MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 1/1983

Kontinuumsmechanik fester Körper

2.1. bis 8.1.1983

Die Tagung stand unter der Leitung der Herren G. Herrmann (Stanford) und H. Lippmann (München). Die Teilnehmer waren einhellig der Meinung, das sie wie die früheren Tagungen erfolgreich und fachlich anregend verlaufen ist. Es wurde der Wunsch geäußert, im Tagungsbericht die Chronik der Tagungen "Kontinuumsmechanik fester Körper" in Oberwolfach festzuhalten. Hier die Daten:

a) 31.1.-6.2.1971; b) 14.1.-20.1.1973; c) 5.1.-11.1.1975;
d) 16.1.-22.1.1977; e) 6.1.-12.1.1980.

Bis 1977 waren W. Günther (Karlsruhe) und H. Lippmann Tagungsleiter, seit 1980 G. Herrmann und H. Lippmann.

Die Vorträge und intensiven Diskussionen zeigten, daß das weitgespannte Tagungsthema auch heute noch aktuell ist und breites Interesse findet. Der Bogen der Vortragsthemen reichte wie schon bei den früheren Tagungen von der Angewandten Mathematik bis zu ingenieurmäßigen Aspekten der Kontinuumsmechanik. Neben der Anwendung von Methoden der Angewandten Mathematik auf Elastizitätsprobleme und Suspensionen sowie der Wellenausbreitung in elastischen Körpern lag ein weiterer Schwerpunkt bei Variations- und Extremalprinzipen unter Einschluß der Numerik. Eine weitere Gruppe von Vorträgen befaßte sich mit der Formulierung von Stoffansätzen und der Beschreibung der Deformationen. Neben theoretischen Überlegungen wurde auch die Verknüpfung mit experimentellen Daten eingehend behandelt. Aus dem Gebiet der Bruchmechanik kamen

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komplexe Fälle der Rißbildung und Rißausbreitung zur Sprache, dazu auch der duktile Bruch. Beim Schwerpunktthema Strukturen standen Nichtlinearität und Randeffekte im Vordergrund.

Vortragsauszüge

J.F. BESSELING:

Finite element properties, based upon elastic potential interpolation

The principle of virtual work requires the virtual work of the external forces to be zero for all infinitesimal rigid body motions. Absence of virtual deformations due to virtual displacements can be expressed in terms of subsidiary conditions. Introduced into the virtual work condition with the aid of multipliers the resulting contribution to the functional can be interpreted as the virtual work of deformation, where the multipliers are components of a stress field.

For a discret model it is sufficient to define for each finite element the necessary and sufficient condition for rigid body motion in terms of the nodal virtual displacements, or equivalently to define the deformation modes by properly chosen generalized strain measures. The dual stress multiplier must be related to these strains by constitutive equations in order to turn the equations of motion, obtained by the principle of virtual work, into a system of equations by which the physical deformation problem can be solved.

In the case of elasticity the constitutive equations are derived from the notion of the elastic potential. For a finite element, rather than resort to numerical integration in each step of the deformation process, first an approximation of the elastic potential of the element as a whole as a function of the generalized strains can be determined by simple interpolation. The derivatives of this potential with respect to the generalized strains then provide the constititive equations for the generalized stresses of the finite element. The accuracy of the elastic potential interpolation approach is found to be the same (or better in the nonlinear case) as the much more complicated so-called conforming finite element method.

J.P. BOEHLER

On isotropic and anisotropic hardening

The classical concepts of isotropic and kinematic hardening are analyzed in detail and it is shown that they cannot describe some important mechanical phenomena. A unified and rational formulation of isotropic and anisotropic hardening is developed within the framework of the tensor functions representation theory. Within the proposed scheme, the generalized concepts of isotropic and anisotropic hardening are defined by, respectively, isotropic and anisotropic transformations of the stress tensor. It is shown that under generally accepted mechanical hypotheses, the proposed procedure permits to transform an arbitrary initial yield condition into an arbitrary isotropic or anisotropic subsequent criterion. The full range of observed mechanical phenomena are thus taken into account and the classical concepts of isotropic and kinematic hardening appear as special cases of the developed theory.

N. BONTCHEVA, A. BALTOV, ST. TODOROV

A rule of nonsymmetric anisotropic hardening

A rule of nonsymmetric anisotropic hardening for metals and alloys, considered as microcomposite materials is proposed. The nonsymmetry of the experimentally observed subsequent yield surface is considered as a result of the different plastic properties of the components. The proposed mechanical model is compared with experimental results. The effect of the nonsymmetry of the anisotropic hardening rule is discussed in the plane case.

P. CHADWICK

Some aspects of the propagation of waves in constrained elastic bodies

The effects of kinematic constraints on wave propagation in elastic media depend strongly on the type of wave and the nature of the constraint. Some general results for simple nonlinear waves and linearized body and surface waves are discussed and illustrated by particular solutions for single- and doubly-constrained isotropic materials.

G. FICHERA

On the Principle of Fading Memory

The Fading Memory Principle introduced by Coleman and Noll is viewed from the standpoint of existence theory in linear viscoelasticity. An alternative Fading Memory Principle is proposed by using an axiomatic approach and an equivalent quantitative theorem proved. It is shown by a significant example how the Fading Memory Principle heavily depends on the topology introduced in the space of the admissible functions. The problem of deciding, on the basis of physical considerations, what is the topology to be chosen is posed.

U. GAMER

On the Rotating Elastic-Plastic Disk

The displacement field belonging to the elastic-plastic stress field in a rotating solid disk that can be found, derived with the help of Tresca´s yield condition, in textbooks on plasticity is discontinuous at the elastic-plastic interface. Tresca´s yield condition cannot be applied to this problem since its associated flow rule predicts a negative plastic strain caused by a tensile stress.

A. GOLEBIEWSKA-HERRMANN, G. FRANCFORT

A new variational principle and conservation laws for thermoelasticity

The main goal of the presentation is to construct a Lagrangian function such that not only the well-known equations of thermoelasticity, but also material conservation laws can be derived. As action variables, the position $\underset{\sim}{x}$ of a material particle and a scalar function η related to temperature are used. The material momentum for thermoelasticity as derived here, by contrast to the purely elastic case depends on a time interval rather than on an instant of time. The balance of material momentum is integrated over time to produce a relation reminiscent of the impulse-momentum relation in classical mechanics.

G. GRIOLI

On the stress in rigid bodies

It is possible to show that the traditional indeterminacy of the stress in a rigid body can be eliminated. The possibility is based on the fact that in the constitutive relations of each different material, as elastic materials and material with memory, it is possible to specify a structural parameter that can assume arbitrarily large values and is such that the body to which it refers is rigid on the limit when the parameter is infinite. On may think a rigid body as an ideal structure obtained as limit of deformable bodies. The constitutive equations of all isotropic rigid bodies have a unique analytic expression and their knowledge permits to determine the stress.

H.G. HAHN, H.A. RICHARD

Gemischte Moden in der Bruchmechanik

Die Anwendung der linear-elastischen Bruchmechanik zur Vorhersage des Sprödbruchverhaltens von Bauteilen mit Rissen unter komplizierten Belastungen ist heute noch weitgehend unmöglich. Die Verhältnisse bei Überlagerung der grundlegenden Rißbeanspruchungsarten (sog. Moden) I und II, d.h. Zug und ebener Schub an der Rißfront, werden diskutiert. Die Aussagen verschiedener Bruchkriterien weichen z.T. voneinander ab. Am Lehrstuhl für Technische Mechanik der Universität Kaiserslautern wurde eine Belastungsvorrichtung entwickelt, die zusammen mit einer neuartigen CTS-Probenform die einwandfreie Ermittlung bruchmechanischer Kenngrößen gestattet. Bisher vorliegende Versuchsergebnisse für Brucheintritt und Rißablenkungswinkel bei PMMA werden mitgeteilt.

E.W. HART

Effects of Material Rotations in Tension-Torsion Testing

The most common test procedure for the investigation of the multiaxial aspects of inelastic deformation is the testing of thin walled cylinders under combined tension and torsion. The analysis of the results of such tests has almost always ignored the effect of the material rotations (vorticity) that accompany the torsional shear. These effects are not small.

We present a rather general method for carrying out the analysis. This is applied to recent experiments on Nickel.

A novel vector method is shown for describing tension-torsion tensor relations.

P. HAUPT

Intermediate configurations and the representation of visco-elastic-plastic materials

The considerations start from the general constitutive equation of a simple material. An intermediate configuration is defined as the solution of a functional equation; the formal definition of the intermediate configuration corresponds to an equilibrium state, which is the asymptotic limit of a local unloading process. The intermediate configuration or the recoverable part of the strain depends on the past history of the total strain. It implies two different possibilities to decompose the strain, strain rate and the stress power into recoverable and inrecoverable parts.

In view of a representation of viscoelastic-plastic material properties the main constitutive assumption is a functional relation between the stress and the recoverable strain. In this constitutive equation the recoverable strain tensor as well as the stress tensor have to relate the intermediate state and the actual configuration.

G. HERRMANN, A. GOLEBIEWSKA-HERRMANN

Statistical Aspects of Local Deformation

Considered is the displacement gradient $u_{i,j}$ at a point for plane deformations. The four independent components may be interpreted, in the case of infinitesimal deformations, as elongations and rotations of fibers parallel to the coordinate lines. The elongation and rotation of a line having an arbitrary orientation α can be readily expressed in terms of components of $u_{i,j}$. The average rotation and average elongation of all fibers passing through the point are calculated and shown to represent two independent invariants of $u_{i,j}$. The standart deviation for both rotation and elongation, however, turns out to be the same and proportional to the maximum shear. The eigenvalues of the tensor $u_{i,j}$ (which exist if the maximum shear is larger than the average rotation) represent elongations of fibers which have not rotated and the eigenvalues of $u_{i,j}$ identify those two fibers. In a "dual" manner one can determine the rotations of those fibers which have not elongated by formulating and solving the "dual" or "secondary" eigenvalue problem. Such non-elongated fibers exist provided the maximum shear is larger than one half of the first invariant of the symmetrized tensor $u_{i,j}$ (i.e. $u_{i,i}/2$). These findings shed considerable light on the simple kinematics of infinitesimal deformation at a point and also generalize the classical eigenvalue problem for non-symmetric tensors.

K.P. HERRMANN

Crack path prediction in different shaped two-phase media under thermal loading

Curved thermal cracks are considered running along special principal stress tarjectories of thermal stress fields originated in different shaped two-phase composite structures due to a steady cooling process. The crack path prediction as well as the determination of the fracture mechanical data governing the quasistatic growth of a curved thermal crack have been performed by applying the concepts of linear elastic fracture mechanics together with the finite element method. Thereby the numerically calculated crack path shows an excellent agreement with a special principal stress trajectory of the corresponding uncracked specimen and with crack paths obtained from cooling specimens, respectively. Finally, a comparison was made between experimentally determined stress intensity factors at the tip of a quasistatic extending curved thermal crack gained by the optical method of caustics in disk-like two-phase glass specimens and theoretically determined stress intensity factors, respectively, obtained by a finite element calculation. Thereby a reasonable agreement could be stated between the theoretical and experimental values of the stress intensity factor.

R.J. KNOPS

Saint-Venant's principle for nonlinear elasticity

Some recent work, undertaken jointly with L.E. Payne, on a formulation and proof of Saint-Venant's principle for nonlinear elasticity is described. The problem considered is that of a semi-infinite cylinder whose base is loaded by forces with zero resultant and whose lateral sides are stress free. It is not assumed a priori that the stresses and displacements vanish at infinity, but merely that they are required to grow not faster than a given exponential. By means of weighted integrals it is shown that under certain hypotheses the L_2-integral of the displacement taken over a cross-section of the cylinder either grows faster than some exponential or decays at least exponentially with distance from the base of the cylinder. The strain energy of the cylinder is assumed a priori not to be bounded.

W.T. KOITER

The complementary energy principle in nonlinear elasticity: useful or useless?

Stable equilibrium of an elastic body under conservative loading conditions is characterized by a proper minimum of the potential energy functional. In terms of the truly complementary energy, expressed as a functional of the asymmetric Piola stress tensor, we cannot ascertain more in general than a stationary value in the actual stress field. Important exceptions arise, however, whenever none of the actual principal stresses is compressive. In such cases the complementary energy has a proper minimum in the actual stress field. For example, stable equilibrium configurations of perfectly flexible nonlinear elastic membranes are characterized by a proper minimum of the complementary energy.

D. KOLAROV

A method of solving the partial differential equations in mechanics and physics in the case of complicated boundary conditions

The basic disadvantages of all analytic methods, employed for the solution of systems of linear partial differential equations, lie in the significant and often insurmountable difficulties that one could come across attempting to satisfy the boundary conditions. The existing methods are applicable at simple boundary surfaces given in the corresponding coordinate systems. We propose a method of obtaining a great number of particular solutions having a lot of free parameters and integration constants (functions). These particular solutions are the base for the formal building of the general solution and can satisfy the boundary conditions around complicated surfaces (contours). One could effectively and formally treat all plane or spatial problems of the mathematical theory of elasticity for complex bodies and general material anisotropy, problems of the micropolar Cosserat continuum etc. This method is useful in every case of systems of linear partial differential equations with coefficients which are constants or functions of a single variable.

TH. LEHMANN

Some remarks on the definition of material isotropy

In the literature different definitions of material isotropy can be found. Sometimes isotropy is defined with respect to the initial configuration. In other cases isotropy is defined with respect to the actual state. The question is, wheater or not these definitions are equivalent and how they are related to one another. It can be shown that these questions can be answered in the simplest way by introducing a body-fixed (co-moving and co-deforming) coordinate system. Then it becomes immedia_tly evident that both definitions are equivalent. Particular considerations, however, are necessary in those cases where the total deformation can be decomposed into different parts and isotropy is restricted only to a certain component of the total deformation.

O. MAHRENHOLTZ

FEM-Ansätze zur Berechnung starr-plastischer Formänderungen

Die Berechnung großer plastischer Formänderungen, z.B. in der Umformtechnik, verlangt eine gute Kenntnis des Spannungs- und Geschwindigkeitsfeldes. Bei instationären Vorgängen können dabei besondere Probleme auftreten, wie die Änderung der freien Oberfläche. Die Finite-Element-Methoden sind hier in hohem Maße anpassungsfähig.

Es gibt verschiedene FE-Techniken zur Berechnung des plastischen Fließens. Sie unterscheiden sich - für dasselbe Stoffgesetz - durch unterschiedliche Modellbildung und unterschiedliche Lösungsansätze. So kann die Volumenkonstanz über Lagrangesche Multiplikatoren oder über "Penalty"-Funktionen erzwungen werden; bei Lösungsansätzen mit Stromfunktionen ist diese Bedingung implizit erfüllt. Eine andere Technik läßt leichte Inkompressibilität zu, um den numerischen Aufwand zu begrenzen. Für Probleme mit freien Oberflächen ist ein spezielles Verfahren entwickelt worden.

Hier werden diese Methoden vorgestellt und neue Entwicklungen diskutiert.

V. MANNL, M. HOLZNER

Tiefziehen rotationssymmetrischer Teile mit elastischem Kissen

Beim Kissentiefziehen wird Blech zwischen einem (nahezu) starren Stempel und einem nachgiebigen Polyurethan-Kissen umgeformt.

Finite-Element-Berechnungen mit den Programmsystemen Hondo II und Dyna2d werden für große elastische Deformationen des Kissens und elastisch-plastisch verfestigendes Tiefziehblech vorgestellt. Für die Kontaktfläche zwischen Polyurethanblock und Blech wurde ein an experimentelle Ergebnisse angepaßtes Reibgesetz implementiert.

Die Rechenergebnisse zeigen, daß die Reibbedingungen und das Gleichgewicht in den Kontaktflächen schlecht erfüllt sind und die Behandlung des Kontaktproblems verbessert werden muß. Hingegen besteht bei den Formänderungen des umgeformten Blechs eine qualitative Übereinstimmung mit experimentellen Ergebnissen für ein ähnliches konventionelles Tiefziehverfahren.

K. MARKOV

Volterra Expansions in Mechanics of Heterogeneous Solids

The problem of specifying the bulk properties for a random suspension of spheres is discussed, for definiteness, in the particular case of thermal conductivity. It is noted first of all that the random differential equation of heat conduction in the medium defines implicitly a nonlinear operator which transforms the random field of thermal conductivity into the random temperature field. According to the general idea of Volterra and Wiener, this operator can be expressed as a Volterra series and, in order to get approximate solutions, it is natural to truncate this series. The first such an approximation is considered here in detail; it corresponds to the case of dilute suspensions. As an illustration it is shown how the obtained results allow to account explicitly for the two-clustering of the spheres when predicting the bulk conductivity of the medium in the limit of small concentration.

M. MIKKOLA

Finite rigid body displacements for deformable bodies

The objective is to find a rigid body displacement which fits most closely the given displacement field of a deformed body. Displacements can be arbitrarily large. The approach suggested is following. The displacement vector of a point of the body is decomposed into two parts: a rigid body motion $\bar{u}_R=\bar{c}+\underline{T}\cdot\bar{r}$ and a deformation part $\bar{u}_D$. $\bar{c}$ is the translation and $\underline{T}$ is the rotator defining the rotation of the body. $\bar{r}$ is the position vector of the point in initial configuration. $\bar{c}$ and $\underline{T}$ are determined by minimizing the expression $I=I(\bar{c},\underline{T})=\int_v \bar{u}_D\cdot\bar{u}_D dv$. The necessary condition for the minimum, $\delta I=0$, leads to two vector equations for the solution of $\bar{c}$ and $\underline{T}$ which decouple if the principal coordinate system of the body is employed. The method is illustrated by some simple examples in 2- and 3-dimensions.

I. MÜLLER

Dynamic Theory of Memory Alloys

A previously formulated structural model of memory alloys is subjected to a time-dependent load and a time-dependent temperature. The resulting deformation as a function of time is calculated by a numerical evaluation of a set of rate laws for temperature and for the fractions of the martensitic and austenitic phases in the model. The rates of change of these quantities are determined by transition probabilities between energetic wells in which the elements of the model reside. Creep and yield of the model thus appear as thermally activated processes.

J. NAJAR

Comparative analysis of dynamic fracture continuous modelling with application to coal burst prevention problems

A contiuous model of fracturization based on the concept of an elastic imperfection criterion for rock damage is proposed in application to contained explosions in coal mining at destressing of burst-prone seams. The coefficients of elasticity degrade in the model due to internal damage in dynamic processes, complete failure occuring, when the deviation from perfect elasticity reaches a certain limit. The imperfection enrgy is made responsible for new surface creation in fragmentation and is shown to be necessarily strain-rate dependent. Fragmentation developes only when inherent damage is present.

The type of rock failure depends both on the inherent damage and on sensitivity of elastic modulae to damage development. A relationship between inherent damage and porosity can be found, as well fitting of data on fracture toughness and strength dependence on strain-rate with Weibull's distribution of flaws in coal is possible. The model explains the discrepancy between the amount of energy spent on fragmentation and the energy of new surface formation, calculated according to Griffith's criterion.

Z. OLESIAK

Application of the associated Weber transforms in the theory of elasticity

Departing from Weber-Orr expansion formula the associated expasions have been derived, firstly on a formal way, secondly by a method based on finding the eigenvalues and the expansion of the solutions of the corresponding differential equation in terms of eigenfunctions. The transforms of the type $W_{\nu+1,\nu}[\,;\,]$, $W_{\nu-1,\nu}[\,;\,]$ and $W_{\nu-2,\nu}[\,;\,]$ have been considered and formulae for the transforms of certain operators of functions have been derived. The kernels of the transforms are functions

$$C_{\mu\nu}(\xi r,\xi a)=J_{\mu}(\xi r)Y_{\nu}(\xi a)-J_{\nu}(\xi a)Y_{\mu}(\xi r),\ \mu=\nu-2,\nu-1, \text{ or } \nu+1,\ r>a>0.$$

Next the examples of applications in the theory of elasticity have been shown for an axial symmetry and an unbounded body with a cylindrical hole. In order to generalize Abel transforms the corresponding integrals were computed for the lower limit in the integrals equal to a certain positive "a" instead of 0.

A. SAWCZUK

Lower bounds to deflections of plastic structures at finite bending

A method of bounding from below the load-displacement relation is developped for rigid-plastic structures at moderately large deflections. The strain rates involve then terms nonlinear in strain gradients. The internal dissipation in the post yield range involves the displacements produced by the plastic flow and configuration changes of the structure. Employing the principle of virtual work the load-displacement relation can be estimated by bounding the internal dissipation. A bounding theorem is proposed and justified. Examples for static and dynamic cases of loading of plates are worked out for stable post yield behavior. Lower bounds to maximum deflections at the finite range of plastic flexure are compared with available experimental results and existing exact and approximate solutions.

Z. SOBOTKA

Unsymmetrische Verformungsvorgänge von nichtlinearen viskoelastischen und viskoplastischen Körpern

Auf Grund von Tensorentwicklungen führt der Verfasser eine neue Art von funktionalen konstitutiven Beziehungen der nichtlinearen Rheologie ein. Diese Beziehungen haben eine endliche Anzahl von Gliedern mit einfachen Integralen.

Der Spannungstensor von isotropen viskoelastischen Körpern, die das Gedächtnis von Verzerrungstensoren in ihren vergangenen Zuständen in N Zeitintervallen besitzen, ist eine Tensorfunktion von N+1 Verzerrungstensoren, welche nach den Regeln der Tensoralgebra in eine Beziehung mit einer endlichen Anzahl von Gliedern entwickelt werden kann. Für die anisotropen Stoffe hat der Verfasser den transformierten Verzerrungstensor eingeführt, der dieselben Hauptachsen wie der Spannungstensor besitzt.

Es wird bewiesen, daß die anisotropen Stoffe mit verschiedenen Verformungseigenschaften im Zug und Druck sich auch auf verschiedene Weise in zwei entgegengesetzen Richtungen von Schub benehmen.

E. STECK

Ein stochastisches Modell für die Hochtemperatur-Plastizität von Metallen

Die Hochtemperaturplastizität, d.h. Kriechen und Spannungsrelaxation metallischer Werkstoffe bei Temperaturen >0,5 *

Schmelztemperatur, wird zumindest für technisch interessante Spannungsbereiche auf thermisch aktivierte Versetzungsbewegungen zurückgeführt, wobei die "Struktur" des Werkstoffs, d.h. Verteilung und Höhe der inneren Barrieren, die diesen Bewegungen entgegenstehen, den Ablauf der Vorgänge maßgeblich beeinflußt.

Die Berücksichtigung dieser Struktur in Form von Übergangswahrscheinlichkeiten einer diskreten Markov-Kette ergibt ein stochastisches Modell, das wesentliche, für die Kriechvorgänge typische Eigenschaften, wie stationäres Kriechen, Abhängigkeit der Struktur von Spannung und Temperatur und Übergangszeiten zum stationären Kriechen bei Änderung dieser Größen, zumindest qualitativ zutreffend wiedergibt, und eine Untersuchung der Auswirkung von Annahmen über die Mechanismen im Mikrobereich zuläßt.

E. STEIN

Thermo-mechanisches Kriechen und Kriechbruch von Steinsalz

Zunächst wird eine Übersicht der heute üblichen Kriechversuche (Übergangskriechen, stationäres Kriechen) an zylindrischen Proben und Würfelproben in eindimensionalen und triaxialen Versuchen, jeweils mit relavanten Temperaturen gegeben.

Anschließend werden gängige, teils rein phänomenologische analytische Beschreibungen der Deformationen kritisch beleuchtet, insbesondere deren Extrapolationsfähigkeit über den Beobachtungszeitraum hinaus. Es folgen Materialgleichungen, die durch Integration der Evolutionsgleichungen im Rahmen der Thermodynamik der Deformationen unter Verwendung der freien Enthalpie und der Versetzungsdichte als innerer Variablen gewonnen wurden und deren Koeffizienten aus verfügbaren Versuchsdaten berechenbar sind. Auch die Zusammenhänge mit anderen Stoffgleichungen, z.B. Dehnungsverfestigung oder unter Verwendung der Arrheniusfunktion zur Beschreibung der thermischen Aktivierung der Versetzungen werden erörtert.

Dann wird kurz auf einen thermodynamisch begründeten gestaffelten FE-Algorithmus zur Berechnung der zeitabhängigen Temperaturen, Verschiebungen und Spannungen infolge mechanischer und thermischer Beanspruchung von Kavernen eingegangen.

Es folgen Anmerkungen zu den aus dem Metallkriechen bekannten Bereichen I, $II_{a,b}$ und $III_{a,b}$ in Abhängigkeit von Spannung und Temperatur mit jeweiliger Dominanz von Versetzungsgleiten, Versetzungsklettern (mit Volumen- bzw. Korngrenzdiffusion) und Diffusion.

Abschließend wird auf das beschleunigte Kriechen (sog. tertiäres Kriechen) und den Kriechbruch eingegangen. Nach Erörterung einiger experimenteller Ergebnisse wird die Problematik der theoretisch-numerischen Beschreibung aus der Sicht der Bruchmechanik beleuchtet.

W. SZCZEPINSKI

A theoretical and experimental study of the progressing process of ductile fracture of metals

Various idealized mechanisms of perfectly ductile fracture of metals are considered in order to obtain a deeper insight into the complex phenomena of non-brittle fracture of real ductile metals. These mechanisms include: the mechanism of internal necking of ligaments between parallel crack, the mechanism of local ductile fracture at the places of high strain concentrations caused by various configuration of adjacent cracks and the mechanism of intercrystalline ductile sliding and fracture. Experiments with the use of various specimens with prepared defects of various configuration substatiate the theoretical analysis. An interesting feature of the investigated models of the possible ductile fracture mechanisms is the appearence of instabilities of the loading process followed by the recovery of the bearing capacity due to strain hardening of the matrix material.

F.Y.M. WAN

Boundary Conditions for Plate Theories

The classical plate theory of Kirchhoff is known to be the leading term of the outer (asymptotic expansion) solution (in a small thickness parameter) for the linear elastostatics of thin, flat, isotropic bodies. Neither this leading term nor the full outer solution alone is able to satisfy arbitrarily prescribed edge conditions. For stress edge-data, St. Venant's principle may be invoked to generate a set of stress boundary conditions for the classical plate theory as well as for some higher order terms in the outer expansion without any reference to the complementary inner (asymptotic expansion) solution. Attempts in the literature to derive the corresponding boundary conditions for displacement edge-data have not been successful.

By an unusual application of the Betti-Rayleigh reciprocity theorem, R.D. Gregory and F.Y.M. Wan have derived the correct set of boundary conditions for classical and higher order plate theories with arbitrary edge-data. In this talk, we work out these conditions for an infinite plate strip with edgewise uniform data. The boundary conditions obtained for the semi-infinite plate case are rigorously correct and the result for the stress data case rigorously justifies the application of St. Venant's principle. Applications of the displacement boundary conditions obtained are illustrated by two simple problems:
(i) The shearing of an infinitely long rectangular block, and
(ii) A clamped infinite plate strip under uniform face pressure.

H.J. WEINITSCHKE, F.Y.M. WAN

<u>Boundary layer solutions for some nonlinear elasticity problems</u>

Problems with a small parameter ε are considered which are of the boundary layer type but different from standart singular perturbation problems due to a difficulty associated with the limit solution when $\varepsilon=0$. Two methods are presented, one is a more heuristic approach, the second is more rigorous in the sense of singular perturbation theory. The first method is illustrated for the problem of a circular membrane under vertical pessure and a small edge tension ε. Here the leading term layer solution is numerically matched at a point with an outer solution. The second method is applied to the annular membrane with a small central stress free hole. Like in the first problem, the inner solution in the boundary layer near the hole can be found explicitly. It can be properly matched

- through introduction of an intermediate variable - in an overlap domain with the outer solution which in first order approximation is the cicular membrane solution without hole. The same procedure can be used to treat a circular elastic plate with a stress free hole.

Z. WESOLOWSKI

String of asymmetric cross-section

On the inextensible string there acts the gravity field and the constant axial force. The principal moments of inertia of the cross-section are different. Position of each cross-section is defined by two displacement components and the rotation angle. The material being elastic, the principal bending moments are propotional to the principal curvatures and the torque is proportional to the twist. The set of equations of motion is linear in the displacements, and non-linear in the rotation angle. This set reduces to the sine-Gordon equation. In connection with the Backlund and Lie transformations various solutions are shown.

W. WUNDERLICH

Nonlinear Analysis of Shells of Revolution

The usual approach for linear shell of revolution problems using a Fourier expansion of loads and unknowns is applied also for geometrically and/or physically nonlinear problems. The cost-effective decoupling of the different harmonics is achieved by treating the nonlinear effects as pseudoloads and solving the nonlinear problem iteratively. The theory is formulated in N systems of ordinary diff. equns. and itegrated along the meridian yielding the stiffness of ring elements which are then assembled in usual finite element approach. The method is applied for shells of revolution which respond nonaxisymmetric due to the loads or initial imperfections, in addition to being linear. Results are shown for the buckling behavior and imperfection sensitivity of cone-torus-cylinder systems and dished end closures.

H. ZORSKI

Bifurcation of Elastic Waves

The relation between the intensity of a shock wave and its speed of propagation is important in determining properties of the waves themselves and in investigations of stability of materials. On the basis of the dynamic and kinematic compatibility conditions for an elastic body a system of algebraic equations is set up for the wave intensity and its speed of propagation with the side condition that the entropy be positive. The singular points and bifurcation properties are studied for various relations between the mechanical and thermal properties of the body.

It is found that besides the usual curves bifurcating for the linear acoustic speeds there are branches with no linear counterparts at all.

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