MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

# Tagungsbericht 21/1982

## Universelle Algebra

17. bis 21. Mai 1982

Die Tagung, die unter der Leitung der Herren Professoren G. Grätzer (Winnipeg) und R. Wille (Darmstadt) stattfand, brachte Mathematiker aus zehn Ländern Ost- und West-Europas sowie Nordamerikas zusammen. Von besonderem Interesse war die Vielfalt der Anregungen und Methoden aus diversen Gebieten wie etwa Logik, Geometrie, Informatik, Kombinatorik und Gruppentheorie, die zur Lösung alter Probleme und zur Erforschung neuer Richtungen beitrugen. Die Teilnehmer führten die Tagung im Gedenken an Jürgen Schmidt, dem Leiter der ersten Oberwolfacher Tagung über Universelle Algebra, durch. Jürgen Schmidt verstarb im Oktober 1980.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

## Vortragsauszüge

K. A. BAKER:

### Markov constructions of algebras

For a finite set S and subset T of $S \times S$, let $M_{S,T}$ denote the Markov chain $\{\vec{s} \in S^{\mathbb{Z}} : (s_i, s_{i+1}) \in T$ for all $i\}$. It is useful to consider the case where S is an algebra and T is a subalgebra of $S \times S$, or even more generally, where S is a partial algebra and T is an induced partial subalgebra. Then $M_{S,T}$ becomes an algebra or a partial algebra. This construction provides a unifying framework in which to consider varied examples: (1) arbitrarily long non-shortenable projectivities in varieties generated by finite nondistributive lattices; (2) McKenzie's proof that these same varieties lack definable principal congruences; (3) Park's construction of a non-finitely based finite idempotent commutative algebra; (4) Shallon's graph algebras; (5) a finite 2-unary algebra whose class of subdirect powers is not finitely axiomatizable (Gimpel).

H. BAUER:

### Semilattices of compact congruences

Definition: A semilattice with 0 is called distributive if the lattice of all ideals is distributive.

Theorem: Each distributive semilattice is isomorphic to the semilattice of all compact congruences of a lattice.

More precisely: Let H be a distributive semilattice. Then it is consistent with ZFC, that there is a lattice V, such that $\mathrm{Con}^c(V) \cong H$.

G. BRUNS:

### Varieties of modular ortholattices

Let MOn be the modular ortholattice consisting of 2n pairwise incomparable elements and the bounds $(n \geq 1)$ and let MO0 be the one-element ortholattice.

Theorem: If $\mathfrak{K}$ is a variety of modular ortholattices which is not contained in the variety [MO2] generated by MO2 then $MO3 \in \mathfrak{K}$ .

Conjecture: If $\mathfrak{K}$ is a variety of modular ortholattices different from all [MOn] $(0 \leq n \leq \omega)$ then $\mathfrak{K}$ contains a projective plane (with orthocomplementation).

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P. BURMEISTER:

## Some recent developments in the theory of partial algebras

During the last years some interest in partial algebras and their theory has started in Computer Science because of some applications in this field. This has given new impact to the development of a theory for partial algebras, which needs model theoretic concepts already on a very early stage. A good basis for this seems to be the concept of "existence-equality" ($\stackrel{e}{=}$): For any two terms $t, t^*$ (in the usual sense) a partial algebra $\underline{A}$ satisfies $t \stackrel{e}{=} t^*$ with respect to a valuation $h : X \to A$ of the set X of variables ($\underline{A} \models t \stackrel{e}{=} t^*[h]$) if the values of the induced partial X-ary term functions $t^{\underline{A}}$, $t^{*\underline{A}}$ exist in $\underline{A}$ on the sequence h and are equal: $t^{\underline{A}}(h) = t^{*\underline{A}}(h)$.
For formulas of the form $\bigwedge_{i=1}^{n} t_i \stackrel{e}{=} t_i \Rightarrow t_o \stackrel{e}{=} t_o^*$ (ECE-equations) or $\bigwedge_{i=1}^{n} t_i \stackrel{e}{=} t_i \Rightarrow t_o \stackrel{e}{=} t_o^*$ (QE-equations) or simply $t \stackrel{e}{=} t^*$ (E-equations), respectively, Birkhoff-type theorems exist (semantical and syntactical ones). Moreover, within this language together with a model theoretical interpretation of the category theoretical concept of a factorization system, one gets good descriptions for the most important attributes for homomorphisms between partial algebras. (For more details see TH-Darmstadt-Preprint No. 582 (Dept. of Math.).)

B. CSÁKÁNY:

## Three-element groupoids with minimal clones

Three-element groupoids $\langle \underline{3} ; f \rangle$ are considered where $\underline{3} = \{0, 1, 2\}$ and f is essentially binary. Two such groupoids are called essentially distinct if they are neither isomorphic nor antiisomorphic. The binary operation with Cayley table

| | 0 | 1 | 2 |
|---|---|---|---|
| 0 | 0 | $n_5$ | $n_4$ |
| 1 | $n_3$ | 1 | $n_2$ |
| 2 | $n_1$ | $n_o$ | 2 |

is denoted by the integer $\sum_{o}^{5} 3^i n_i$ .
There exist twelve essentially distinct three-element groupoids whose clones of term functions are minimal (i.e., are atoms of the lattice of all clones on $\underline{3}$), namely, the groupoids $\langle \underline{3} ; f \rangle$ with

$$f = 0, 8, 10, 11, 16, 17, 26, 33, 35, 68, 178, 624 .$$

($\langle \underline{3} ; 0 \rangle$ and $\langle \underline{3} ; 10 \rangle$ are semilattices, $\langle \underline{3} ; 178 \rangle$ is the triangle, and $\langle \underline{3} ; 624 \rangle$ is the three-element affine space.)

A. DAY (with D. PICKERING):

Coordinatizing Arguesian Lattices

A spanning n-diamond in a (modular) lattice L is a sequence $(x_1,\ldots,x_n,x_{n+1})$ such that $\bigvee_{j\neq i} x_j = 1$ for all i and $x_i \wedge \bigvee_{k\neq i,j} x_k = 0$ for all $i \neq j$. Using an assymetric notation $(x_1,\ldots,x_{n-1},z,t)$ for a spanning n-diamond and $h = \bigvee x_i$, $w = h \wedge (z \vee t)$ and $D = \{a : a \vee w = z \vee t,\ a \wedge w = 0\}$ we prove:

Theorem 1: If L is Arguesian, then $(D, \oplus, z, \otimes, t)$ is a ring for $n \geq 3$.

Theorem 2: If L is Arguesian and $n \geq 3$ then there is a meet-preserving $F : [0, n] \to \mathcal{L}({}_D D^{n-1})$. Moreover F is injective if $F|[0, w]$ is injective and F is join preserving if L satisfy (h − UC) : $[p \vee h = 1 \Rightarrow \exists\, q \cdot \ni \cdot q \leq p,\ q \vee h = 1$ and $q \wedge h = 0]$.

T. EVANS:

Lattice-ordered loops

In a variety of lattice-ordered algebras, the fully-ordered algebras generate a subvariety, every algebra of which is a subdirect product of fully-ordered algebras. Characterizations of these subvarieties in terms of identities are known for groups, rings etc. In this joint work with P. Hartman, we describe the identities satisfied by the fully-ordered algebras in the variety of lattice-ordered loops. The known results for groups and commutative groups are consequences of this result.

P. GORALČÍK and V. KOUBEK:

Isomorphism-complete and group-universal semigroup varieties

A variety $\underset{\sim}{V}$ is group-universal if for every group G there is $A \in \underset{\sim}{V}$ with $G = \mathrm{Aut}(A)$. A pseudovariety $\underset{\sim}{P}$ is isomorphism-complete if there is a polynomial reduction of the graph-isomorphism problem to $\underset{\sim}{P}$.
We prove that a semigroup pseudovariety is isomorphism-complete if it is not contained in $(Z_r \vee C) \cup (Z_l \vee C) \cup (Z_r \vee Z_l \vee V_g)$, where $Z_r = [xy = y]$, $Z_l = [xy = x]$, $C = [xy = zt]$, and $V_g$ is a variety containing only groups.
We give a complete list of minimal group-universal semigroup varieties.
We prove that a semigroup pseudovariety is isomorphism-complete if it is not contained in $(Z_r^f \vee C^f \vee G) \cup (Z_l^f \vee C^f \vee G) \cup (Z_r^f \vee Z_l^f \vee G)$, where $Z_r^f$, $Z_l^f$, $C^f$ are the pseudovarieties of the finite members of $Z_r$, $Z_l$, C, respectively, and G is the pseudovariety of all finite groups. We give a complete list of minimal isomorphism-complete semigroup pseudovarieties.

M. GOULD:

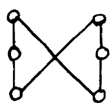
Embedding in Globals of Groups and Semilattices

The global of a semigroup S is the family gl(S) of all nonvoid subsets of S, with the natural multiplication: AB = {ab | a in A, b in B}. In the case where S is a group G, every factor group of G is a subsemigroup of gl(G), but gl(G) has many other subsemigroups. We shall survey the recent literature on subsemigroups of gl(G), emphasizing Trnkova's embedding of an arbitrary commutative semigroup into the global of a direct power of the group of integers. There will be some discussion of recent attempts by the author and others to establish a finite version of Trnkova's result. An analogous problem concerning globals of semilattices will be discussed, and a connection will be elaborated between the semilattice question and the group question.

G. GRÄTZER and D. KELLY:

Some infinitary free lattices I and II

In Part I, the free m-lattice freely generated by H is described. Here, m is an infinite regular cardinal, an m-lattice is a poset L in which $\bigwedge X$ and $\bigvee X$ exist for all $X \subseteq L$ with $0 < |X| < m$; finally, H is the poset



This lattice, $CF_m(H)$ is put together from three building blocks: $CF_m(\underset{\sim}{2} + \underset{\sim}{2})$, A, and B, where $A = \{<r,s> \mid r < s;\ r, s$ dyadic rationals; $0 \leq r, s \leq 1;$ and $s - r = 2^{-n}$ with $n \geq \text{ord}(s)\}$; B is defined similarly with $r > s$. The ordering of A and B is componentwise.

Theorem: Let P be a countable poset. T.F.A.E.

(1) $CF_m(P)$ does not contain $F_m(3)$;

(2) P does not contain $\underset{\sim}{1} + \underset{\sim}{5}$, $\underset{\sim}{2} + \underset{\sim}{3}$, $\underset{\sim}{1} + \underset{\sim}{1} + \underset{\sim}{1}$;

(3) $CF_m(P)$ can be embedded into $CF_m(H)$.

In the special case: P is finite and $m = \aleph_0$, this theorem is due to I. Rival and R. Wille.

H. P. GUMM:

Two sides of modular varieties

Using the commutator operation $[\alpha, \beta]$ of two congruences $\alpha$ and $\beta$ we say that a congruence relation $\rho$ is prime if $[\alpha, \beta] \leq \rho$ implies $\alpha \leq \rho$ or $\beta \leq \rho$. Setting $\xi(\theta) = \bigvee \{\alpha \mid \exists_{\beta \not\geq \theta} [\alpha, \beta] \leq \theta\}$ we find that for $\theta$ finitely $\wedge$-irreducible $\mathfrak{A}/_{\xi(\theta)} \in HSP_u^{\neq}(K)$ whenever $\mathfrak{A} \in HSP(K)$. In particular this is true, if $\theta$ is prime. Moreover, since $\theta$ is prime iff $\xi(\theta) = \theta$, on defining the prime radical $\sqrt{\mathfrak{A}} := \bigcap \{\rho \mid \rho \text{ prime}\}$ we find for arbitrary $\mathfrak{A} \in HSP(K)$, that $\mathfrak{A}/\sqrt{\mathfrak{A}} \in P_s HSP_n(K)$.

On the other side of modular varieties we look at congruences $\alpha \geq \beta$ with $[\alpha, \beta] = 0$. In a natural way we are associating affine algebras with the $\beta$-classes, such that the $\beta$-classes in one and the same $\alpha$-class carry isomorphic affine algebras. In the case $\alpha = 1$ those affine algebras $\mathfrak{A}^{\nabla}[\beta]$ are in $\mathcal{V}(\mathfrak{A})$, in particular, $\beta \cong \mathfrak{A} \times \mathfrak{A}^{\nabla}[\beta]$.

C. HERRMANN:

Definability, generation and decidability problems for varieties of modular lattices

The short comings of the axiomatic approach to lattices of submodules of a module can be illustrated by the following facts: 1) The word problem for the modular lattice in four free generators is recursively unsolvable. 2) No modular lattice variety containing an infinite dimensional projective geometry can be both finitely based and generated by its finite dimensional members. In contrast, the lattice variety generated by all lattices of submodules has a solvable word problem for free lattices and is generated by its finite members. Who minds that it is not finitely based?

W. HODGES (joint with J. BALDWIN, J. BERMAN, A. GLASS):

A combinatorial property of free algebras

We give a common generalisation of many known results of the following type: A free boolean algebra contains no uncountable chain (Horn 1968). We define as endomorphism base of the algebra A to be a set $X \subseteq A$ such that every map $f : X \to X$ extends to an endomorphism $f^* : A \to A$ so that $f^* g^* = (fg)^*$, $1_X^* = 1_A$.

This X is an endomorphism base over $Y \subseteq A$ iff moreover each $f^*$ fixes Y pointwise. THEOREM: Let A be a free algebra in a variety with countable language, and let $X, Y \subseteq A$, $|Y| < \kappa \leq |X|$ where $\kappa$ is a regular uncountable cardinal. Then some subset of X of cardinality $\kappa$ is an endomorphism base over Y.

A. HUHN:

On the representation of distributive semilattices

E. T. Schmidt proved that every distributive lattice with 0 is isomorphic with the lattice of compact congruences of a lattice. P. Pudlâk gave a new proof and a category theoretical generalization of Schmidt's theorem by considering the distributive lattice to be represented as a direct limit of its finite sublattices containing 0 and representing this sublattices simultaneously. Our main theorem asserts that a considerable part of Pudlâk's program, namely the simultaneous representation of two objects, can be carried out by working with distributive semilattices instead of distributive lattices. Among the corollaries we have the results of Schmidt and Pudlâk as well as Bauer's unpublished theorem that every countable distributive semilattice with 0 is the semilattice of finitely generated congruences of a lattice. Our technique combines the technique of a proof of E. T. Schmidt (for the relatively pseudocomplemented case) with the methods in the theory of free distributive products.

B. JÔNSSON:

Varieties of relation algebras

In the variety RA of all relation algebras (in the sense of A. Tarski) every subdirectly irreducible member is simple. Every finite, simple relations algebra is splitting. In particular, this is true of R(n), the full relation algebra on n elements.

Thm 1. Every embedding of R(n) into a simple relation algebra is an isomorphism.

Thm 2. A simple relation algebra A is isomorphic to R(n) iff A has an element a with

$$a_j a \leq a, \quad a + a^n \geq 0', \quad a^n = 0, \quad a^{n-1} \neq 0 .$$

Thm 3. The conjugate variety of R(n), $RA^*(n)$, is a dual atom in the lattice of all subarieties of RA.

Thm 4. The identity $a^{n-1} \leq 1; ((a;a)a^- + a^- a^{n-} 0' + a^n); 1$ constitutes an equational basis for $RA^*(n)$ mod RA.

H. KAISER:

Approximation in universal algebra

When analyzing the interpolation problem for functions over the field of real numbers from the topological view one is led to the following definition (due to G. Kowol):
Let A be a topological universal algebra. A has the approximation property if the algebra $P_k(A)$ of all k-ary polynomial functions over A is dense in the full function algebra $F_k(A)$ for every positive integer k.
Some properties of algebras with the approximation property will be discussed and the problem of describing all topological universal algebras satisfying $T_2$ and having the approximation property will be solved for the case of congruence permutable varieties. As an application Jacobson's density theorem for rings of linear transformations of vectorspaces over skewfields will be derived in this setting.

E. W. KISS:

Finitely Boolean representable varieties

A subalgebra $\mathcal{L}$ of an algebra $\mathfrak{A}$ is called very skew if $\mathcal{L}$ is skew in each direct decomposition of $\mathfrak{A}$. It is proved that a finite neutral simple algebra $\mathfrak{A}$ contained in a modular variety is quasi-primal iff there is a bound on the cardinalities of the very skew subalgebras of the finite direct powers of $\mathfrak{A}$. With the help of this characterisation a short, elementary proof of a result of S. Burris and R. McKenzie stating that each variety Boolean representable by a finite set of finite algebras is the join of an abelian and a discriminator variety is obtained.

P. KÖHLER:

The finite congruence lattice problem

There are three good reasons to claim that Group Theory will play a crucial rôle in any attempt to solve the "finite congruence lattice problem". The first one is - of course - the Pudlák-Pálfy result that every finite lattice is isomorphic to the congruence lattice of a finite algebra if and only if every finite lattice is isomorphic to an interval in the subgroup lattice of a finite group. The second one is the recent example - due to Walter Feit - of a finite algebra with congruence lattice $M_7$. The third one is the - possibly decisive - special case of the lattices $M_n$, n-1 not a prime power. Here some restrictions on the structure of the possible finite algebras having such $M_n$'s as congruence lattices can be proved quite easily by applying some - not always trivial - results from Group Theory.

G. F. McNULTY (joint work with C. SHALLON):

Inherently nonfinitely based finite algebras

A variety V is inherently nonfinitely based provided V is locally finite and whenever W is a locally finite variety including V then W is not finitely based. A finite algebra is said to be inherently nonfinitely based if the variety it generates is. A graph algebra is a groupoid with universe A z where z A such that az = za = zz = z for all a in A and ab has either the value a or the value z for every a, b in A.

THEOREM. Let V be a locally finite variety of groupoids. If either every finite idempotent graph algebra belongs to V or every finite graph algebra without idempotents belongs to V, then V is inherently nonfinitely based.

THEOREM. Every nonassociative, nonabsorptive finite groupoid that has a multiplicative zero and a unit is inherently nonfinitely based.

Lyndon's nonfinitely based groupoid fails to be inherently nonfinitely based. The work of Murskii and Perkins inspired ours.

R. PADMANABHAN:

Geometric universal algebras

Let us consider the following phenomenon: If $<G, \Omega>$ is a mathematical structure admitting a "natural" binary operation $\mu : G \times G \to G$ with two sided identity e, i.e. $\mu(x,e) = \mu(e,x) = x$ for all $x \in G$, then the set G has a commutative semi-group structure (actually a group in many interesting cases) naturally associated with $\Omega$ and $\mu$. This phenomenon occurs in several "disjoint" areas of mathematics: (1) If G is a completely irreducible algebraic curve over an algebraically closed field k and $\mu$ a morphism then $<G, \mu, e>$ is an abelian (algebraic) group. (2) If G is a topological space and $\mu$ continuous then the fundamental group $\Pi(G, e)$ is abelian. (3) If G is an affine algebra and $\mu$ itself is an affine operation of G then $<G, \mu, e>$ is an abelian group. Thus it is natural to ask for a common universal algebraic formulation of this implication

$$\{\mu(x,e) = \mu(e,x) = x\} \vDash_G \quad \{\mu \text{ is an abelian group operation}\}.$$

With this in mind we give a few formal rules of derivation for an equational theory such that (i) these rules of derivations are formally valid for all the mathematical systems mentioned above and (ii) under these rules of derivations, one can derive the abelian group laws for $\mu$ from the one-variable law $\{\mu(x,e) = \mu(e,x) = x\}$ .

D. PIGOZZI:

Varieties with equationally definable congruences - a study of the deduction theorem in algebraic logic

A variety V has equationally definable principal congruences (EDPC) if there is a system of equations $p_i(x,y,z,w) = q_i(x,y,z,w)$ for $i = 1,\dots,n$ such that a is congruent to b modulo the principal congruence generated by (c,d) iff $p_i(a,b,c,d) = q_i(a,b,c,d)$ for all i. V is 1-regular if each congruence relation is completely determined by its 1-equivalence class. EDPC is characteristic of the varieties that arise from logic, but there are many varieties with the property that have no apparent relation to logic. It is shown that if V has EDPC and is 1-regular and congruence-permutable, then it has a structure very close to that of the familiar varieties of non-classical logic. In particular, V has terms - and that behave very much like the implication and conjunction of intuitionistic logic.

R. W. QUACKENBUSH:

Affine Equational Classes and Affine Equational Logic

Let K be an equational class. An algebra $\mathfrak{A} = \langle A;F \rangle$ is quasi-affine if for some abelian group $\langle A;+ \rangle$, each $f \in F$ is an affine transformation with respect to +; $\mathfrak{A}$ is affine if in addition $x - y + z$ is a term function. K is (quasi-) affine if each $\mathfrak{A} \in K$ is (quasi-) affine.
Theorem (Ch. Herrmann): K is affine iff K is modular and for each $\mathfrak{A} \in K$, $\Delta(\mathfrak{A}) = \{(a,a) \mid a \in A\}$ is a congruence class of $\mathfrak{A}^2$.
Generalization is the following rule of inference in equational logic: for terms $t, \alpha_i, \alpha'_i$ $(1 \le i \le n)$, $\beta_j, \beta'_j$ $(1 \le j \le n)$, from $t(\underline{\alpha}, \underline{\beta}) = t(\underline{\alpha}, \underline{\beta}')$ infer $t(\underline{\alpha}', \underline{\beta}) = t(\underline{\alpha}', \underline{\beta}')$ .
ET(K) is the equational theory of K and G(K) is the smallest equational theory containing K and closed under generalization.
Theorem (R. McKenzie): If K is permutable and ET(K) = G(K), then K is affine.
Theorem (W. Taylor): If K is n-permutable and ET(K) = G(K), then K is permuatable.
Theorem: If K is n-modular and ET(K) = G(K), then K is n-permutable (same n).
Theorem: If Mod(G(K)) is the class of all quasi-affine algebras in K and $\mathfrak{A} \in K$, then $\mathfrak{A}$ is quasi-affine iff $\Delta(\mathfrak{A})$ is a congruence class of $\mathfrak{A}^2$.

I. RIVAL:

Probability, linear extensions, and distributive lattices

Ordered sets and even distributive lattices occur often in scheduling and sorting problems. A set of inequalities (e.g. $a < b$, $c < d$, etc.) in an ordered set P can be regarded as an "event" and can then be identified with the set of all linear extensions of P in which these inequalities are satisfied. If all linear extensions of P are taken as equally likely we have a probability measure. Recently, L. A. Shepp proved this conjecture of I. Rival and B. Sands:

$$\Pr (a < b \mid a < c) \geq \Pr (a < b) ,$$

where $\Pr (a < b)$ equals the number of linear extensions of P in which $a < b$ and $\Pr (a < b \mid a < c)$ is the usual conditional probability. The proof is a clever use of distributive lattices.

I. G. ROSENBERG:

Delayed algebras

Gates in real switching circuits perform operations which to the input values $x_i$ (over an k-valued alphabet) assign the output value $f(x_1,\ldots,x_n)$ with a certain delay. The presence of unwanted phenomena like races and hazards means that the delay should not be ignored in fast circuits. One of the simplest models is formed by uniformly delayed operations requiring restricted compositions and hence a treatment different from the usual one. The primality (i.e. completeness) problems were studied by Kudrjavcev and Birjukova in the 50ties (k=2) and more recently by Nozaki and Hikita (k>2). Surprisingly, the latter lead to very interesting relational problems.

J. SCHMID:

Semigroups of Quotients of (semi-) lattices

If S, T are commutative semigroups, $S \subseteq T$, T is called a semigroup of quotients of S (written $S \leq T$) iff for all $t_1 \neq t_2$, $t \in T$ there exists $s \in S$ such that $st_1 \neq st_2$ and $st \in S$. There exists a maximal semigroup Q(S) such that $S \leq Q(S)$; whenever $S \leq T$, T embeds into Q(S) over S. Fact: If S is a (meet-) semilattice, so is T whenever $S \leq T$. In fact, T is then a sup-extension of S. We investigate the structure of Q(S) for different classes of semilattices. Sample of results:

(i) S is a distributive lattice. Then Q(S) is a distributive lattice also, and the canonical embedding of S into Q(S) preserves joins. If S is Boolean, so is Q(S), and Q(S) is the McNeille completion of S (Lambek). (ii) S is a finite semilattice, Q(S) is a finite lattice, and Q(S) is isomorphic with the lattice of ideals of the lower set generated in S by the join-irreducibles. In contrast to the general case, Q(S) may be constructed "internally" as a semilattice of certain lower sets of S if S is a semilattice.

E. T. SCHMIDT:

Kongruenzverbände der komplementären modularen Verbände

Problem: is every distributive algebraic lattice isomorphic to the congruence lattice of a complemental modular lattice? For the finite case we have:
THEOREM. For every finite distributive lattice D there exists a complemental modular lattice K such that the congruence lattice of K is isomorphic to D and K is a sublattice of the lattice of all subspaces of a countably infinite dimensional vector space over a finite field.
For infinite D we follow Pavel Pudlák's approach which reduce the problem to investigations of the representations of finite distributive lattices. We need to use continuous geometries instead of the subspace lattices of vectorspaces. Some remarks were made for solving the problem.

J. SCHULTE MÖNTING:

The notion of codimension for Heyting algebras

The codimension of a Heyting algebra H is a pair $\langle d,c \rangle$, $c, d \in \omega \cup \{\infty\}$ where c is the number of minimal prime filters on H, and d is the number of those minimal prime filters which do not contain the filter D of dense elements of H. In a Heyting algebra of codimension $\langle d,c \rangle$ there exists an orthogonal antichain $\langle a_\mu \mid \mu < c \rangle$ satisfying $a_\mu \vee a_\nu = 1$ $(\mu \neq \nu)$, $a'_\mu = 0$ for $\mu < d$, $a_\mu$ strictly regular (i.e. $[a_\mu, 1] \cap D = \{1\}$) for $d \leq \mu < c$. The elements of such an antichain generate a subalgebra of the form $\underline{2}^d \times \underline{2}^{c-d}$, "the" prime algebra of codimension $\langle d,c \rangle$. Codimensions are partially ordered by the product order. This concept seems to be useful for the structure theory of Heyting algebras. As an example, one can prove the
Theorem: A Heyting algebra can be embedded into every algebraically closed Heyting algebra of finite codimension $\langle d,c \rangle$ if and only if it is countable and locally finite and has a codimension not greater than $\langle d,c \rangle$.
A similar theorem holds for the infinite case.

D. SCHWEIGERT:

Congruences of relational systems

For $(A;\rho)$, $\rho$ n-ary, $n > 1$ an equivalence $\Pi$ is a congruence if for all $\rho(a_1,\dots,a_n)$, $a_1 \,\Pi\, b_1,\dots,a_{n-1} \,\Pi\, b_{n-1}$ there exists $b_n \in A$ such that $\rho(b_1,\dots,b_n)$ and $a_n \,\Pi\, b_n$. For $(A;\rho)$, $(B;\overline{\rho})$ $f : A \to B$ is a relational homomorphism if 1) $\rho(a_1,\dots,a_n) \Rightarrow \overline{\rho}(f(a_1),\dots,f(a_n))$, 2) for $\overline{\rho}(f(d_1),\dots,f(d_n))$ there is $c \in A$ such that $f(c) = f(d_n)$ and $\rho(d_1,\dots,d_{n-1},c)$.
We show homomorphism theorems, the connection to algebras, and that the lattice $C(A;\rho)$ of the congruences of $(A;\rho)$ is complete. Then we confine us to relations $\rho$ which are flexible i.e.: $\rho(a_1,\dots,a_{n-1},x)$ is solvable for all $a_1,\dots,a_{n-1} \in A$. In this case $C(A;\rho)$ is algebraic and we can develope a subdirect product theorem. Furthermore we study classes of flexible relational system which are closed under flexible subsystems, relational homomorphisms and direct products. To describe these classes we consider formulas for predicate symbols $R_i$ of the following form: 1) $R_j(x_1,\dots,x_n)$, 2) $R_{j_1}(x_1,\dots,x_{n_{j_1}}) \wedge \dots \wedge R_{j_s}(x_1,\dots,x_{n_{j_s}}) \to R_t(x_1,\dots,x_{n_t})$ ordered in such a way that $x_{n_{j_r}}$ does not appear in any $R_{j_k}(x_1,\dots,x_{n_k})$ for $1 \le k < r$.

J.D.H. SMITH (with A. B. ROMANOWSKA):

Idempotent Entropic Algebras

An idempotent entropic algebra is an algebra in which each element forms a singleton subalgebra (idempotence) and for which each operation is a homomorphism (entropicity). Typical models are semilattices, and convex subsets of a finite-dimensional Euclidean space under weighted means. Semilattice words on an alphabet may be regarded as finite subsets of the alphabet, and weighted means as finite probability distributions. Thus idempotent entropic algebras furnish a universal algebraic approach to "choice and chance".
Current work on these algebras involves investigation of their subalgebra systems, including freeness results, an abstract consideration of approximation of sub-algebras by finitely generated subalgebras, and structure theorems (decomposition and extension).

L. SZABÓ:

Compatible orderings of lattice ordered algebras

By a compatible ordering of an algebra $\langle A;F\rangle$ we mean a partial order $\rho$ on A preserved by every operation in F.

Theorem. Let $\mathfrak{A} = \langle A;F\rangle$ be an algebra having two binary local algebraic functions $\wedge$ and $\vee$ such that $\langle A; \wedge, \vee\rangle$ is a lattice and every operation in F preserves the natural ordering $\leq$ of $\langle A; \wedge, \vee\rangle$. If $<$ is a compatible ordering of $\mathfrak{A}$ then $\Theta_1 = (< \cap \leq) \circ (< \cap \leq)^{-1}$ and $\Theta_2 = (< \cap \geq) \circ (< \cap \geq)^{-1}$ are congruence relations of $\mathfrak{A}$ with $\Theta_1 \cap \Theta_2 = \omega$. Thus $\mathfrak{A}$ is a subdirect product of $\mathfrak{A}/\Theta_1$ and $\mathfrak{A}/\Theta_2$. Moreover, $a < b$ iff $[a]\Theta_i <_i [b]\Theta_i$, $i = 1,2$, where $<_1 = ((\Theta_1 \vee \Theta_2)/\Theta_1) \cap \geq_1$ and $<_2 = ((\Theta_1 \vee \Theta_2)/\Theta_2) \cap \leq_2$. (Here $\leq_i$ is the natural ordering of $\langle A/\Theta_i; \wedge, \vee\rangle$, $i = 1,2$.) If $<$ is a lattice ordering (i.e. $\langle A;<\rangle$ is a lattice) then $\Theta_1 \circ \Theta_2 = A \times A$, and thus $\mathfrak{A} \cong \mathfrak{A}/\Theta_1 \times \mathfrak{A}/\Theta_2$ and $<_1 = \geq_1$, $<_2 = \leq_2$.

This theorem is an extension of a joint result of G. Czédli, A.P. Huhn and myself.

Á. SZENDREI:

Completeness theorems for finite algebras with semiregular automorphism groups

A finite algebra $\underline{\underline{A}} = (A;F)$ is called demi-primal iff $\underline{\underline{A}}$ has no proper subalgebra and every operation on A admitting the automorphisms of $\underline{\underline{A}}$ is a polynomial of $\underline{\underline{A}}$. Clearly, the automorphism group of a demi-primal algebra is semiregular.

In order to get a criterion for the demi-primality of algebras with a fixed semiregular automorphism group G on A we have to determine the maximal subclones of the clone Pol G consisting of all operations on A which commute with the permutations in G. Two special cases will be discussed:

1. G is of prime order / joint result with I.G. Rosenberg /;
2. G is transitive.

S. TULIPANI:

Congruence lattice size of models of a first order theory

Given a first-order theory T in a language without relation symbols, denote by $C_T(\lambda)$ and by $L_T(\lambda)$ the supremum of cardinalities and lengths, respectively, of the congruence lattices of all models of T of cardinality $\lambda$. If $\lambda$ is any infinite cardinal, one of the following cases holds:

$L_T(\lambda) = \text{ded}(\lambda) \leq C_T(\lambda) \leq 2^\lambda$; there exists a positive integer n such that $L_T(\lambda) = n$ and $C_T(\lambda) = \lambda$; there are positive integers m and n such that $C_T(\lambda) = m$ and $L_T(\lambda) = n$.

Further properties of $L_T(\lambda)$ and $C_T(\lambda)$ can be proved for $\chi_o$-categorical theories or for theories with Definability of Compact Congruences (DCC). Moreover, there are examples of $\chi_o$-categorical theories for which $C_T(\lambda) = \text{ded}(\lambda)$. However, if T is a stable theory which has DCC, then $C_T(\mu) > \mu$ for some infinite $\mu$ implies $C_T(\lambda) = 2^\lambda$ for every infinite cardinal $\lambda$.

It is open if $C_T(\mu) > \text{ded}(\mu)$, for some infinite $\mu$, implies always $C_T(\lambda) = 2^\lambda$.

J. TŮMA:

Planes in Dilworth truncations

Let us consider a finite geometric (i.e. point and semimodular) lattice L. Denote by $L_k$ the lattice obtained from L by identifying all elements with rank $\leq k - 1$. In general, $L_k$ will not be geometrical lattice. Dilworth found a canonical construction which extends $L_k$ to a new geometrical lattice $D(L_k)$ having the same points, and which preserves as many properties of $L_k$ as it can: covering relation, meets, and joins which do not damage semimodularity. If B is the boolean lattice of all subsets of a finite set, then $D(B_2)$ is isomorphic to a partition lattice. We give a partition-like representation of elements in $D(B_k)$ for all k.

A geometric lattice is a minor of a geometric lattice L if it is a join-subsemilattice of L preserving covering relation. Tutte's deep characterization of graphic matroids gives a finite list of all minimal forbidden minors of partition lattices (i.e. of all minimal geometric lattices which are not minors of any partition lattice). We show that for all $k \geq 3$ there are infinitely many minimal geometric lattices of rank 3, which are forbidden in all $D(B_k)$. Further properties of minors of $D(B_k)$ are given.

A. URSINI:

Ideals in Universal Algebra

Fix a variety K having a nullary operation 0. An ideal term in $\vec{y}$ is a term $t(\vec{x}, \vec{y})$ such that $t(\vec{x}, \vec{0}) = 0$ holds identically in K. A non empty subset I of $A \in K$ is an ideal of A if $t(\vec{a}, \vec{b}) \in I$ for all ideal terms $t(\vec{x}, \vec{y})$ in $\vec{y}$, $\vec{a} \in A$, $\vec{b} \in I$. K has a good theory of ideals if each ideal of any A K is the class of 0 for exactly one congruence of A, abbreviated: g.i.t.

A g.i.t. is a Mal'cev Condition.

A g.i.t. implies that congruences are modular, but not necessarily permutable nor regular.

A commutator term is a term $t(\vec{x}, \vec{y}, \vec{z})$ such that $t(\vec{x}, \vec{0}, \vec{z}) = 0 = t(\vec{x}, \vec{y}, \vec{0})$ identically in K. The commutator of two ideals I, J of $A \in K$ is the set of $t(a, i, j)$ for t a commutator term, $\vec{a} \in A$, $\vec{i} \in I$, $\vec{j} \in J$. If K has a g.i.t. this corresponds to the commutator of congruences as usually defined in modular varieties.

R. WILLE:

Congruence relations of concept lattices

Lattices can be interpreted as hierarchies of concepts. This fundamental interpretation may be formalized as follows: A context is understood as a triple (G,M,I) where G and M are sets, and I is a binary relation between G and M; the elements of G and M are called objects and attributes, respectively. If gIm for $g \in G$ and $m \in M$ we say: the object g has the attribute m. Following traditional philosophy we define a concept of (G,M,I) as a pair (A,B) with $A \subseteq G$, $B \subseteq M$, $A = \{g \in G \mid gIm \text{ for all } m \in B\}$, and $B = \{m \in M \mid gIm \text{ for all } g \in A\}$. The hierarchy is captured by the definition $(A_1, B_1) \leq (A_2, B_2) :\Leftrightarrow A_1 \subseteq A_2$ $(\Leftrightarrow B_1 \supseteq B_2)$. All concepts of (G,M,I) together with the order $\leq$ form a complete lattice, the concept lattice $\underline{\mathfrak{L}}$(G,M,I). A basic problem is to determine the concept lattice for a given context. With respect to this problem the study of congruence relations and subdirect decompositions of $\underline{\mathfrak{L}}$(G,M,I) leads to a reduction if $\underline{\mathfrak{L}}$(G,M,I) can be subdirectly decomposed. The main result is that congruence relations and subdirect decompositions of $\underline{\mathfrak{L}}$(G,M,I) can be directly obtained from (G,M,I) without knowing $\underline{\mathfrak{L}}$(G,M,I).

B. WOJDYLO:

Some properties of partial algebras:

This lecture bases on the common considerations by P. Burmeister (Darmstadt) and B. Wojdylo (Toruń). The properties of homomorphisms and quomorphisms between partial algebras are discussed. It is presented a lattice of concepts for homomorphisms between partial algebras. (Details - see Preprint Nr. 657, FB Mathematik, TH Darmstadt). Moreover, it is given the meaning of category theoretical notions in selected categories of partial algebras.

Berichterstatter: J. D. H. Smith (Darmstadt)

# Liste der Tagungsteilnehmer

Professor M. E. Adams
Dept. of Mathematics
State University College
New Patts, New York 12561, U.S.A.

Professor K. A. Baker
Dept. of Mathematics
University of California
Los Angeles, California 90024, U.S.A.

Dr. H. Bauer
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Prof. Dr. B. Bosbach
Math. Institut der GHS
Heinrich-Plett-Str. 40
D-3500 Kassel

Prof. Dr. P. Burmeister
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Prof. G. Bruns
Dept. of Mathematics
McMaster University
Hamilton. Ontario

Prof. D. Clark
Dept. of Mathematics
State University College
New Palts, New York 12561, U.S.A.

Prof. B. Csákány
József Attila Tudományegyetem
Bolyai Intézste
Szeged
Aradi Vértanúk Tere 1, UNGARN

Dr. Alan Day
Dept. of Mathematics
Lakehead University
Thunderbay/Ontario, CANADA

Dr. M. Dichtl
Abteilung für Mathematik
Universität Ulm
Oberer Eselsberg
D-7900 Ulm

Prof. T. Evans
Dept. of Mathematics
Emory University
Atlanta, Georgia 30322, U.S.A.

Prof. Dr. W. Felscher
Fachbereich Mathematik
Auf der Morgenstelle 10
D-7400 Tübingen

Prof. E. Fried
ELTE TTK
Muzeum krt. 6-8
1088 Budapest, Hungary

Prof. Dr. B. Ganter
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Dr. P. Goralčík
Charles Univ.
Sokolovská 83, 186 00 Praha 8
ČSSR

M. Gould
Dept. of Mathematics
Vanderbilt University
Nashville, Tennessee 37203
USA

Prof. George Grätzer
Dept. of Mathematics
University of Manitoba
Winnipeg, Manitoba R3T 2N2
Canada

Dr. H.-P. Gumm
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Dr. C. Herrmann
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Prof. W. Hodges
Dept. of Mathematics
Bedford College
Regent's Park
London NW1 4NS, ENGLAND

Dr. A. P. Huhn
Research Group on Math. Logic
Hungarian Academy of Sci.
Szeged
Somogyi Béla u. 7
UNGARN

Prof. Dr. B. Jónsson
Dept. of Mathematics
Vanderbilt University
Nashville, Tennessee 37203
USA

Prof. Dr. K. H. Kaiser
Technische Hochschule Wien
Institut für Algebra und Math. Strukturtheorie
Argentinierstr. 8
A - 1040 Wien

Prof. Dr. O. H. Kegel
Math. Institut
Universität Freiburg
Herrmann-Herder-Str. 10
D-7800 Freiburg

Prof. D. Kelly
Dept. of Mathematics
University of Manitoba
Winnipeg, Manitoba
Canada

Dr. E. W. Kiss
Dept. of Alg. and Number Theory
Eötrös L. Univ.
Muzeum krt. 6-8
1088 Budapest
Hungary

Dr. P. Köhler
FB Mathematik
Universität Gießen
Arndtstr. 2
D-6300 Gießen

Prof. Dr. P. Krauss
FB 17 - Mathematik der GHS
Heinrich-Plett-Str. 40
D-3500 Kassel

Prof. George McNulty
Dept. of Mathematics
University of South Carolina
Columbia, South Carolina 29208
USA

Prof. Dr. W. B. Müller
Institut für Mathematik
Universität Klagenfurt
Universitätsstraße
A-9010 Klagenfurt

Prof. R. Padmanabhan
Dept. of Mathematics
University of Manitoba
Winnipeg, Manitoba
Canada

Prof. D. Pigozzi
Dept. of Mathematics
Iowa State University
Ames, Iowa 50010
USA

Prof. A. F. Pixley
Dept. of Mathematics
Harvey Mudd College
Claremont, Cal.
USA

Prof. M. Pouzet
Dep. de Math.
Univ. C. Bernard, Lyon 1
43 Blvd. du 11 Novembre 1918
69622 Villeurbanne Cédex
France

Dr. P. Pudlak
Math. Inst. Acad.
Žitná 25, 115 67 Praha 1
ČSSR

Prof. R. Quackenbush
Dept. of Mathematics
University of Manitoba
Winnipeg, Manitoba
Canada

Dr. Ivan Rival
Dept. of Mathematics
University of Calgary
Calgary/Alberta T2N 1N4
Canada

Dr. A. Romanowska
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Dr. Ivo Rosenberg
Centre de Recherche de Math.
Appliquées - 354
Université de Montréal
Case Postale 6128, Succursale "A"
Montréal, P.Q.
KANADA

Prof. Dr. J. Schmid
Math. Institut
Universität Bern
Bern / Schweiz

Prof. E. T. Schmidt
Math. Forschungsinstitut der
Akademie
Budapest V
Reâltanoda 13-15, UNGARN

Dr. J. Schulte Mönting
Mathematisches Institut
Auf der Morgenstelle 10
D-7400 Tübingen

Prof. Dr. D. Schweigert
FB Mathematik
Universität Kaiserslautern
Pfaffenbergstraße
D-6750 Kaiserslautern

J. D. H. Smith
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Dr. L. Szabó
József Attila Tudomànyegyetem
Bolyai Intézete
Aradi Vértanúk Tere 1
Szeged, Hungary

Dr. A. Szendrei
József Attila Tudomànyegyetem
Bolyai Intézete
Aradi Vértanúk Tere 1
Szeged, Hungary

Prof. Dr. T. Traczyk
Inst. Matematyki
Politechnika Warszawska
Plac Jednośei Robotniczej 1
00644 Warszawa, Polen

Dott. S. Tulipani
Via F. Puccinotti 18
50129 Firenze, Italy

Dr. I. Tůma
Math. Inst. Acad.
Žitná 25
11567 Praha 1, ČSSR

Dr. A. Ursini
FSMFN U, Math. Inst.
Via del Capitano 15
53100 Siena, Italy

G. H. Wenzel
Mathematische Fakultät
Universität Mannheim
Seminargebäude A5
D-6800 Mannheim

Prof. Dr. H. Werner
Fachbereich 17 - Mathematik
Gesamthochschule Kassel
Heinrich-Plett-Str.
D-3500 Kassel

Prof. Dr. R. Wille
TH Darmstadt
FB 4 - AG 1
Schloßgartenstr. 7
D-6100 Darmstadt

Dr. B. Wojdyło
Institut Mathematiky
Kopernikus University
ul. Chopina 12/18
87-100 Toruń, Poland