MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 25/1982

Darstellungstheorie und l-adische Kohomologie

13.6. bis 19.6.1982

Im Rahmen der Arbeitsgemeinschaft Algebra fand in diesem Jahr unter der Leitung von J.C. Jantzen (Bonn) und T.A. Springer (Utrecht) eine Tagung zum Thema "Darstellungstheorie und l-adische Kohomologie" statt, an der 39 Mathematiker aus verschiedenen Ländern teilnahmen. Diese Tagung bot sowohl jüngeren Mathematikern die Gelegenheit, anhand von (häufig mehrteiligen) Überblicksvorträgen einen Einblick in die Anwendungen von l-adischer Kohomologie und (in aller neuester Zeit) von Schnittkohomologie in der Darstellungstheorie zu gewinnen, als auch dem mit diesen Methoden bereits vertrauten Zuhörer eine Übersicht über neueste Forschungsergebnisse.

So wurde ausführlich über die Kohomologietheorien referiert (I.G. Macdonald, A. Borel) und die wesentlichen Anwendungen dieser Methoden auf die Darstellungstheorie endlicher Chevalley-Gruppen vorgestellt (R.W. Carter). Über weitere Anwendungen auf die Darstellungen von Weylgruppen und die Bestimmung der Green-Funktionen berichteten P. Slodowy and T.A. Springer. Die erst kürzlich gefundenen Resultate von T. Shoji and N. Spaltenstein über die explizite Berechnung der Green-Funktionen gehörten ebenso zum Vortragsprogramm wie von J.-L. Brylinski neuestens gefundene Zusammenhänge zwischen Fourier-Transformationen und Darstellungen der Weylgruppen.

C.W. Curtis trug über Anwendungen der Hecke-Algebren in der Darstellungstheorie vor und berichtete gemeinsam mit T.A. Springer über die von Alvis beschriebene Dualität von Darstellungen endlicher

Chevalley-Gruppen. G. Lusztig referierte selbst seine neuesten Ergebnisse über die Zerlegung der Deligne-Lusztig-Charaktere $R_{T,\theta}$. Über modulare Darstellungen endlicher Chevalley-Gruppen berichteten J.C. Jantzen (in der beschreibenden Charakteristik) und B. Srinivasan (über Blöcke in anderer Charakteristik).

Wie bei früheren Tagungen dieser Art zeigt es sich, daß die Berücksichtigung didaktischer Gesichtspunkte das wissenschaftliche Niveau der Tagung nicht beeinträchtigte.

Vortragsauszüge

I.G. MACDONALD:

Introduction to l-adic cohomology

The conjectures of A. Weil in diophantine geometry (Solutions of equations in finite fields, Bull. AMS 1949) indicated strongly that there should exist a cohomology theory for algebraic varieties defined over a field of arbitrary characteristic; the cohomology groups $H^i(X)$ should be vector spaces of finite dimension over some field of characteristic 0, and should have the usual formal properties which would in particular imply the truth of a Lefschetz trace formula expressing the number of fixed points of a morphism $F: X \to X$ as the alternating sum of the traces of F on the $H^i(X)$. Such a theory was developed by Grothendieck, Artin and others in the early 1960's, by generalizing the classical notions of sheaf cohomology to define ètale cohomology for an arbitrary scheme. The lectures were a brief introduction to this subject, designed for the non-expert, and covered the statements of some of the main theorems, including that of the trace formula in its general form, for an arbitrary l-adic sheaf of coefficients.

R.W. CARTER:

Introduction to Deligne-Lusztig theory

Let G be a connected reductive group defined over the field with q elements with Frobenius map $F: G \to G$. Let $G_F = \{g \in G; g^F = g\}$. The Deligne-Lusztig theory studies the characters of the finite group G_F. For each F-stable maximal torus T of G and each character θ of T_F the Deligne-Lusztig generalized character $R_{T,\theta}$ of G_F was defined. The character formula for $R_{T,\theta}$ in terms of Green functions on certain subgroups of G was stated, and also the scalar product formula for $(R_{T,\theta}, R_{T',\theta'})$. The degrees of the $R_{T,\theta}$ and their character values on semisimple elements of G_F were also described. The way in which the $R_{T,\theta}$ give a partition of the set of all irreducible characters of G_F into geometric conjugacy classes was described, each class containing just one character of degree prime to p (if p is not a bad prime for G). The degrees of these latter characters of G_F were described in terms of the semisimple classes in the dual group G_F^*. Finally a brief discussion of Lusztig's work on the unipotent characters of G_F was given.

P. SLODOWY:

Trigonometric Sums and Representations of Weyl Groups

According to Springer's hypothesis (proved by Kazhdan) the character values $R_{T,\theta}(u) = R_{T,1}(u)$ on unipotent elements can be computed as trigonometric sums on the G_F-orbit of a strongly regular element $A' \in (\text{Lie } T)_F$. Using l-adic cohomology and some geometric reductions one can express $R_{T,1}(u)$ as the alternating sum of the traces of a twisted Frobenius $F^* r_B^i(w)^{-1}$ on the cohomology groups $H^i(\mathfrak{B}_u, \mathbb{Q}_l)$, where $\mathfrak{B}_u$ is the set of Borel subgroups of G containing u, and where $r_B^i : W \to \text{Aut}(H^i(\mathfrak{B}_u, \mathbb{Q}_l))$ is a representation of the Weyl group $W = N(T)/T$. A theorem of Springer describes how the top cohomology groups $H^{top}(\mathfrak{B}_u, \mathbb{Q}_l)$, for u varying through a set of representations of the unipotent conjugacy classes, parameterize the irreducible representations of W.

T.A. SPRINGER:

Green functions and Deligne-Lusztig characters (after Kazhdan)

This talk contained a review of a paper by D.A. Kazhdan (Israel J. of Math. 25 (1977), 272-286), which contains another approach to the Deligne-Lusztig characters $R_{T,\theta}$. This approach leads to character formulas on unipotent elements. However, restrictions on p and q are needed.

The main problem is to show that the class function on G_F which is the candidate for $R_{T,\theta}$, is a virtual character of G_F. This is done by a suitable application of Brauer's theorem. The most difficult part of the proof is to show that the restriction of this function to the group U^F of rational points of a maximal connected unipotent $\mathbb{F}_q$-subgroup U of G is a virtual character of U^F. To do this, l-adic cohomology is involved. It is used to prove the following result. Let X be an algebraic variety defined over $\mathbb{F}_q$. Assume there is a closed filtration $X = X_0 \supset X_1 \supset \dots$ such that for all i there exists a morphism $f_i : X_i - X_{i+1} \to Y_i$ whose non-empty fibres are all isomorphic to a fixed affine space A^d (the filtration is not assumed to be defined over $\mathbb{F}_q$!). Then the number $|X^F|$ of rational points of X is divisible by q^d.

N. SPALTENSTEIN:

Determination of Green functions

Let G be a connected reductive group over $\mathbb{F}_q$ with Frobenius endomorphism F. If $\mathrm{char}(\mathbb{F}_q)$ and q are large enough, Springer and Kazhdan have shown that the Green functions of G can be expressed in terms of F and some representations of the Weyl group W on the l-adic cohomology of the varieties $\mathcal{B}_u = \{B \mid B$ Borel subgroup, $B \ni u\}$, $u \in G^F$ unipotent. In the top cohomology groups, the actions of F and W can be computed. It has been noticed by Shoji that these informations and the results of Borho and MacPherson on Springer's representations can be used to transform the orthogonality relations into a system of equations for the Green functions. These can be used to compute the Green functions of exceptional groups. For classical groups, Shoji has a geometric argument which gives more equations and the Green functions can in principle be computed. As applications, we get the results (already known for classical groups) that the Green functions are polynomials and that $\mathcal{B}_u$ has no odd cohomology.

A. BOREL:

Introduction to middle intersection cohomology

M. Goresky and R. MacPherson have associated new topological invariants, in the form of homology or cohomology groups, to certain singular spaces X (e.g. admitting a Whitney stratification with axiom B, in particular algebraic varieties or complex analytic spaces). The space X is endowed with a filtration

$$X = X_n \supset X_{n-2} \supset X_{n-3} \supset \dots \supset X_0 \supset X_{-1} = \emptyset$$

by closed subspaces X_i such that $S_j = X_j - X_{j+1}$ is either empty or a j-manifold (the j-th stratum). In particular S_n is a n-manifold ($X_{n-1} = X_{n-2}$ by convention). There is moreover a local triviality condition: around $x \in S_j$, stratification is a product of S_j by a cone over a stratified space (the link). To this and a suitable sequence of integers (the perversity) are associated cohomology groups. This talk was devoted to one such, the middle intersection cohomology, so far the most important for applications to algebraic varieties. Accordingly it was assumed that $S_j = \emptyset$ for odd j's.

First, the simplicial definition was recalled. Then I went to the sheaf theoretic point of view and gave various characterization (up to quasi-isomorphism) of the intersection cohomology sheaf IC^*, and some of the main properties of the intersection cohomology groups $IH^*(X;R)$, where R is the underlying ground ring. In particular, when R is a field, IC^* is (Verdier)-self dual, hence there is a perfect pairing

$$IH^i(X;R) \times IH_c^{n-i}(X;R) \to R \quad (i \in \mathbb{N}),$$

where $_c$ refers to cohomology with compact supports. Finally, the extension to local coefficients was described: given a locally constant sheaf E on S_n, whose stalks are finitely generated R-modules, there is similarly an intersection cohomology sheaf $IC^*(E)$ and a perfect pairing (when R is a field)

$$IH^i(X;E) \times IH_c^{n-i}(X;E')$$

where E' is the locally constant sheaf contragredient to E.

Ref.: M. Goresky - R. MacPherson; "Intersection homology Theory" Topology 18 (1980), 135-162; Intersection Homology II (preprint); Cheeger, Goresky, MacPherson: L'cohomology and Intersection homology (recent Annals of Math. Studies, ed. by S.T. Yau), various preprints.

C.W. CURTIS:

Hecke Algebras and their application to representations of finite Chevalley groups

In this survey lecture, the properties of the Hecke algebra of a permutation representation of a finite group, on the cosets of a subgroup, were summarized, along with formulas for degrees and character values for irreducible constituents of the permutation character in terms of the character values of irreducible characters of the Hecke algebra. With these ideas as background, the Hecke algebra $H(G,B)$ of a finite Chevalley group G, and a Borel subgroup B, was described along with the presentation of $H(G,B)$ due to Iwahori. Changing the point of view, the generic Hecke algebra H associated with a Coxeter group (W,R) was defined, so that if (W,R) is the Weyl group of G, then $H(G,B)$ and QW are both obtained as specializations of H. This led to the deformation theorems, that $H(G,B) \cong CW$, and the parameterization of characters of $H^{\bar{K}}$ and the components of 1_B^G in terms of irreducible characters of W. Generic degrees associated with characters of W were defined; they turn out (in general) to be polynomials which specialize to give the degrees of components of 1_B^G. A generic multiplicity formula was also given, with an application to determine the effect of the duality operation on components of 1_B^G. The possibility of extending these results to components of $\tilde{\lambda}^G$, for λ cuspidal irreducible character of a Levi subgroup of a parabolic subgroup, was indicated, using Howlett and Lehrer's results that $End_{\mathbb{C}G}\tilde{\lambda}^G$ can be obtained as the specialization of a generic algebra associated with a subgroup of the Weyl group, depending on λ. An introduction was given to Kazhdan and Lusztig's results on representations of generic Hecke algebras, using the idea of a $\mathcal{W}$-graph for a finite Coxeter group $\mathcal{W}$ (Inv. Math. 53, 165-184 (1979)). Finally, Lusztig's theorem, that there exists an explicit isomorphism $H^{Q(\sqrt{u})} \cong Q(\sqrt{u})\mathcal{W}$ was stated. (For further discussion and references on most of this material, the reader may consult a survey article by the author (Bull. Amer. Math. Soc., vol. 1 (NS))).

G. LUSZTIG:

Decomposition of the R_T^θ's.

Let G be a connected reductive algebraic group defined over F_q. Deligne and the author have constructed for each maximal torus $T \subset G$ defined over F_q and for each character $\theta: T(F_q) \to \bar{\mathbb{Q}}_l^*$ a virtual representation R_T^θ of the finite group $G(F_q)$. The lecture was concerned with the problem of decomposing these virtual representations in the case where G has connected centre. The main tool used is the intersection cohomology of Deligne-Goresky-MacPherson. Let X_w be the locally closed subvariety of G/B defined by $\{gB \mid g^{-1}F(g) \in BwB\}$ and let $\bar{X}_w$ be its Zariski closure. The following description of its intersection cohomology was given:

$$\Sigma(-1)^i \mathbb{H}^i(\bar{X}_w)u^{i/2} = \sum_{E \in \hat{W}} Tr(\sum_{y \leq w} P_{y,w} T_y, \tilde{E}) R_E$$

as elements in the representation ring of $G(\mathbb{F}_q)$ tensored by $\mathbb{Q}[u^{1/2}]$. Here $\tilde{E}$ denotes the representation of the Hecke algebra corresponding to an irreducible representation E of W. $P_{y,w}$ are the polynomials defined by Kazhdan and the author for any two elements in a Coxeter group and $R_E = |W|^{-1} \sum_w Tr(w,E)\Sigma(-1)^i H_c^i(X_w)$.

An analogous result in cohomology with compact support of X_w was proved by Asai and Digne-Michel. However use of $\mathbb{H}^i$ gives more precise results and generalizes well to non-unipotent representations.

The following theorem was stated and a proof was indicated. If E, E' are irreducible representations of W, then $R_E, R_{E'}$ are disjoint if and only if E, E' are in distinct two sided cells of W.

This gives rise to a partition of the set of unipotent representations into families one for each two sided cell of W. The representations in a given family can be parameterized by the elements of a set $M(\Gamma)$ where Γ is a finite group associated to the cell and $M(\Gamma) = \{(x,\sigma) \mid x \in \Gamma$ up to conjugacy, $\sigma \in Z(x)^{\wedge}\}$.

The multiplicities of these representations in the R_E, can be expressed in terms of a Fourier transform on $M(\Gamma)$. This implies explicit character formulas for all unipotent representations of $G(F_q)$ on semisimple elements. This generalizes to non-unipotent representations of $G(F_q)$ under the assumption that G has connected centre. Here one uses intersection cohomology of $\overline{X_w}$ with coefficients in local systems of rank 1.

J.-L. BRYLINSKI:

Weyl group representations and Fourier transformation

Kashiwara has recently given a new proof of Borho-MacPherson's result on the decomposition of the cohomology of Springer's resolution of the nilpotent variety of a semi-simple lie algebra, into intersection cohomology groups of closures of nilpotent orbits, with twisted coefficients (Kashiwara-Hotta, to appear). This talk was attempting to relate Kashiwara's approach to Springer's original construction of Weyl group representations; this is easy because both use Fourier transformation (in the case of Springer, Fourier transformation for vector spaces over finite fields, and the corresponding geometric Fourier transformation, using Artin-Schreier coverings; in the case of Kashiwara, the operation $x \to \frac{\partial}{\partial \xi}$, $\frac{\partial}{\partial x} \to -\xi$, familiar in the theory of linear P.D.E). The basic fact I use is that a Borel subalgebra is orthogonal to its nilpotent radical.

J.C. JANTZEN:

Modular representations of finite Chevalley groups (in equal characteristic)

Let G be a connected semi-simple algebraic group defined over $\mathbb{F}_q$ with Frobenius endomorphism F. For the sake of simplicity assume G to be simply connected and split. This talk gave a survey over the representation theory of the finite group G^F over $\bar{\mathbb{F}}_q$ and described its relations with the representation theories of G and of its Frobenius kernel $_FG$. It contained a description of Lusztig's conjecture how to express the formal characters of the irreducible G-modules in terms of the formal characters of the Weyl modules and how to obtain from this information the composition factors of the Deligne-Lusztig characters reduced mod p (for G^F) and of the universal highest weight modules for $_FG$. Furthermore, it showed how the characters of the principal indecomposable modules for G^F as well as for $_FG$ might be computed from a knowledge of these character formulas. Finally some results on the decomposition of the reduction mod p of unipotent characters of G^F were mentioned.

B. SRINIVASAN:

Blocks in classical groups (Unequal characteristic)

This talk is an exposition of some results obtained (jointly with P. Fong) on the r-blocks of general linear, unitary, symplectic and orthogonal groups over $\mathbb{F}_q$, where r is an odd prime not dividing q. First, let $G = GL(n,q)$ and let e be the order of q mod r. The unipotent characters of G are parametrized by partitions of n. If λ is a partition of n, let χ^λ be the corresponding unipotent character. The first theorem is that χ^λ, χ^μ are in the same r-block if and only if λ, μ have the same e-core. Then, the r-blocks are classified. There is a "Jordan decomposition theorem" for blocks similar to the Jordan decomposition of characters of GL(n,q). Finally the characters in a block can be classified. These theorems were stated and the main ideas in the proofs were indicated. Analogous theorems hold for the unitary groups. Finally some work in progress for symplectic and orthogonal groups was described; for example the r-blocks can be classified in these groups also.

C.W. CURTIS/T.A. SPRINGER:

Duality operation for representations of finite Chevalley groups

A duality operation in the character ring ch $\mathbb{C}H$ of a finite group H is a Z-automorphism of period 2 preserving the inner product of characters, and hence permuting, up to sign, the irreducible characters. For a finite Coxeter system (W,R), such an operation is given by $\mu \to \mu\varepsilon$, where ε is the sign representation, which can also be expressed as $\mu\varepsilon = \sum_{J \subseteq R} (-1)^{|J|} (\mu|_{W_J})^W$. Let G be a finite Chevalley group, with Weyl group (W,R). For a standard parabolic subgroup P_J, $J \subseteq R$, operations of truncation T_J: ch $\mathbb{C}G \to$ ch $\mathbb{C}L_J$, and I_j: ch $\mathbb{C}L_j \to$ ch $\mathbb{C}G$ were defined, for a Levi subgroup L_J of P_J, and the operation $\zeta \to \zeta^* = \sum_{J \subseteq R} (-1)^{|J|} I_j T_J \zeta$. Then we have $T_J(\zeta^{*_G}) = (T_J\zeta)^{*_{L_J}}$ for all $J \subseteq R$, and using this fact it can be proved that $\zeta \to \zeta^*$ is a duality operation. Some applications to character theory were indicated, including Alvis' interpretation of Springer's formula for the characteristic function on the unipotent set, and Alvis' proof of MacDonalds' conjecture that $\zeta(1)^{-1} \sum_{u \in U} \zeta(u) = \pm q^m$, for $\zeta \in \mathrm{Irr}\, G(q)$. (See D. Alvis, J. Algebra 74 (1982), 211-222.)

In the second part a brief review was given of some additional results on the duality, viz.

(a) a homological interpretation of the duality, in terms of homology of a system of coefficients on the Tits building of G (after Deligne-Lusztig, J. Alg. 74 (1982), 204-291);

(b) a result of N. Kawanaka for the (similarly defined) duality operation for class functions on the finite Lie algebra $\mathfrak{g}$ associated with G. This result connects the Fourier-transform on $\mathfrak{g}$, restricted to the nilpotent set, with the duality operation (N. Kawanaka, Fourier transforms of nilpotently supported invariant functions on a simple Lie algebra over a finite field, preprint).

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