MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1982

Lokale Algebra und lokale analytische Geometrie

23.5. bis 29.5.1982

Die Tagung fand unter Leitung von R. Berger (Saarbrücken), J. Lipman (Lafayette) und G. Scheja (Tübingen) statt.
Den Teilnehmern sollte Einblick in neueste Ergebnisse der lokalen Algebra und lokalen analytischen Geometrie gegeben werden. Den starken Beziehungen, die zwischen den beiden Gebieten bestehen, trugen gemeinsame Diskussionen Rechnung. In den Vorträgen wurde eine Vielzahl von Themen angesprochen, von denen hier die Deformationstheorie und die Verbindung zur komplexen Analysis hervorgehoben seien.
Zusätzlich zu dem normalen Tagungsablauf fanden an drei Abenden weitere informelle Vorträge in kleinerem Kreise statt. Das internationale Interesse an dieser Tagung zeigt sich darin, daß über 1/3 der Teilnehmer aus dem europäischen und nicht-europäischen Ausland waren. Die ruhige Atmosphäre des Instituts, in der viele fruchtbare Diskussionen entstehen konnten, trug sicher zum Gelingen der Tagung bei.

Teilnehmer

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Vortragsauszüge:

Differential calculus and characteristic classes

(B. Angeniol)

The given explicit formulas for the fundamental class of a Cohen Macaulay subvariety of a given smooth variety generalizing the formula $\begin{bmatrix} df_1,\dots,df_p \\ f_1,\dots,f_p \end{bmatrix}$ of the complete intersection case.

For this we use the following basic construction:
if $L_1 \xrightarrow{\varphi} L_o \longrightarrow M$ is a presentation of a R-module M, choose bases of L_1, L_o such that the matrix of φ is $A = (a_{ij})$. Then the morphism $d\varphi \in \mathrm{Hom}(L_1, L_o \otimes \Omega_R^1)$ has an image γ_M^1 in $\mathrm{Ext}^1(M, M \otimes \Omega_R^1)$ which is independant of the different choices: it is the Atiyah class of M (which corresponds to the extension of the principal parts of M). Then one shows that $\frac{(\partial \frac{1}{A})^p}{p!}$ is the fundamental class of A in $\mathrm{Ext}^p(A, \Omega_R^p) \simeq \mathrm{Ext}^p(A, \Omega_R^p \otimes A)$.
One can also relate the local intersection theory of modules with the evaluation of Chern classes.
This construction has a global analogy which can be related to the Chern classes of sheaves and which gives explicit formulas for the Chern classes in Čech cohomology. Comparing the local and global theories, one obtains easily the Grothendieck formula relating the Chern class of the ring of a subvariety and the fundamental of this subvariety.

Growth of Betti numbers and of Bass numbers of local rings.

(L.L. Avramov)

Two series of numerical invariants defined by a local ring $(R,\underline{m},k)$ are considered: the Bass series $I_R(t) = \Sigma_{i\geq 0} \mu_i t^i$, μ_i being the i-th Bass number $\dim_k \mathrm{Ext}^i_R(k,R)$, and the Poincaré series $P_R(t) = \Sigma_{i\geq 0} b_i t^i$, b_i being the i-th Betti number $\dim_k \mathrm{Tor}^R_i(k,k)$.

<u>Theorem 1</u>: (joint work with J. Lescot). Let c_i denote the dimension of the i-th homology group of the Koszul complex on a minimal set of generators of $\underline{m}$. Then for every non-regular R the coefficientwise inequality

$I_R(t) \leq (\Sigma^{n-1}_{i=0} c_{n-i} t^i - t^{n+1})/(1 - \Sigma^n_{i=1} c_i t^i)$ holds $(n = \dim_k \underline{m}/\underline{m}^2)$. Moreover, the upper bound is attained if and only if R is a non-regular Golod ring.

<u>Theorem 2</u>: Suppose the characteristic of k is zero. If R is not a complete intersection, there exist an integer N and real numbers $C > 1$, $D > 1$ such that for all $i > N$ the inequalities $C^i < b_i \leq D^i$ hold. (It is well-known that for a complete intersection b_i is given for $i > n$ by a polynomial in i of degree $n-\dim(R)-1$ with integer coefficients.)

Infinitesimal Deformations of Two-Dimensional Cusps

(K. Behnke)

Let (X,x) be the two-dimensional cusp singularity, defined by the complete lattice M in the real quadratic number field K, and the cyclic group U of units of K. Using a lemma of E. Freitag on the sheaf cohomology of quotient spaces, we give the following description of the vector space T_X^1 of infinitesimal deformations of X:

Let $\mathcal{O}$ be the space of convergent Fourier-series $\sum_{\gamma \in M^*} c_\gamma \exp 2\Pi i(\gamma z_1 + \gamma' z_2)$ on the product of upper half planes $H \times H$. Let $\mathcal{D}$ be the space of derivations $\mathcal{D} = \mathcal{O}\frac{\partial}{\partial z_1} \oplus \mathcal{O}\frac{\partial}{\partial z_2}$, and let $f_1, \ldots, f_n$ holomorphic functions on $H \times H$, invariant under M and U, such that $i(x) = (f_1(x), \ldots, f_n(x))$ gives a minimal embedding i of X in $\mathbb{C}^n$.

Then there is a commutative diagram

$$
\begin{array}{ccccc}
 & & \mathcal{D} & \xrightarrow{\ \chi\ } & \bigoplus_{\nu=1}^{n} \mathcal{O}\frac{\partial}{\partial x_j} \\
 & & \downarrow{\scriptstyle \varepsilon-1} & & \downarrow{\scriptstyle \varepsilon-1} \\
 & & \mathcal{D} & \xrightarrow{\ \chi\ } & \bigoplus_{\nu=1}^{n} \mathcal{O}\frac{\partial}{\partial x_j} \\
 & & \downarrow & & \downarrow \\
T_X^1 & \hookrightarrow & H^1(X-\{\infty\}, \Theta_X) & \xrightarrow{\ \chi\ } & H^1(X-\{\infty\}, i^*\Theta_{\mathbb{C}^n})
\end{array}
$$

giving T_X^1 as the kernel of the bottom horizontal map χ, defined by $\chi(\mathfrak{d}) = (\mathfrak{d}f_1, \ldots, \mathfrak{d}f_r)$, $\mathfrak{d} \in D$.

Konstruktion verseller Deformationen in der analytischen Geometrie (Bericht über gemeinsame Arbeit mit S. Kosarew)

(J. Bingener)

Es wurden die Grundzüge einer Theorie skizziert, mit der im Prinzip "jedes" analytische Deformationsproblem behandelt werden könnte. Für die meisten bisher bekannten Fälle (Deformationen von kompakten komplexen Räumen, Deformationen von kohärenten Moduln mit kompaktem Träger, Deformationen von isolierten Singularitäten,...) läßt sich das Verfahren bereits jetzt mit Erfolg anwenden.

A generalization of the Zariski discriminant criterion and the extension of derivations on analytic algebras

(E. Böger)

If A is a reduced equidimensional local analytic algebra over $\mathbb{C}$, which is finite over a regular analytic algebra R over $\mathbb{C}$ of the same dimension, one may ask for a given k-derivation $\delta \in Der_k(R)$ if its canonical extension to the total quotient ring of A maps A into itself. The case of a normal A has been settled by Scheja-Storch. Scheja-Regel have investigated the case $A = R[x]$, i.e. A is a "simple extension" of R. On the other hand the classical Zariski discriminant criterion in the nonnormal case $A = R[x]$ suggests that one should look for conditions on δ to map the relative Lipschitz-saturation $\tilde{A}^R$ of A with respect to R into it self (instead of A as above). Such conditions are derived in two cases which are considered separately: Firstly one looks for the most general conditions on A for the Zariski criterion to hold in essentially the original form (δ-stability of the discriminant locus of A over R). Secondly the general case of an arbitrary A is considered and it turns out that not only the discriminant locus of A over R has to be δ-stable but also (eventually) some embedded prime ideals.

A lifting result for finiteness of local cohomology

(M. Brodmann)

Let M be a finetely generated module over a powerseries ring $k[\![x_1,\dots,x_s]\!] = R$. Let $k[\![Z,T]\!] \longrightarrow R$ be a homomorphism, $\mathfrak{m}$ the maximal ideal of R. Assume that f is regular with respect to $M/\Gamma_{\mathfrak{m}}(M)$ for all $f \in (Z,T)k[\![Z,T]\!] - (0)$ and that

$H_{\mathfrak{m}}^{i-1}(M/f)$ is fineteley generated. Then $H_{\mathfrak{m}}^{i}(M)$ is finetely generated. An easy proof of Grothendiecks finiteness theorem follows.

How to make a complex exact: The existence of generic free resolutions and related objects.

(W. Bruns)

In order to focus all the equational conditions, which are satisfied by the entries of the matrices a^k in a finite free resolution (FFR)

$$F: 0 \longrightarrow R^{b_n} \xrightarrow[a^n]{} R^{b_{n-1}} \longrightarrow \cdots \longrightarrow R^{b_1} \xrightarrow[a^1]{} R^{b_0}$$

over a commutative ring R, into a single object, Hochster coined the notion of a <u>generic</u> FFR: A pair (R,F) as above is called generic if every such FFR G over a commutative ring A can be obtained by a ring extension $R \longrightarrow A$ from F. He completely solved the problem for $n \leq 2$ and showed that for these cases there exist even universal FFRs: the extension $R \longrightarrow A$ is always unique then. Moreover he conjectured that generic FFRs exist for every possible type $(b_n,\ldots,b_o)$ and that the underlying ring can always be taken as a finitely generated $\mathbb{Z}$-algebra.

We prove Hochster's conjecture with "finitely" replaced by "countably". The proof is based on a very simple "exactification technique" which can also be used to produce generic models for many other types of objects like complexes with certain exactness conditions, periodic free resolutions, perfect resolutions etc. The existence of one object of a given "type" always ensures the existence of a generic object of that type. For certain acyclic complexes the underlying ring of a generic model can not be chosen as a noetherian ring, we believe however that Hochster's conjecture holds for FFRs.

An deformations of Cones over Flag-Varieties

(R.-O. Buchweitz)

Let $G/\mathbb{C}$ be a semi-simple alg. group, $B \subset G$ a Borelsubgr., $\rho: G \longrightarrow Gl(V)$ a finite-dimensional irreducible representation, $l \subseteq V$ a line fixed by B, $\rho(B)l = l$.

Then $\overline{\rho(G)l} \subseteq V$ is the cone over the embedding

$$G/_{\mathrm{Stab}\,(\langle l\rangle \in \mathbb{P}(V))} \xrightarrow{|\mathcal{L}_\rho|} \mathbb{P}(V)$$

The question (asked by Kraft, Slodowy,...) is:

Which of these cones are rigid ?

We show how to answer this question, using Bott's Thm. and results of M.Demazure on deformations of G/P, P parabolic.

Among the results are:

Thm. 1: For $G = Sl(n+1, \mathbb{C})$, $n \geq 1$, the only non-rigid cones are those corresponding to the adjoint representation and, for $n = 3$, to the Plücker-embedding of Grass (2,4) as well as its second Veronese-embedding.

Thm. 2: The cone over every embedding of G/B, $\dim G/B \geq 2$, is rigid.

PM-Rings

(M. Contessa)

A ring A is a pm-ring if every prime ideal is contained in a unique maximal ideal.

Theorem: A direct product of any family of pm-rings is still a pm-ring.

Two proofs, one topological and one algebraic, are given. The algebraic one is based on a new characterization of these kind of rings.

A structure theorem for noetherian reduced pm-rings is also done.

Embedding of curves and cuspidal rational curves

(D. Eisenbud)

This is a report of recent work of mine with Joe Harris.

Theorem 1: Let C be a general curve of genus g over $\mathbb{C}$, $g \neq 0,1,3$. C can be embedded as a curve of degree d in projective space if and only if $d \geq \frac{3}{4} g + 3$

This and other results on general linear series on general curves can be deduced from corresponding theorems on general (geometrically) rational curves with g ordinary cusps. In particular, a simpler proof of the Brill-Noether-theorem than that due Griffiths-Harris, which used nodal rational curves can be given, and the ramification in general embeddings can be determined.

Equations and Syzygies of projective curves (after Rob Lazarsfeld)

(D.Eisenbud)

Let $C \subset \mathbb{P}^r_k$ be a reduced and irreducible projective curve, $S = k[x_0,\ldots,x_r]$ the homogeneous coordinate ring of $\mathbb{P}^r_k$, and I_C the homogenous ideal of the curve. Following Castelnuovo, Mumford, and others, we say that I_C is p-regular if $H^1(\mathbb{P}^r, I_C(p-1)) = 0$ and $H^2(\mathbb{P}^r, I_C(p-2)) = H^1(\mathbb{P}^r, \mathcal{O}_C(p-2)) = 0$. It is not difficult to show that I_C is p-regular if and only if I_C is generated by forms of degreee $\leq p$ and, for each l, the l^{th} sysygy of I_C as an S-module is generated by forms of degree $\leq p+k$, or again, if and only if $I_C \cap (x_0,\ldots,x_r)^p$ has a linear free resolution (this circle of ideas is exposed, for example, in a forthcoming paper by S. Goto and the author.).

<u>Theorem:</u> (Lazarsfeld) If C as above is contained in no hyperplane, then I_C is [(degree C) - r + 2]-regular.

This result was proved by Castelnuovo for smooth curves in $\mathbb{P}^3$ and by Peskine-Grusan for arbitrary curves in $\mathbb{P}^3$ by different methods. Lazarsfeld's proof is essentially to approximate the resolution of I_C by the Eagon-Northcott complex associated to the presentation matrix of the graded module corresponding to a general line bundle of degree $\deg C - r + g + 1$ on the normalization of C.

Mixed Hodge Structure of an isolated singularity and the purity theorem.

(F. Elzein)

Let $y \in Y$ be an isolated singular point on an analytic germ of variety. Let $p: Y' \longrightarrow Y$ be a desingularization of Y

and S' the normal crossing divisor (N.C.D) over y. Then:

Proposition: We have an exact sequence of mixed Hodge structure (M.H.S.)

$$H^i_y(Y) \xrightarrow{p^* + i^* \circ i_*} H^i_{S'}(Y') \oplus H^i(y) \xrightarrow{i'^* \circ i'_* - p^*} H^i(S') \xrightarrow{\partial} H^{i+1}_y(V)$$

where $i : y \longrightarrow Y$ and $i' : S' \longrightarrow Y'$ are embeddings.

Let $S' = \bigcup_i S'_i$ be union of smooth irreducible and proper components. Consider the complex ${}_wE_1^{\cdot,q}$: $(S'^{(p)}$ denote $\coprod_{i_o < \ldots < i_p} S'_{i_o}$

$$\to H^{2p+q-2}(S'^{(-p)}) \ldots \longrightarrow H^{q-4}(S'^{(1)}) \longrightarrow H^{q-2}(S'^{(0)}) \xrightarrow{\mathcal{I}} H^q(S'^{(0)}) \to H^q(S'^{(p-1)}) \to$$

$$\underbrace{E_1^{p,q}\ (p<0)}_{\text{like for } H^*_{S'}(Y')} \qquad E_1^{-1,q} \qquad E_1^{0,q} \qquad E_1^{1,q} \qquad \underbrace{E_1^{p,q}\ (p>0)}_{\text{Like for } H^*(S')}$$

where to the left differentials are Gysin morphisms like for the logarithmic complex in [H II] by Deligne Publ. Math. IHES N° 40 (1972), and to the right the differentials are restrictions morphisms, dual to those at left. The morphism $\mathcal{I}$ is the dual of the intersection matrix

$I : \oplus_q H_q(S'_i) \longrightarrow \oplus H_{q-2}(S'_i)$ defined by $I(a)_i = a \cap_{Y'} S'_i$.

Proposition: The cohomology of the above complex $E_1^{*,q}$ gives the terms of weight q of $H_y^{*+q}(Y,\mathbb{C})$.

Gabber has proved a purity theorem on McPherson cohomology in car $p > 0$, we refer to notes by Deligne at the IHES. We deduce the following interpretation after discussions with Deligne.

Proposition: Semi-purity. The weights of $H^i_y(Y,\mathbb{C})$ are $\geq i$ for $i > n$, and dually the weights of $H^i_y(Y,\mathbb{C})$ are $< i$ for $i \leq n$.

Corollary: The complex

$$H^{q-2}(S'^{(0)}) \xrightarrow{\mathcal{I}} H^q(S'^{(0)}) \ldots \longrightarrow H^q(S'^{(p-1)}) \ldots \longrightarrow H^q(S'^{(n-1)}) \longrightarrow 0$$

is exact for $q > n$.

A dual statement is true for $q \leq n$.

The Hodge-Index-Theorem in Arakelov's Intersection-Theory

(G. Faltings)

Suppose X/R is a semistable curve of genus g over the integers R of a numberfied K. Arakelov has defined an intersection product for divisors on a compactification of X, obtained by adding fibres over the infinite places of K (see IZV Akad. Nauk. SSSR, 38 (1974)) We prove a Riemann-Roch and a Hodge-index-theorem for this product. The Riemann-Roch deals with the volume of a fundamental domain in $\Gamma(X,\mathcal{O}(D)) \otimes_{\mathbb{Z}} \mathbb{R}$, with respect to the lattice $\Gamma(X,\mathcal{O}(D))$, for a divisor D. For its formulation we construct a canonical volume-form on $\Gamma(X,\mathcal{O}(D)) \otimes_{\mathbb{Z}} \mathbb{R}$. The Hodge-index-theorem is proved by relating the selfintersection of a divisor to its Néron-Tate-height in the Mordell-Weil-group. For this we need the Riemann-Roch.

K-theory for complexes of modules

(H.-B. Foxby)

For any category $\underline{\underline{X}}$ of complexes of modules over a ring A the abelian group $\mathcal{K}(\underline{\underline{X}})$ is presented by generators $[P]$, only depending on the isomorphism class of $P \in \underline{\underline{X}}$, subject to the relation $[P] = 0$ if P is exact, and to the relation $[P] = [\bar{P}] + [\tilde{P}]$ whenever these is an exact sequence $0 \longrightarrow \bar{P} \longrightarrow P \longrightarrow \tilde{P} \longrightarrow 0$ in $\underline{\underline{X}}$. Let $S_1,\dots,S_d \subseteq$ Center A be multiplicatively closed, let $\underline{\underline{P}}_S$ denote the category of bounded complexes P of finite generated (left) A-modules, such that $S_v^{-1}P$ is exact for all $v = 1,\dots,d$, and let $\underline{\underline{P}}_S^d$ be the subcategory consisting of complexes of the form

$$0 \longrightarrow P_d \longrightarrow \dots \longrightarrow P_1 \longrightarrow P_o \longrightarrow 0$$

<u>Theorem 1</u>: The canonical homomorphism : $\text{Æ}(\underline{\underline{P}}^d_s) \longrightarrow \text{Æ}(\underline{\underline{P}})$ is an isomorphism.

For $d = 1$ the inverse can be given explicitly (and this gives rise to an exact sequence of groups

$$K_1(A) \to K_1(S_1^{-1}A) \to \text{Æ}(\underline{\underline{P}}_s) \to K_0(A) \to K_0(S_1^{-1}A)).$$

Now assume that A is local, and that M and N are finite generated A-modules, such that $\operatorname{pd} M < \infty$ and $\dim (M \otimes N) = 0$.

The <u>intersection multiplicity</u> is $\chi(M,N) := \sum_l (-1)^l$ length $\operatorname{Tor}_l(M,N)$

<u>Theorem 2</u>: if grade $M \leq 1$ or $\dim N \leq 1$, then

(0) $m + n \leq d$

(1) $\chi(M,N) = 0$ if $m + n < d$

(2) $\chi(M,N) > 0$ if $m + n = d$

where $d = \dim A$, $m = \dim M$, and $n = \dim N$

<u>Corollary 1</u>: (0), (1) and (2) holds always, if either $\dim A \leq 2$ or A is regular and $\dim A \leq 4$.

(Here the last part has also been proved by Hochster)

<u>Corollary 2</u>: (1) holds always, if A is regular and $\dim A \leq 5$

(This has also been proved by Dutta)

My proof uses:

<u>Lemma</u>: if A is regular and (M,N) is a counter example to (1) with $m + n$ minimal, then $d + m + n$ is even.

The proof of the Lemma, as well as the proof of Theorem 2, use the groups $\text{Æ}(\underline{\underline{X}})$ for various categories $\underline{\underline{X}}$.

<u>Two remarks on flatness and tangential flatness</u>

(M. Herrmann)

This is a report of recent work of mine with U. Orbanz.

The 1. remark is concerned with (necessary and sufficient) conditions for flatness of a morphism $f : X \longrightarrow Y$ having Cohen-Macaulay-fibres. Most of these conditions involve (generalized) Hilbert-functions.

A stronger property than flatness of f is the flatness of the induced morphism in the tangent cones. This property may be called tangential flatness. The geometric implications or interpretations of this stronger property seem to be less clear than for ordinary flatness. The 2. remark gives a (necessary and sufficient) condition for a flat morphism $f : X \longrightarrow Y$ with regular base Y to be tangentially flat. The link between the motivations of these 2 remarks is a recent paper of Shepherd-Barron, in which he gives a numerical criterion for flatness, assuming regular fibres. But the assumption of regular fibres makes a flat morphism tangentially flat. So the results of Shepherd-Barron are special cases of our first remark (and in part they are special cases of a recent preprint of Herzog):

<u>Theorem 1</u>: Let $f : (A,M) \longrightarrow (B,N)$ a local homomorphism of local rings. Let $I = MB$, $\bar{B} = B/I$, $d = \dim \bar{B}$ for any prime $P \subset B$ with $P \supset I$ request $\bar{P} = P/I$. Assume that $\bar{B}$ is Cohen-Macaulay. Then the following conditions are equivalent:

i) f is flat

ii) for any system of parameters $\underline{x}$ for I we have:
$H^{(0)}[\underline{x},I,B] = e(\underline{x},\bar{B})\, H^{(0)}[A]$

iii) for any system of parameters $\underline{x}$ for I we have:
$H^{(0)}[\underline{x}R + I,B] = e(\underline{x},\bar{B})\, H^{(d)}[A]$

iv) for all $P \in \mathrm{Min}(I)$ we have: $H^{(0)}[I B_P \cdot B_P]\ e(\bar{B}_{\bar{P}})\ H^{(0)}[A]$

v) $A \longrightarrow B_P$ is flat for all $P \in \mathrm{Min}(I)$

<u>Theorem 2</u>: (Same notations)

Let $\overline{gr_N B} = gr_N B/gr(f)(gr_M^{+}(A)) \xrightarrow{\ p\ } gr_{\bar{N}}\bar{B}$ the canonical epimorphism (where $gr(f) := gr_M(A) \longrightarrow gr_N B$)

Then if <u>A is regular</u>, the following conditions are equivalent:

i) $gr(f)$ is flat

ii) f is flat and p is an isomorphism

A more general frame of Thm. 2 was indicated.

Cotangent functors of curve singularities

(J. Herzog joint work with R. Waldi)

Let k be a perfect field. We consider a one-dimensional analytic k-algebra $R \cong k[[X,\dots,X_n]]/I$ which is the residue class ring of an analytic k-algebra S modulo a regular sequence $z_1,\dots,z_n$. Let $A = k[[X]] \subseteq R$ be a noetherian normalization of R such that $Q(R)/Q(A)$ is separabel.

Theorem 1: Suppose $S_{\mathfrak{p}}$ is regular for all primes of height 1, and let $l \geq 0$ be an integer, then

$$(-1)^l \sum_{i=0}^{l} (-1)^i \, l(T_i(R/A,R)) \geq (-1)^l \, l(\mathcal{C}(R/A)/R), \quad \text{and}$$

$$(-1)^l \sum_{i=1}^{l} (-1)^i \, l(T^i(R/A,R)) \geq (-1)^{l-1} \, l(\mathcal{C}(R/A)/R)$$

where the T_i, T^i denote the cotangent functors, $\mathcal{C}(R/A)$ the complementary module and $l(--)$ the length of a module.

Corollary: If the defining ideal of R is in the linkage class of a complete intersection, then

$l(\tau\Omega_{R/k}) = l(\mathrm{Co\,ker}\, C_R) + l(\tau I/I^2)$, here $C_R : \Omega_{R/k} \longrightarrow \omega_R$

is the canonical map into the module of regular differentials.

Theorem 2: Suppose $S_{\mathfrak{p}}$ is a Gorensteinring for all primes of height 1, and let $l \geq 1$ be an integer, then

$$(-1)^l \sum_{i=1}^{l} (-1)^i \, l(T^i(R/A,R)) \geq (-1)^{l-1} \sum_{i=0}^{l} (-1)^i \, l(T_i(R/A,R))$$

Residuenkomplex und reguläre Differentialformen

(M. Kersken)

Sei K ein bewerteter Körper der Charakteristik 0, A eine lokale analytische K-Algebra. Als den Residuenkomplex $D_\Omega(A)$ definieren wir den Komplex $\mathrm{Hom}_{\Omega_R}(\Omega_A, \Omega_R^{\cdot+m} \otimes_R C^{\cdot+m}(R))$, wobei $R: = K\langle\langle x_1,\ldots,x_n\rangle\rangle$ eine reguläre Potenzreihenalgebra und $R \xrightarrow{\Pi} A$ ein endlicher Homomorphismus der Codimension m ist. $C^{\cdot}(R)$ bezeichne den Cousinkomplex von R. $D_\Omega(A)$ (Residuenkomplex) ist bis auf kanonische Isomorphie unabhängig von der Darstellung Π und ist ein Komplex graduierter Ω_A-Moduln (Ω_A: De Rham-Algebra) mit äußerer Differentiation d. Die 0-te Kohomologie wird mit ω_A bezeichnet und ist ein Ω_A-Modul mit äußerer Differentiation d. Bezüglich der Restklassendarstellung $R = k\langle\langle x_1,\ldots,x_n\rangle\rangle \longrightarrow A$ der Codimension m kann ω_A als Modul, dessen Elemente gewisse "Residuensymbole" $\left[{x^* \atop F_1,\ldots,F_m}\right]$, $x \in \Omega_R^{\cdot+m}$, $F_1,\ldots,F_m \in \mathrm{Ker}\,(R \longrightarrow A)$ maximale R-Sequenz sind. Es gibt einen Homomorphismus $C_A : \Omega_A \longrightarrow \omega_A$, der bei vollständigem Durchschnitt $A = R/(F_1,\ldots,F_m)$ durch

$$\xi \longrightarrow \left[{\tilde{\xi}_1 dF_1,\ldots,dF_m \atop F_1,\ldots,F_m}\right] \text{ gegeben ist.}$$

Mit Hilfe von Residuensymbolen können einige Aussagen bei vollständigem Durchschnitt mit isolierter Singularität gemacht werden, nämlich:

1) $\mathrm{Der}_K(A)$ wird von der Eulerderivation und den trivialen determinantiellen Derivationen erzeugt.
2) Die De Rham-Kohomologie $M_{DR} = K \oplus (\omega_A{}^{q-1})_0 \oplus (\omega_A{}^q)_0$

<u>Equisingular deformations of Hamburger-Noether expansions</u>

(K. Kiyek)

Let k be an algebraically closed field of characteristic 0, $f_o \in k[[X,Y]]$ irreducible, $B = k[[X,Y]]/(f_o)$. Using Puiseux expansions one can define the characteristic sequence of f_o and show that this sequence, the multiplicity sequence of B and the value-semigroup of B determine each other. If the characteristic of k is arbitrary one may use Hamburger-Noether expansions instead

of Puiseux expansions to get analogous results.
Nobile studied equisingular deformations of $f_0(ch(k) = 0)$: Let A be a local complete k-algebra with residue field k, $f \in A[[X,Y]]$, $\bar{f} = f_0$, $x,y \in A[[t]]$ a parametrisation of f sucht that x,y and $\bar{x},\bar{y}$ have the same characteristic sequence.
We show that by using Hamburger-Noether expansions one can prove nearly all the results of Nobile in the case of arbitrary characteristic.

Ein Verschwindungssatz für gewisse Kohomologiegruppen

(S. Kosarew)

Sei $Y \subset X$ ein kompakter komplexer Unterraum des komplexen Raumes X, der durch das kohärente Ideal $I \subset \mathcal{O}_X$ definiert sei. Genügt das Normalenbündel von Y in X gewissen Krümmungsvoraussetzungen, so gilt ein Verschwindungssatz des Typs $H^r(W, I^k F) = 0$ für gewisse kohärente $\mathcal{O}_X$-Moduln F und $k \gg 0$, wobei W eine geeignete offene Umgebung von Y in X ist.
Hierdurch wird ein Satz von P.A. Griffiths auf den Fall mit Singularitäten und nicht mehr notwendig lokal freiem F verallgemeinert.

Ein Satz über endliche Erweiterungen von normalen analytischen Algebren und einige Anwendungen

(E. Platte)

Sei k ein bewerteter Körper der Charakteristik Null; ist $A \longrightarrow B$ eine endliche Erweiterung von normalen analytischen k-Algebren, so läßt sich zeigen, daß der Modul der

Zariski-Differentiale auf A ein direkter A-Summand des Moduls der Zariski-Differentiale auf B ist. Als erste Anwendung bewiesen wir eine Aussage über analytisch-verzweigte Überlagerungen isolierter Hyperflächensingularitäten. Als zweite Anwendung des obigen Satzes bewiesen wir einige Aussagen über solche normale Singularitäten A, die von einem konvergenten Potenzreihenring $B = k\langle x_1,\dots,x_s\rangle$ überlagert werden, derart, daß der Strukturhomomorphismus $A \longrightarrow B$ nicht ausgeartet ist. Es wird gezeigt, daß der Modul der k-Derivationen auf A sowie der Modul der Zariski-Differentiale auf A Macaulay Moduln sind. Hieraus folgt dann, daß die Kotangentenmoduln $\mathrm{Ext}^i_A(D_k(A),A)$ sowie $\mathrm{Ext}^i_A(D_k(A),\omega_A)$, ω_A: = kanonischer Modul von A, für i = 1..., d-2 verschwinden, falls der singuläre Ort von A von der Kodimension $d \geq 3$ ist. Insbesondere sind solche Singularitäten A starr, die in der Kodimension 2 regulär sind. Ferner folgt: Ist A nicht regulärer vollständiger Durchschnitt, so ist der singuläre Ort von A rein-2-kodimensional. Eine ähnliche Aussage gilt für fast vollständige Durchschnitte A.

Epimorphism Problems

(A. Sathaye)

We discuss partial results about the following question raised by Abhyankar (at least in special cases).

Question (Epimorphism problem). Let $x_1,\dots,x_n \in k^{[m]}$ = the polynomial ring in m variables over a field k.
Let $k[x_1,\dots,x_n] = k^{[m]} = k[u_1,\dots,u_m]$. Does there exist an automorphism $\sigma = k[X_1,\dots,X_n] \longrightarrow k[X_1,\dots,X_n]$ (the polynomial ring in n variables over k) such that
$(\sigma(X_1),\dots,\sigma(X_n))_{X_i \rightarrow x_i} = (u_1,\dots,u_m, 0,\dots,0)$?

There are easy counterexamples if cha $k > 0$, namely $x_1 = u_1^{p^2}$, $x_2 = u_1 + u_1^p$, in $n = 2$, $m = 1$ and this can be easily generalized to arbitrary n,m.

So <u>Assume</u> cha $k = 0$ and without loss of generality k-algebraically closed.

The following cases are known.

1. $n = 2$, $m = 1$ - Abhyankar-Moh epimorphism theorem
2. $n = 3$, $m = 2$. Let us rewrite the notation.

$$\varphi : k[X,Y,Z] \longrightarrow k[u,v] \longrightarrow 0$$

$(f(X,Y,Z)) = \operatorname{Ker} \varphi$. Consider f in $k[X,Y][Z]$

2.1 If f is linear in Z over $k[X,Y]$ then yes

2.2 If $f = a_0 z^d + a_1 z^{d-1} + \ldots + a_d$ and $a_0,\ldots,a_{d-1}$ have a common factor in $k[X,Y]$ then yes. (With Russell)

2.3 If $f = az^2 + bz + c$ and a,b do not meet as curves i.e. $(a,b) = (1)$ then yes. (Student of Russell)

3. $n = 3$, $m = 1$. Notation. $\varphi = k[X,Y,Z] \longrightarrow k[t] \longrightarrow 0$ $P = \ker \varphi$

$\Delta = \{r \mid \deg \varphi(h) = r, h \text{ variable in } k[X,Y,Z]\}$

Object is to show $\Delta = \{0,1,2,\ldots\}$.

3.1 It is possible to arrange that $\varphi(\alpha z + \beta) = t$

Where $\alpha,\beta \in k[X,Y]$, $\alpha \in k[X]$. Then $\deg \varphi(\sqrt{\alpha}) + s \in \Delta$ for all $s \geq 0$ where $\sqrt{\alpha}$ denotes the reduced expression of .

In particular Δ misses only finitely many numbers if any.

3.2 Moreover one can arrange $\varphi(XZ + f(Y)) = t$ for suitable $f(Y) \in k[Y]$ (after automorphism). If $\deg_Y f(Y) \leq 2$ then <u>yes</u>.

Un theoreme de Riemann Roch local

(L. Szpiro)

Il s'agit de l'énoncé suivant: (qui n'est démontré a cet instant que pour les anneaux locaux des variétés algébriques par W. Fulton, et pour les anneaux gradée par C. Pestrine et votre serviteur).

Soit L. un complexe parfait sur un anneau local noethérien A $X = \operatorname{Spec} A$, $y = \operatorname{Supp}(H(L.))$, K.(-) le foncteur groupe de Grothendieck des modules de type fini sur -, et A.(-) le groupe de Chow de - tensorisé par $\mathbb{R}$ alors on a un diagramme commutatif

$$\text{R.R.} \qquad \begin{array}{ccc} K.(X) & \xrightarrow{\Omega_X(\cdot)} & A.(X) \\ \chi(\cdot)\downarrow & & \downarrow \operatorname{Op}(L.)(\cdot) \\ K.(Y) & \xrightarrow{\Omega_Y(\cdot)} & A.(Y) \end{array}$$

où Ω est l'opérateur de Todd et Op(L.) est un opérateur gradué. Ω et Op(L.) possédent les propriétés fonctionelles qu'on devine.

On peut après avoir compris (même démontré) ce théorème se pencher sur mes deux conjectures favorites:

Soit L. un complexe parfait a homologie de longueur finie alors

C.1 $\chi(L.^{v}) = (-1)^{\dim A}\,\chi(L.)$ où $L.^{v} = \operatorname{Hom}(L.,A)$

C.2 $\chi(L.^{(p)}) = p^{\dim A}\,\chi(L.)$ quand la charactéristique de A vaut p et $L.^{(p)}$ le "frobéniasé" de L.

En particulier R.R. implique que

$$\chi(L.^{(p^n)})/p^{nd} \xrightarrow[n\to\infty]{} \text{limite} = \bar{\chi}(L)$$

et C1 et C2 se ramenent à montrer que

$$(*)\ \bar{\chi}(L) = \chi(L).$$

On peut remarquer que si on connaissait (*) on aurait-grace à R.R. - une démonstration de $\chi(L. \otimes N) = 0$ dès que dim N < dim A

Les points doubles rationels des surfaces

(J.L. Verdier)

Il s'agit de points singuliers isolés de surface obtenus en passant au quotient dans $\mathbb{C}^2$ par l'action d'un sous-groupe fini G de $SL(2,\mathbb{C})$. La surface S obtenue peut être désingularisée en $\tilde{S} \xrightarrow{q} S$. Le graphe dual du diviseur exceptionnel $\Gamma(G)$ est du type A_n, D_m, E_6, E_7, E_8 . Soit $c: G \hookrightarrow SL(2,\mathbb{C})$ la représentation canonique de G . En examinant l'action de la multiplication par c sur l'ensemble des représentations irréductibles non trivial de G , Mac Kay construit un diagramme qui n'est autre que $\Gamma(G)$. Par suspection de ces diagrammes on peut donc associer à toute représentation irréductible ρ de G une composante irréductible du diviseur exceptionnel $d\rho$. De plus le cycle $Z = \Sigma\, rg(\rho)\, d\rho$ n'est autre que le cycle fondamental de la singularité. Dans un travail commun avec G. Gonzalez nous donnons une déscription géométrique de cette correspondance.

Microgéométrie

(J.L. Verdier)

Soient X un espace analytique et $Y \subset X$ un sous espace fermé. Pour tout faisceau F sur X on définit Sp(F): un complexe de faisceaux sur le cône normal de Y dans X , dont la cohomologie est localement constante sur les génératrices épointées du cône. Ce complexe spécialisé redonne, lorsque Y est un diviseur de X définit par une équation, le complexe des cycles évanescents de F .

Soient Y un espace analytique et $E \longrightarrow Y$ un fibré vectoriel complexe. A tout complexe de faisceaux $F^\bullet$ sur E , à cohomologie localement constante sur les génératrices épointées on peut associer le transformé de Fourier géométrique de $F^\bullet$ qui est un complexe de même nature sur le fibré dual.

Soient X une variété analytique , Y une sous-variété, F un complexe de faisceaux sur X . Le micro localisé de F est le complexe $\mathcal{F}$(SpF) . C'est un complexe de faisceaux sur le fibré conormal de Y dans X . Lorsqu'on prend par F le faisceau $\mathcal{O}_X$, on obtient ainsi le faisceau $C^{\mathbb{R}}_{Y\,K}$ introduit par Sato, Kawai, Kashiwara.

The depth of module of differentials of a generic determinantal singularity

(U. Vetter)

Let K be a field, (X^i_u) an (m,n)-matrix of indeterminates over K , r an integer such that $1 \leq r < \min\{m,n\}$ and $R: = K[X^i_u]_{(X^i_u)}/I_{r+1}$, where $I_\rho = I_\rho(X^i_u)$ denotes the ideal generated by all ρ-minors of (X^i_u) . By $D_K(R)$ we will denote the module of Kähler-differentials of R over K . Then one can prove:

$$\text{depth } D_K(R) = \dim K[X^i_u]/ I_r (X^i_u) + 2$$

It follows that $D_K(R)$ is a second syzygy. Since one easily gets that depth $D_K(R)_{\mathfrak{p}} = 2$ for the (prime) ideal $\mathfrak{p}$ of the singular locus of R, $D_K(R)$ is not a third syzygy. The formula also implies a negative answer to the question of Buchsbaum and Robbiano resp., concerning the behaviour of depth $I_{r+1}^{s}/I_{r+1}^{s+1}$ for $s \geq 2$.
The formula has been proved by using the results on "algebras with straightening laws" due to De Concini, Eisenbud and Procesi.
These results also can be used in order to obtain results on depth $\mathrm{Hom}_R\,(D_K(R),R)$ and vanishing of $\mathrm{Ext}_R^i\,(D_K(R),R)$.

Berichterstatter: S. Mittelbach

Liste der Tagungsteilnehmer

Prof. B. Angeniol
Universite Paris Sud
Departement de Math.

91405 - Orsay Cedex
F r a n k r e i c h

Prof. Dr. J. Bingener
Universität Regensburg
Fachbereich Mathematik
Universitätsstr. 31

8400 Regensburg

Prof. Dr. L. Avramov
Bulgarian Academy of Sciences
Institute of Mathematics
P.O. Box 373

1000 Sofia, BULGARIEN

Prof. Dr. E. Böger
Universität Bochum
Abt. f. Mathematik
Universitätsstraße 150

4630 B o c h u m

Frau Dr. R. Barattero

Hammer-Str. 215

4400 Münster

Dozent Dr. M. Brodmann

Pappelweg 7

CH-5200 Windisch

Prof. Dr. S. Barcanescu
Increst
Bdul Pacii 220

79622 Bucharest

Rumänien

Prof. Dr. W. Bruns
Universität Osnabrück
Abteilung Vechta
Fach Mathematik
Driverstraße 22

2848 V e c h t a

Dr. K. Behnke
Fachbereich Mathematik
der Universität
Bundesstr. 55

2000 Hamburg 13

Dr. R.-O. Buchweitz
Universität Hannover
Institut f. Mathematik
Welfengarten 1

3000 Hannover

Prof. Dr. R. Berger
Fachbereich Mathematik
der Universität d. Saarlandes

6600 Saarbrücken

Prof. Dr. M. Contessa
Istituto Matematico
Universita di Roma
Citta Universitaria

00100 Roma
I t a l i e n

Prof. Dr. J. Herzog
Gesamthochschule Essen
Fachbereich 6
Universitätsstr. 2

4300 E s s e n

Prof. Dr. F. Elzein
Universite Paris VII
Maths. Tour 45-55
2, Place Jussieu

75221 Paris Cedex 05

Frankreich

Dozent Dr. G. Faltings
Universität Münster
Mathematisches Institut
Roxeler Str. 64

4400 M ü n s t e r

Prof. Dr. H. Flenner
Universität Göttingen
Mathematisches Institut
Bunsenstr. 3-5

3400 Göttingen

Prof. Dr. H.-B. Foxby
Kobenhavns Universitets
Matematiske Institut
Universitetsparken 5

21000 Kopenhagen
D ä n e m a r k

Prof. Dr. G.M. Greuel
Universität Kaiserslautern
Fachbereich Mathematik
Erwin-Schrödinger-Str.

6750 Kaiserslautern

Prof. Dr. M. Herrmann
Universität Köln
Mathematisches Institut
Weyertal 86-90

5000 Köln 41

Prof. Dr. D. Eisenbud
Brandeis University
Dept. of Mathematics

Waltham. Mass. 02154

U.S.A.

Dr. M. Kersken
Universität Bochum
Abteilung f. Mathematik
Universitätsstr. 150

4630 Bochum

Prof. Dr. K.-H. Kiyek
Gesamthochschule Paderborn
Fachbereich 17 - Mathematik
Warburger-Str. 100

4790 Paderborn

Dr. S. Kosarew
Fachbereich Mathematik
Universitätsstr. 31

8400 Regensburg

Prof. Dr. E. Kunz
Universität Regensburg
Fachbereich Mathematik
Universitätsstr. 31

8400 Regensburg

Prof. Dr. J. Lipman
Purdue University
Division of Math. Sciences

West Lafayette, Indiana 47907

U.S.A.

Dipl.-Math. S. Mittelbach
Fachbereich Mathematik
Universität des Saarlandes

6600 S a a r b r ü c k e n

Prof. Dr. H.-J. Nastold
Universität Münster
Mathematisches Institut
Roxeler Str. 64

4400 M ü n s t e r

Dr. E. Platte
Universität Osnabrück
Abteilung Vechta
Fach Mathematik
Driverstr. 22

2848 Vechta

Dipl.-Math. M. Regel
Universität Tübingen
Fak. f. Mathematik
Auf der Morgenstelle 10

7400 Tübingen

Prof. Dr. O. Riemenschneider
Universität Hamburg
Mathematisches Seminar
Bundesstr. 55

2000 Hamburg 13

Doz. Dr. Ch. Rotthaus
Math. Institut
Universität Münster
Roxeler Str. 64

4400 Münster

Prof. Dr. A. Sathaye
Purdue University
Division of Math. Sciences
West Lafayette, Indiana 47907
U.S.A.

Prof. Dr. G. Scheja
Universität Bochum
Abt. f. Mathematik
Universitätsstr. 150

4630 B o c h u m

Herrn G. Schneider
Universität Kaiserslautern
Fachbereich Mathematik
Erwin-Schrödinger-Str.

6750 Kaiserlautern

Prof. Dr. U. Storch
Universität Bochum
Abt. f. Mathematik
Universitätsstr. 150

4630 Bochum

Prof. Dr. L. Szpiro
3 Rue de la Butte aux Cailles

P a r i s 13
Frankreich

Dr. B. Ulrich
Purdue University
Division of Math. Sciences
West Lafayette, Indiana 47907
U.S.A.

Prof. Dr. J.-L. Verdier
Centre de Mathematique
Ecole Normale Superieure
45, rue d'Ulm

F-75230 Paris Cedex 05
Frankreich

Prof. Dr. U. Vetter
Universität Osnabrück
Abteilung Vechta
Fach Mathematik
Driverstr. 22

2848 Vechta

Doz. Dr. R. Waldi
Universität Regensburg
Fachbereich Mathematik
Universitätsstr. 31

8400 Regensburg

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