G. BARON:

Binomial Systems on Free Monoids

Since the original papers of G. C. ROTA and his team several generalizations of binomial polynomials were considered.

We want to develop a noncommutative analogue of these structures.
Starting with an at most countable alphabet A considered as a poset we define a generalized shuffle product and a generalized concatenation for the words of A*. The coefficients appearing in the shuffle product are identified as an analogue of the ordinary and EILENBERG binomial coefficients and lead to a noncommutative version of the binomial theorem. From this point of view we study related polynomial systems and operator systems and also formal power series on A. Some algebraic results derived on these structures will be presented.

D. BRESSOUD:

A constructive proof of the q - analog of Pfaff - Saalschütz

It is proved by constructive methods that the generating function for pairs of partitions, Π and Ψ, satisfying
(1) Π has n distinct parts, all ≥ 1 and $\leq m + n - k$,
(2) Ψ has m parts, zero permitted, all parts $\leq s + k$,
(3) the crossing numbers of Ψ with respect to Π starting at s is r; is

$$q^{\binom{n+1}{2}} \begin{bmatrix} s \\ r \end{bmatrix} \begin{bmatrix} m+n-s \\ n-s+r \end{bmatrix} \begin{bmatrix} m+n+k-r \\ m+n \end{bmatrix} q^{(m-r)(s-r)} .$$

Summing over all r, and recognizing that the generating function for pairs of partitions satisfying (1) and (2) is

$$q^{\binom{n+1}{2}}\begin{bmatrix} m+n+k \\ m+n \end{bmatrix}\begin{bmatrix} m+s+k \\ m \end{bmatrix}.$$

yields the q - analog of the Pfaff - Saalschütz summation.

R. CANFIELD:

Interplay between combinatorics and topology

We consider a class of regular hypergraphs whose vertices are all the k - element subsets of a given set and whose edges are families of subsets which are pairwise disjoint. A problem is to determine the chromatic number of these. Following a method used by Lovasz, we relate this coloring problem to questions about coverings by open sets of certain topological spaces.

J. CIGLER:

Some methods for q - identities

Some results obtained by students in Vienna are given. PAULE gives a simple transformation

$$\sum a_k \begin{bmatrix} a+b \\ a+k \end{bmatrix}\begin{bmatrix} a+b \\ b+k \end{bmatrix} = \sum \frac{[a+b]!}{[a-j]![b-j]!}\frac{q^{j^2}}{[2j]!} S_{2j}$$

with $S_{2j} = \sum\limits_k \begin{bmatrix} 2j \\ j+k \end{bmatrix} q^{-k^2} a_k.$

For suitable values of a_k finite forms of known q - identities are derived such as Rogers - Ramanujan, Rogers - Selberg, Göllnitz - Gordon, etc. .

E. g. $a_k = (-1)^k q^{\frac{k}{2}(5k+1)}$ gives the finite form of R. - R. - I.:

$$\sum_k (-1)^k q^{\frac{1}{2}k(5k+1)} \begin{bmatrix} a+b \\ a+k \end{bmatrix} \begin{bmatrix} a+b \\ b+k \end{bmatrix} = \sum_j \frac{[a+b]!}{[a-j]![b-j]!} \frac{q^{j^2}}{[j]!} (1-q)^j$$

KRATTENTHALER has given a q - Lagrange formula, a q - generalization of Egorychev's inverse relation results and further q - identities.

M. CLAUSEN:

Pictures and standard tableaux

Pictures appeared first in papers by James/Peel (J. Alg. 56) and Zelevinsky (J. Alg. 69). Roughly speaking a picture is a bijection $T : A \to B$ $(A,B \subseteq \mathbb{N} \times \mathbb{N})$ such that T and T^{-1} both satisfy the same standard property. A problem which arises in representation theory is to compute explicitly all pictures between two skew diagrams A and B. We establish an algorithm which constructs by suitable hook deformations all those pictures.

This work was done jointly with F. Stötzer.

D. I. A. COHEN:

A Schur Bet

Having previously (Oberwolfach 1980) proved that the number of partitions of n into parts ≡ 1,4 (mod 5) called b(n) is

$$b(n) = \Sigma\ (-1)^k\ p(n - k/2\ (5k \pm 1))$$

we now continue by defining a(n,m) = the number of partitions of n into parts $\leq$ m such that any two parts differ by at least 2.

Schur noted that

$$\begin{cases} a(n,m) = a(n,m-1) + a(n-m,m-2) \\ a(n,0) = 0 \text{ unless } n = 0 \text{ when } a = 1 \\ a(n,1) = 0 \text{ unless } n = 0,1 \text{ when } a = 1. \end{cases}$$

Now let P(x,y,z) be the number of partitions of z into at most y parts each $\leq$ x. Obviously

$$P(x,y,z) - P(x-1,y,z) = P(x,y-1,z-x)$$

$$P(x,y,z) - P(x,y-1,z) = P(x-1,y,z-y).$$

Now we show

$$a(n,2m) = \Sigma\ \{P(m+5i+1,m-5i,n-10i^2-i) - P(m+5i+3,m-5i-2,n-10i^2-11i-3)\}$$

$$a(n,2m+1) = \Sigma\ \{P(m+5i+1,m-5i+1,n-10i^2-i) - P(m+5i+4,m-5i-2,n-10i^2-11i-3)\}.$$

Now take m = n. The P(x,y,z) turn into p(z) and the distinction between even and odd vanishes leaving

$$a(n) = \Sigma (-1)^k p(n - k/2 (5k \pm 1)).$$

Hence

$$a(n) = b(n)$$

and the Rogers - Ramanujan - Schur identities are proved. Bijections and congruence were also given.

P. FLAJOLET:

Combinatorial Aspects of Continued Fractions and Orthogonal Polynomials

We show that the universal continued fraction of the Stieltjes - - Jacobi type is equivalent to the characteristic series of labelled paths in the plane. The equivalence holds in the set of series in non - commutative indeterminates. Using it, we derive direct combinatorial proofs of continued fraction expansions for series involving known combinatorial quantities: the Catalan numbers, the Bell and Stirling numbers, the tangent and secant numbers, the Euler and Eulerian numbers ... . We also show combinatorial interpretations for the coefficients of the elliptic functions, the coefficients of inverses of the Tchebycheff, Charlier, Hermite, Laguerre and Meixner polynomials. Other applications include cycles of binomial coefficients and inversion formulae. Most of the proofs follow from direct geometrical correspondences between objects.

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D. FOATA:

Combinatorics of the Laguerre polynomials

The Laguerre polynomials $L_n^{(\alpha)}(x)$ $(n \geq 0)$ defined by

$$\sum u^n L_n^{(\alpha)}(x) = (1 - u)^{-\alpha-1} \exp(-xu/(1 - u)) \quad (n \geq 0).$$

have a combinatorial interpretation that can be used to prove most of the classical identities such as the Hille - - Hardy formula, the Erdélyi formula. Furthermore a multilinear version of the latter identity can be proved that is the analog for the Laguerre polynomials of the Kibble - - Slepian formula for the Hermite polynomials. See D. Foata & V. Strehl, Une extension multilinéaire de la formule d'Erdelyi pour les produits de fonctions hypergéométriques confluentes, C. R. Acad. Sc. Paris, 293 (1981), 517 - 520 and Combinatorics of the Laguerre polynomials, Proc. Waterloo Conference, 1982, to appear.

I. GESSEL:

Combinatorial Proofs of Saalschütz's Theorem and Multivariable Lagrange Inversion

If we equate coefficients in the identity

$$\frac{(1+x)^a(1+y)^b}{(1-xy)^{a+b+1}} = \sum_{m,n=0}^{\infty} \binom{a+n}{m}\binom{b+m}{n} x^m y^n,$$

we obtain a form of Saalschütz's Theorem.

This identity can be proved by counting paths between nodes of two colors in two different ways.

A multivariable Lagrange Inversion formula can be proved by counting functions on colored vertices; first directly, and second by connected components.

C. GREENE:

Some Properties of the Majorization Order

The majorization order $\lesssim$ on the set P_n of partitions of n is defined as follows: if $\theta = \{\theta_1 \geq \theta_2 \geq \ldots\}$ and $\lambda = \{\lambda_1 \geq \lambda_2 \geq \ldots\}$ then $\theta \lesssim \lambda$ iff $\theta_1 + \theta_2 + \ldots + \theta_i \leq \lambda_1 + \lambda_2 + \ldots + \lambda_i$ for $i = 1,2,\ldots$.

We obtain simple combinatorial characterizations of two important functions on the lattice $(P_n,\lesssim)$:

(i) the Möbius function and (ii) the height function. The first is based on a combinatorial decomposition of partitions (called the "staircase decomposition"), over which, in a certain sense, the Möbius function is multiplicative. Our arguments refine and "explain" earlier results of Brylawski and Bogart, which state that μ_{P_n} assumes only the values ±1, with certain periodicities modulo 3. The height function of P_n is characterized by special maximal chains (called "HV - - chains"), in which all covers of a certain kind ("H - steps") precede all covers of another kind ("V - steps"). Our main result (obtained jointly with D. J. Kleitman) is that any pair $\sigma \lesssim \lambda$ can be linked by an HV - chain, all of these chains have the same length, and this length is maximal.

J. HOFBAUER:

On q - analogs of the Lagrange inversion theorem and of Catalan numbers

Let us consider the following q - analogs of notion of n - th power of a formal power series: $\varphi_n'(z) = [n]\ \varphi_n(z)\bar{\varphi}(z)$ and $\psi_n'(z) = q^{-n}[n]\ \psi_n(qz)\bar{\psi}(z)$. Then the coefficients in the expansion

$$f(z) = \Sigma a_n \frac{z^n}{\varphi_n(z)\psi_n(z)}$$

are given by $a_n = \frac{1}{[n]} f'(z)\varphi_n(z)\psi_n(qz)\Big|_{z^{n-1}}$.

(In this general form, this q - analog of the Lagrange inversion theorem, is due to Chr. Krattenthaler).

The most important special cases are:

a) $\varphi_n(z) = (1-z)(1-qz)\ldots(1-q^{n-1}z) = (z)_n$, $\psi_n = 1$ (Carlitz 1973)

b) $\varphi_n(z) = e(a[n]z)$, $\psi_n = 1$ (Cigler 1980)

c) $\varphi_n(z) = (az)_n$, $\psi_n(z) = (q^{-n}z)_n$.

The last example is closely related to the q - analogs of the classical orthogonal polynomials (e. g. the little q - Jacobi polynomials of Andrews and Askey) and may be applied to obtain nice q - analogs of all inverse relations of Legendre and Čebyshev type in Riordan's book.

To illsustrate the differences of this q - Lagrange formula with that of Garsia, where $\varphi_n(z) = \varphi(z)\varphi(qz)\dots\varphi(q^{n-1}z)$, two different versions of q - Catalan numbers were presented.

P. KIRSCHENHOFER:

Lagrange Inversion and Sheffer Systems on free monoids

We present a generalization of the well known inversion formula of Lagrange - Good, which works for a special type of systems of formal power series on free monoids. The proof makes use of a result on binomial systems on free monoids corresponding to the classical or multivariate Steffensen formula. A generalization of the classical Rodrigues formula will be presented, too.
In the second part we present a short development of the "Sheffer" systems related to these binomial families, which seems to present a natural way of defining noncommutative analogues of classical polynomial systems such as e. g. Hermite polynomials.

A. LASCOUX & M. P. SCHÜTZENBERGER:

Ordering permutations and generalizing Schur functions

We refine the order on permutations due to Ehresmann (and called Bruhat order). By reading the edges of the graph of this order in different ways, one can describe the ring of polynomials modulo the ideal generated by the totally symmetric polynomials - which is, in geometry, the cohomology ring of the flag manifold - and generate different families of polynomials which have a geometrical and combinatorial interpretation. As a special case, one finds back the Schur functions.

P. LEROUX:

Jacobi Polynomials: Combinatorial interpretation and generating function

(With D. Foata. to appear in Proc. Amer. Math. Soc.)

The classical generating function for Jacobi polynomials

$$\sum_{n\geq 0} P_n^{(\alpha,\beta)}(x)\, u^n = 2^{\alpha+\beta} R^{-1}(1 - u - R)^{-\alpha} (1 + u + R)^{-\beta}$$

with $R = (1 - 2xu + u^2)^{1/2}$

is derived by purely combinatorial methods.

The combinatorial interpretation comes from the explicit expression

$$P_n^{(\alpha,\beta)}(x) = \sum_{j=0}^{n} \binom{n+\alpha}{n-j}\binom{n+\beta}{j}\left(\frac{x-1}{2}\right)^j \left(\frac{x+1}{2}\right)^i .$$

Setting $X = \frac{x-1}{2}$, $Y = \frac{x+1}{2}$, we define

$$P_n^{(\alpha,\beta)}(X,Y) = \sum_{i+j=n} \binom{n}{i} (\alpha + 1 + j)_i\ (\beta + 1 + i)\ X^i Y^j$$

This counts "Jacobi endofunctions" on $[n] = \{1,2,\dots,n\}$

that is • ordered partitions (A,B) of [n]

• injective functions $f: A \to [n]$

$g: B \to [n]$

with weights $(\alpha + 1)^{c(f)}(\beta + 1)^{c(g)}\, X^{|A|}\, Y^{|B|}$.
c(f) stands for the number of cycles of f. The generating function of $P_n^{(\alpha,\beta)}(X,Y)$ is obtained by standard techniques of combinatorial generating functions.

S. C. MILNE:

Schur functions and the invariant polynomials characterizing U(n) tensor operators

We give a direct formulation of the invariant polynomials ${}_uG_g^{(n)}(,\Delta_i,i,x_{i,i+1},)$ characterizing U(n) tensor operators $\langle p,q,\dots,q.0,\dots,0\rangle$ in terms of the symmetric functions S_λ known as Schur functions. To this end we show after the change of variables $\Delta_i = \gamma_i - \delta_i$ and $x_{i,i+1} = \delta_i - \delta_{i+1}$, that ${}_uG_g^{(n)}(,\Delta_i,i,x_{i,i+1},)$ becomes an integral linear combination of products of Schur functions $S_\alpha(,\gamma_i,)\cdot S_\beta(,\delta_i,)$ in the variables $\{\gamma_1,\dots,\gamma_n\}$ and $\{\delta_1,\dots,\delta_n\}$, respectively. By making further use of basic properties of Schur functions such as the Littlewood - Richardson rule we prove several remarkable new symmetries for the yet more general bisymmetric polynomials ${}_u^mG_g^{(n)}(\gamma_1,\dots,\gamma_n;\delta_1,\dots,\delta_m)$. In addition we derive the polynomial formula

$$ {}_u^mG_1^{(n)}(\gamma;\delta) = (-1)^{\binom{u+1}{2}} \sum_{\substack{\lambda=(\lambda_1,\dots,\lambda_{u+1})\\ \lambda_1\le u+1+m-n}} (-1)^{|\lambda|} S_{\lambda'}(\delta) S_{(u+1+m-n)-\lambda}(\gamma), $$

where $(u + 1 + m - n) - \lambda$ denotes the partition $\nu = (\nu_1, \nu_2, \ldots, \nu_{u+1})$ with $\nu_i = (u + 1 + m - n) - \lambda_{u+2-i}$.

W. OBERSCHELP:

Proof concepts for almost - all results

We introduce results of Fagin (J. Symb. Log. 41, 50 - 58) and Blass - Harary (J. Graph. Th. 3, 225 - 40) concerning 0 - 1 laws for relative frequencies of first - order - - defined n - element models in relations.
Then we interprete a generalization of Lynch (Ann. Math. Log. 18, 91 - 135), where 0 - 1 laws are proved, if a superimposed structure (e. g. the successor mod n) can be used. The idea of "richness" (technically: k - extendibility with respect to the Ehrenfeucht game) is explicated, and a negative result for finite linear order is compared with the successor case.
As a second generalization we consider relative frequencies with respect to special relation theories defined by a condition ϑ. ϑ is called Blass - Fagin (BF), if the limit exists and is 0 or 1 for every first order condition $\mathcal{L}$.
We exhibit the proof idea of Blass in the case, that ϑ is graph theory, and sketch, how things work in the framework of parametric relations (W. Oberschelp, Lecture Notes Math. (Springer) 579 (ed. Foata), 297 - 307). The concept of richness appears again in the BF - proof for parametric

conditions (W. Oberschelp, Oberwolfach 1980 and DMV - Meeting 1980 Dortmund, not yet published completely). Beyond that we present results of K. J. Compton (Dissertation 1981). So far the exponential generating power series had convergence radius $R = 0$. But Compton's results apply to $R > 0$. Here exactly the case $R = \infty$ yields BF - conditions. We interprete this case "at the other end of the convergence scale" as "poorness" in structure, contrasted to richness in the former cases. Finally we interprete Compton's most interesting positive BF - example, viz. equivalence relations, and correspondingly partitions of n in the analogous unlabelled case.

D. RAWLINGS:

Enumeration of permutations by descents, idescents, imajor index, and basic components.

Multi - variable extensions of classic permutation results are obtained by counting permutations by descents, idescents, imajor index, and basic components. For instance, the generating function

$$A(n;s,q,z) = \sum_{\sigma} s^{\operatorname{des}\sigma} q^{\operatorname{imaj}\sigma} z^{\operatorname{bc}\sigma}$$

for permutations σ of $\{1,2,\ldots,n\}$ by descents, imajor index, and basic components satisfies the recurrence

$$A(n+1;s,q,z) = zA(n;s,q,z) + sq \sum_{k=1}^{n} \left[{n \atop k-1}\right] q^{n-k}(1-s)^{n-k} A(k;,s,q,z)$$

where $A(0;,s,q,z) = 1$. In the case $s = 1$ the recurrence yields the classic identity due to Gould

$$A(n;1,q,z) = \prod_{k=0}^{n-1} (z+q[k])$$

for the q - Stirling numbers of the first kind. When $z = 1$ the recurrence defines the q - Eulerian numbers of Stanley.

D. P. ROBBINS :

Alternating sign matrices and descending plane partitions

An alternating sign matrix is a square matrix such that (i) all entries are 1, -1, or 0, (ii) every row and column has sum 1, and (iii) in every row and column the non - - zero entries alternate in sign.
I have discovered striking numerical evidence of a connection between these matrices and the descending plane partitions introduced by Andrews, but I have been unable to prove the existence of such a connection. However this evidence did suggest a method of proving the Andrews Conjecture on descending plane partitions, which in turn suggested a method of proving the Macdonald Conjecture on cyclically symmetric plane partitions.

In this talk we discuss alternating sign matrices and descending plane partitions, and present several conjectures and theorems about them.

D. P. ROBBINS:

Proof of the Macdonald Conjecture

A plane partition is cyclically symmetric if its Ferrers graph is invariant under the cyclic permutation $(x,y,z) \to (y,z,x)$ of the coordinate axes. Let $M(m,n)$ be the number of cyclically symmetric partitions of n whose Ferrers graphs are contained in the box $[1,m] \times [1,m] \times [1,m]$. Macdonald has conjectured that the generating function $\Sigma_{n \geq 0} M(m,n)q^n$ is a certain product of cyclotomic polynomials. The special case $q = 1$ has been settled by Andrews. In the course of his proof Andrews introduced a new class of partitions that he called descending plane partitions and made a similar conjecture about these partitions. The object of this paper is to prove both of these conjectures.

B. SAGAN:

Enumeration of partially ordered sets with hooklengths

An algorithm of Hillman and Grassl [J. Comb. Thy.(A) 21 (1976), 216 - 221] for reverse plane partitions is extended to shifted reverse plane partitions and rooted trees. This is motivated by a desire to explain why all three families have generating functions of the form $\prod_{i=1}^{h} \frac{1}{1 - X^{h_i}}$ in a combinatorial manner (the h_i are called hooklengths and have a combinatorial interpretation in each case). This method is seen to apply to several other types of partitions whose generating function is a product, notably: plane partitions without fixed shape, partitions restricted in the number & size of its parts, and multi - - variable generating functions which also keep track of the sum of diagonal elements in a plane partition; We note that various other algorithms have been generalized to shifted shapes and trees as well.

V. STREHL:

Three observations concerning Hermite polynomials

The usual combinatorial model for Hermite polynomials is extended in order to derive several classical identities on orthogonal polynomials within a common combinatorial framework. Three examples are given:

1) The Szegö - relations connecting Hermite - and Laguerre - - polynomials.

2) An identity of Tricomi's connecting polynomials defined by

$\left(\frac{d}{dx}\right)^n (1 - x^2)^{-\lambda} = Q_n^{\lambda}(x) \; (1 - x^2)^{-\lambda-n}$ to Gegenbauer -

polynomials.

3) Two different relations between Gegenbauer and Jacobi - - polynomials (making use of the combinatorial ideas of Foata - Leroux).

E. TRIESCH:

On an adjacency property of graphs

A graph G has property A(m,n,k) if for any sequence of m + n distinct points of G, there are at least k other points, each of which is adjacent to the first m and not adjacent to the last n points of the sequence. The property is important in the first - order theory of graphs. Let a(m,n,k) denote the minimum order among all graphs with property A(m,n,k). The talk provides a survey of the known inequalities for a(m,n,k). Some new inequalities are derived.

G. VIENNOT, M. FRANCHI - ZANNETTACCI:

The number of convex polyominoes

A convex polyomino is a union of elementary cells (defined up to a translation) of the "plane" $\mathbb{Z} \times \mathbb{Z}$ which is simply connected, with no cut point, and such that the intersection with any vertical and horizontal line is a connected segment. We prove that the number p_{2n} of convex polyominoes with perimeter 2n is:

$$p_4 = 1 , p_6 = 2$$

and $p_{2n+8} = (2n + 11)\, 4^n - 4(2n + 1)\binom{2n}{n}$ for every $n \geq 0$.

The proof is in 3 steps:

1) Coding of a convex polyomino with a word of an <u>algebraic language</u>,
2) Resolution of systems of algebraic equations,
3) Expanding the generating function.

G. VIENNOT:

<u>Some "simple" bijections related to the Rogers - Ramanujan - - Schur identities</u>

We give some interpretations for the inverse of the left hand sides of the Rogers - Ramanujan - Schur identities. The bijections are particular cases of a general bijection proving the MacMahon Master theorem and the cofactor - - determinant classical formula for matrix inversion (see a forthcoming paper with S. Dulucq). In particular the ratio of the two identities is interpreted in terms of weighted paths and Ramanujan continued fraction deduced. The connection with bijections for general orthogonal polynomials is made explicit.
A pictorial model is introduced, related to q - polynomial identities of Andrews from which the Rogers - Ramanujan -

- Schur identities can be deduced. With different weights for the paths, we have the left hand sides of the identities, and also the ones of the 14 identities of the hard hexagonal model in statistical physics (Baxter, Andrews). The numbers involved in the corresponding infinite products are easily seen on the picture. The problem is to find a nice bijective way to make explicit these infinte products.

D. ZEILBERGER:

How to get "cute" bijective proofs from dull inductive proofs

Recently Garsia and Milne, and Garsia and Remmel used general ideas of the Lotharingian school to translate manipulative proofs of q - series identities into nice bijective proofs. We are going to offer a general algorithm for getting bijective proofs of results of the form $F(m_1,\dots,m_n) = G(m_1,\dots,m_n)$ where both F and G are counting functions of finite sets depending on n discrete parameters, each of them having a natural linear partial recurrence equation with polynomial coefficients. It is shown how this algorithm leads in a natural way to the Schensted correspondence, Sagan's correspondence, a bijective proof that the Gaussian polynomials enumerate partitions restricted by size and number of elements and the author's correspondence establishing the hook - lengths formula. However, the chief interest of the method is that we are guaranteed the existence of bijective proofs in a wider variety of situations, in particular, every binomial identity has a bijective proof.

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