Mathematisches Forschungsinstitut Oberwolfach

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Mathematische Optimierung
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Diese dritte Tagung über Mathematische Optimierung in Oberwolfach hat wie schon ihre beiden Vorgänger wiederum einen sehr großen Anklang in der Fachwelt gefunden. Dies wird durch die relativ hohe Zahl von 63 Teilnehmern aus dreizehn verschiedenen Ländern dokumentiert. Bemerkenswert ist weiterhin, daß zwanzig Teilnehmer aus Nordamerika angereist waren, was den Stellenwert und die Attraktivität dieser Tagung unterstreicht.

Der Themenkatalog der 50 Vorträge war weit gespannt und überdeckte das gesamte Gebiet der diskreten und nichtlinearen Optimierung. Die Diskussion spezieller kombinatorischer Strukturen hinsichtlich Dualitätsbeziehungen, algorithmischer Fragestellungen sowie Anwendungsmöglichkeit auf praktische Probleme stand im Mittelpunkt einer Reihe von Vorträgen zur diskreten Optimierung. Darüber hinaus wurden neue Ergebnisse der Graphen- und Polyedertheorie, der Charakterisierung von Facetten sowie neue (Simplex-)Methoden für spezielle diskrete Optimierungsprobleme vorgestellt. Die Beiträge zur nichtlinearen Optimierung befaßten sich u.a. mit der Schätzung "dünner" Hesse-Matrizen im Zusammenhang mit der Lösung partieller Differenzialgleichungen, neuen exakten und approximativen Verfahren zur globalen Optimierung, der semi-infiniten Optimierung, Verfahren zur Lösung konvexer und quadratischer Programme sowie Ansätzen zur LP- und Tschebycheff Approximation.

Die Veranstalter planen auch dieses Mal die Herausgabe thematisch zusammengefaßter Proceedings als Sonderband der Zeitschrift "Mathematical Programming".

Zusätzlich zu den Vorträgen fanden eine Reihe von informellen abendlichen Diskussionen statt. Dabei war zu beobachten, daß obwohl die Mathematische Optimierung erst zum dritten Mal Gast in Oberwolfach war, es auch in dieser Disziplin schon eine Reihe von sogenannten "Oberwolfach-Problemen" gibt, die inzwischen zu Theorien weiterentwickelt, erneut von den Teilnehmern diskutiert wurden.

Veranstalter und Teilnehmer danken besonders herzlich dem Direktor des Mathematischen Forschungsinstitut und seinen Mitarbeitern für die ausgezeichnete Betreuung und - wie immer - einzigartige Atmosphäre.

Es folgen Kurzfassungen der Vorträge (in alphabetischer Reihenfolge der Vortragenden) sowie eine vollständige Adressenliste der Teilnehmer.

Vortragsauszüge

A. Bachem:

Adjoints of Oriented Matroids

The cocircuits of an oriented matroid can be used to define face lattices of oriented matroids. Two different matroids are known, the Las Vergnas and the Edmonds / Mandel face lattice which are polar to each other. Unfortunately the set of Las Vergnas face lattices differ from the set of all Edmonds / Mandel face lattices which implies that oriented matroids do not have a polar in general.

In this talk we give some reasons why polarity must fail in general in oriented matroids and give characterizations of those oriented matroids having a polar. It turns out that adjoints of matroids play an important role and we give a new definition of an oriented adjoint of oriented matroids. We show how to embedd an oriented adjoint into the extension lattice of the corresponding oriented matroid which allows us to construct a polar for oriented matroids having an oriented adjoint.

M. L. Balinski:

Dual Transportation Polyhedra and a New Method for the Assignment Problem

This talk will describe several properties of dual transportation polyhedra, prove that its diameter is at most $(m-1)(n-1)$ (the Hirsch conjecture) and give a new, simple method for the assignment problem that takes at most $(n-1)(n-2)/2$ steps.

M. J. Best:

Parametric Quadratic Programming and Related Topics

Several active set QP algorithms (Whinston and van de Panne - Dantzig, Fletcher, Gill and Murray, Best and Ritter) are known to construct identical sequences of points when applied to a common problem. We extend these methods to solve a parametric quadratic programming problem. We show that the complementary pivot method of Lemke and Howson solves a QP by solving a certain parametric QP. It does so by choosing the parametrized vectors in such a way that the origin is optimal for large values of the parameters. The method proceeds by reducing the parameter to zero at which point the given QP is solved. It is observed that using the parametric extension of the active set QP algorithms is computationally more attractive than the original solution format of Lemke and Howson.

R. E. Bixby:

The Partial Order of a Polymatroid Extreme Point

Let E be a finite set and let $f: 2^E \rightarrow \mathbb{R}$. f is called a (polymatroid) rank function if $f(\emptyset) = 0$, $A \subseteq B$ implies $f(A) \leq f(B)$, and $f(A) + f(B) \geq f(A \cup B) + f(A \cap B)$ for all $A, B \subseteq E$. The last condition is called submodularity. The associated polymatroid is the polyhedron $P(f) = \{x \in \mathbb{R}^E: x \geq 0, x(A) \leq f(A) \ (A \subseteq E)\}$ where $\mathbb{R}^E = \{(x(e): e \in E): x(e) \in \mathbb{R}, (e \in E)\}$ and $x(A) = \Sigma (x(e): e \in A)$.

Given a subset $B \subseteq E$ and an ordering $e_1,\ldots,e_n$ of B $(n = |B|)$ let $B_i = \{e_1,\ldots,e_i\}$ $(0 \leq i \leq n)$. The greedy "algorithm" for f and B defines $x \in \mathbb{R}^E$ by $x(e_i) = f(B_i) - f(B_{i-1})$ $(1 \leq i \leq n)$, $x(e) = 0$ $(e \in E / B)$. It is easy to show $x \in P(f)$.

Given a rank function f and its corresponding polymatroid $P(f)$, we associate with each extreme point of $P(f)$ a certain partial order. We show that this partial order is efficiently constructible, and that it characterizes all the orderings with which the greedy algorithm can be used to generate the given extreme point. We give several applications, including one to the still open problem of finding an efficient combinatorial procedure for testing membership in polymatroids. Our results can also be applied to convex games.

(joint work with W.H. Cunningham and D.M. Topkis)

R. E. Burkard:

Scheduling Periodic Events

Given r periodic events the problem occurs to schedule them such that the average or the maximal time between two occurrences, which follow immediately one the other, becomes minimal. This problem occurs in constructing timetables for urban transportation systems using the same routes. They are equivalent to arrange r regular polygons on a circle such that

- the minimal distance between two adjacent vertices on the circle becomes maximal (problem 1)

or

- the maximal distance between two adjacent vertices becomes minimal (problem 2).

Problem 1 is equivalent to find a minimal integer N such that certain congruence classes can be packed within the interval $[1,\ldots,N]$. Problem 1 and 2 admit the same unique solution, if all polygons have the same number of vertices. In the case that there are ℓ_1 polygons with m_1 vertices and ℓ_2 polygons with m_2 vertices the optimal solution of problem 1 is described by

$$N = \left(\left\lceil \frac{\ell_1 d}{m_2} \right\rceil + \left\lceil \frac{\ell_2 d}{m_1} \right\rceil \right) \frac{m_1 m_2}{d}$$

where d is the gcd (m_1, m_2) and an optimal arrangement can explicitely be given. In this case the optimal solution of problem 2 is described by

$$z = \max \left(\ell_1 m_1, \ell_2 m_2, \left(\left\lfloor \frac{\ell_1 d}{m_2} \right\rfloor + \left\lfloor \frac{\ell_2 d}{m_2} \right\rfloor \right) \frac{m_1 m_2}{d} \right)$$

and again an optimal arrangement can explicitely be described. If there are more than two different types of polygons the above results can be used to derive bounds for the optimal value. A bound for problem 1 with ℓ_i polygons of m_i vertices is $N = k_o M$ where $M = \mathrm{lcm}\,(m_1,\ldots,m_k)$ and

$$\max_i \left\lceil \frac{m_i}{M} \left(\ell_i + \sum_{j<i} \left\lfloor \frac{\ell_i d_{ij}}{m_i} \right\rfloor \frac{m_j}{d_{ij}} \right) \right\rceil \leq k_o \leq \max_i \left\lceil \frac{m_i}{M} \left(\ell_i + \sum_{j<i} \left\lceil \frac{\ell_i d_{ij}}{m_i} \right\rceil \frac{m_j}{d_{ij}} \right) \right\rceil$$

where d_{ij} is the gcd (m_i, m_j) . For N given above a feasible solution can be constructed by a (polynomial) algorithm, attaining this objective value.

A. R Conn:

Non-smooth Optimization - and Worse

We consider the problem of minimizing a function subject to constraints. The functions and / or the constraints may be non-differentiable or even discontinous. However, they must not be too pathological. In particular, we must be able to partition the problem into regions (cells) bounded by a finite number of continous constraints. This is formalised by the introduction of the local concept of f having the partition property at a point. In particular, this concept rules out the possibility of a neighbourhood covering an infinite number of cells. Furthermore, it implies that all discontinuities occur on cell boundaries.

In practice, we need better than C^0. We thus introduce the global concept of ╫ - piecewise ✱ functions, where ╫ refers to the degree of continuity of the cell boundary functions and ✱ does the same for f on any given cell interior.

We suggest a conceptual algorithm that depends upon solving subproblems to convergence. Clearly this is undesirable in practice. Thus, we would like to relax the algorithm. Such an approach is possible. It depends upon i) relaxing the concept of neighbourhood cells ii) admitting approximate optima of the subproblems and iii) relaxing the cell boundaries.
A practicable algorithm and applications are given.

W. H. Cunningham:

<u>A Generalization of the Network Simplex Method</u>

Let $G = (V, E)$ be a directed graph. For $F \subseteq V$ and $j \in E$ let

$$a_j^F = \begin{cases} 1 & \text{if } j \text{ enters } F \\ -1 & \text{if } j \text{ leaves } F \\ 0 & \text{else .} \end{cases}$$

Let $\mathcal{F}$ be a crossing family of subsets of V and let b be a function submodular on crossing pairs of members of $\mathcal{F}$. The optimal submodular flow problem (OSFP) is

$$\text{minmize} \quad cx \quad \text{s.t}$$

$$a^F x \leq b(F)\ ,\ F \in \mathcal{F}\ ;\ \ell \leq x \leq u\ .$$

This problem was introduced by Edmonds and Giles, who established important integrality properties for it. The special case in which $\mathcal{F}$ is the power set of V, b is modular, and $b(V) = 0$, is the well-known optimal network flow problem. An important algorithm for the latter problem is the network simplex method, a combinatorial specialization of the LP simplex method. We describe an analogous simplex method for OSFP.

After adding slack variables a basis of OSFP is determined by a set $T \subseteq E$ and a family $\mathcal{N} \subseteq \mathcal{F}$ with $|T| = |\mathcal{N}|$. We restrict attention to feasible bases determined by (T, N) for which N is cross-free. Then N determines a diagraph H obtained from G by identifying certain subsets of V, and T is a spanning tree of H. Our algorithm performes pivots by operating on T, H, but H can change when a slack variable is involved in a pivot. The fact that such a restricted class of bases is sufficient, leads to characterizations of vertices and of vertex-adjacency for OSFP. Finally, we generalize the notion of strongly feasible trees from the network simplex method, give a pivot rule for preserving this structure, and show that "potential" values associated with such bases behave monotonely during degenerate pivots, leading to a proof of finiteness for the algorithm.

(joint work with F. Barahona)

J. Dennis:

Secant Methods for Noisy and almost Sparse Nonlinear Equations

This talk will present current research into the effects of inaccurate function values and function differences on sparse Broyden methods. We will present a theorem relating noise to attainable accuracy. We will present and analyse a technique for updating a Jacobian approximation more sparse than the Jacobian. These results give some insight into the role of the quasi-Newton or secant equation.

(joint work with H. Walker)

U. Derigs:

Combinatorial Background and Near-Equivalence of Matching Algorithms

It is known that the standard algorithms for solving min-cost flow problems (Simplex / Out-of-Kilter / Dual Simplex / Complementary Pivot) when using the approximate start solutions and pivoting rules are only different implementations of one general method - the so-called "shortest augmenting path technique".

In this talk we show that for the existing algorithms for the min-cost perfect matching problem an analog result is true:

- Edmonds' (primal - dual) Blossom-algorithm
- the primal method of Cunningham and Marsh (with Big M-start) and
- the shortest augmenting path method proposed by the author

are performing - modulo ties - the same sequence of augmentations and dual changes.

A. Dress:

Matroids and Valuations

Ist $M = (E, \mathcal{B})$ ein auf der Menge E definiertes Matroid vom Range n mit $\mathcal{B} \subseteq \mathcal{P}_n(E)$ als Menge aller Basen und ist $\varphi: E \to K^n$ eine Realisierung von E, so definiert jede Bewertung $v: K^x \to \Gamma$ von K eine Abbildung $\mathcal{B} \to \Gamma: \{e_1,\ldots,e_n\} \mapsto v(\det(\varphi(e_1),\ldots,\varphi(e_n)))$. Es werden die kombinatorischen Eigenschaften dieser Abbildung und deren Zusammenspiel mit der gegebenen Matroidstruktur in der für die Theorie der Matroide charakteristischen abstrakt-axiomatischen Form diskutiert.

J. Fischer:

The Convergence of the Discrete Linear L_p Approximation as $P \to 1$

It is well known that the solution of the discrete linear L_p approximation problem converges to a special Chebyshev solution as $p \to \infty$. Here it is shown that the corresponding result for the case $p \to 1$ is also true. Furthermore, the special L_1 solution obtained in the limit is characterized as the unique optimal solution of a nonlinear programming problem on the set of all L_1 solutions.

J. Fonlupt:

Some New Polynomial Algorithms Related to Totally Unimodular Matrices

A matrix $A \in R^{m\times n}$ is equivalent to a matrix A' if it is possible to transform A into A' by pivoting operations. We prove that any matrix A is equivalent to a matrix A' with the following property:
All the elements of any column of A' have the same sign. A finite, pivoting but not polynomial algorithm is given.

In the case of totally unimodular matrices, this problem can be solved by a polynomial algorithm.

It is also proved that any totally unimodular matrix which does not contain a special matrix R_{10} as a minor can be transformed into a 0 - 1 totally unimodular matrix which does not contain certain classes of submatrices.

(joint work with M. Raco)

A. Frank:

On Directed and Odd Cuts

There is a certain analogy between directed cuts of directed graphs and odd cuts of undirected graphs. The Lucchesi - Younger theorem states that the maximum number of pairwise edge-disjoint directed cuts is equal to the minimum number of edges meeting all directed cuts. Seymour's theorem says the same for odd cuts of a bipartite graph. Here we go further on this line. Among others, a min-max formula is proved for the minimum number of edges in a covering of directed / odd / cuts which meet a fixed directed / odd / cut. The following result is a corollary: The minimum number of edges in an undirected graph, such that the graph becomes Eulerian by doubling these edges, is equal to $\max \Sigma q(V_i)$ where the maximum is taken over all partitions $\{V_1,V_2,\ldots,V_k\}$ of the node set and $q(V_i)$ denotes the number of components in $G - V_i$ in which the sum of degrees is odd.

(these results were obtained jointly with É. Tardos)

S. Fujishige:

Submodular Systems and Related Topics

Let R be the set of reals and $\mathcal{D}$ be a distributive lattice formed by subsets of a finite set with set union and intersection as the lattice operations. A submodular system $(\mathcal{D}, f)$ is a pair of a distributive lattice $\mathcal{D}$ and a submodular function $f: \mathcal{D} \to R$. We consider polyhedral and algorithmic aspects of submodular systems in connection with boundary hypermatroids induced by networks, submodular functions on crossing families, submodular flows, strongly connected orientations of graphs, Lovász's extension of set functions, minimization of submodular functions, etc.

D. M. Gay:

A Trust-Region Approach to Linearly Constrained Optimization

This paper suggests a class of trust-region algorithms for solving linearly constrained optimization problems. The algorithms use a "local" active-set strategy to select the steps they try. This strategy is such that degeneracy and zero Lagrange multipiers do not impair convergence and that no anti-zigzagging precautions are needed.

D. Goldfarb:

Optimal Estimation of Sparse Jacobian and Hessian Matrices that Arise in Finite Difference Calculations

The problem of estimating Jacobian and Hessian matrices arising in the finite difference approximation of partial differential equations is considered. Using the notion of "computational molecule" or "stencil", schemes are developed that require the minimal number of differences to estimate these matrices. The approach used is related to the problem of tiling the plane. A tearing procedure applicable to more complicated structures is also given.

A. A. Goldstein:

Complexity of Tchebycheff Approximation

Let A be an $m \times n$ matrix and b, an m-vector. Assume x^{∞} minimizes $\| Ax - b \|_{\infty}$. Given $\varepsilon > 0$ there is a p such that the ℓ_p method (which is based on approximating $\| \cdot \|_{\infty}$ by $\| \cdot \|_p$) will generate a point $x(k)$ such that:

$$\| Ax(k) - b \|_{\infty} \leq \| Ax^{\infty} - b \|_{\infty} (1+\varepsilon)$$

in less than k steps, where k is bounded by a polynomial in m, n and $\log (1/\varepsilon)$.

B. Gollan:

Inner Estimates for the Generalized Gradient of the Value Function in Nonlinear Programming

We consider the value function V of parameterized constraint optimization problems and the generalized gradient $\partial V(.)$. New results of the following type are proved: If for some optimal solution x of the unperturbed problem the set of Lagrange multipliers $\Lambda(x)$ is non-empty and compact, then $\partial V(0) \cap \Lambda(x) = \emptyset$.

As a novelty, similar statements are proved for multiplier sets defined by second order necessary optimality conditions.

M. D. Grigoriadis:

Generalized Network Flows

We consider linear programs whose coefficient matrix has at most two nonzero elements per column. By associating the rows and columns of this matrix with a vertex set V and edge set E respectively, the problem may be stated as a "generalized network flow" problem over the graph $G(V, E)$. By referring to the sign and magnitude of the ratio of its nonzero elements, an edge may be classified as "flow transforming" (with positive loss or gain, or with unit gain) or "flow absorbing or generating". A basis for the problem is a forest of spanning trees or "one-trees" (trees with exactly one cycle). Basic solutions are computed by using appropriate data structures to solve the upper triangular system by backsubstitution, and then by solving the "cycle-matrix system" which is upper Hessenberg. An error analysis gives an attractive bound for partial pivoting which is likewise implemented by using efficient data structures. The problem is then treated by specializing the revised simplex method is a manner similar to the network simplex method. Computational results indicate that the method is 20-50 times faster than a linear programming code. In many instances, additional linear constraints are given. This case is treated by partitioning the overall basis. It is shown that the "working basis" for such problems is usually one order of magnitude more dense than the coefficient matrix of the additional constraints. An LU factorization and a partial pivoting strategy for manipulating the working basis is adopted. The factorization is updated by both row and column elementary transformations and permutations to reduce the bump which is created to upper Hersenberg and then to upper triangular. This is done in such a manner as to minimize the computational burden for each step of the partitioning algorithm and to improve numerical accuracy.

M. Grötschel:

New Facets of the Bipartite Subgraph Polytope

The bipartite subgraph polytope $P_B(G)$ of a graph $G = [V,E]$ is the convex hull of the incidence vectors of all edge sets of bipartite subgraphs of G. We show that all complete subgraphs of G of odd order and all so-called odd bicycle wheels contained in G induce facets of $P_B(G)$. Moreover, we describe several methods with which new facet defining inequalities of $P_B(G)$ can be constructed from known ones. Examples of these methods are contraction of node sets in odd complete subgraphs, odd subdivision of edges, certain splittings of nodes, and subdivision of all edges of a cut. Using these methods we can construct facet defining inequalities of $P_B(G)$ having coefficients of order $|V|^2$.

P.L. Hammer:

Decompositions of Independence Systems

The concept of matroidal monotone Boolean functions (describing independence systems forming a matroid) and of the matroidal number of a monotone Boolean function (the min. number of matroidal functions whose conjunction the given function is) are introduced, and an " obstruction removal procedure" for finding a matroidal decomposition of a function is described. Graphs whose family of stable sets is 1-matroidal or 2-matroidal are characterized, and a class of k-matroidal graphs (k-arbitrary positive integer) is constructed.

(The work was jointly done with C. Benzaken)

S. P. Han:

On a Successive Projection Method

The paper is concerned with finding the closest point in the intersection of a finite number of closed convex sets to a given point with respect to an elliptic norm. A method is proposed, which is based on successive projection on the individual sets. The method has convergence properties when the interior of the intersection is nonempty. It is also shown that the SOR method and the method of multiplier for a definit quadratic program turn out to be special cases of the method.

K.-H. Hoffmann:

Semi-Infinite-Programming and Free Boundary Value Problems

A semi-infinite-programming problem of the following type is considered: Given compact indexsets $T_0, T_1, T_2 \subset \mathbb{R}^m$ with empty intersections and $T_0 \neq \emptyset$. Define $T := T_0 \cup T_1 \cup T_2$ and let $X \subset \mathbb{R}^n$ be an open set. Consider a twice continously differentiable function $F: T \times X \to \mathbb{R}$ and set $f_t(x) := F(t, x)$, $t \in T$, $x \in X$. The problem is:

$$\text{Minimize} \quad f(x) := \sup \{f_t(x) \mid t \in T_0\} \quad \text{subject to}$$
$$x \in X \quad \text{and} \quad f_t(x) \leq 0 \quad \text{for} \quad t \in T_1 \quad \text{and} \quad f_t(x) = 0 \quad \text{for} \quad t \in T_2 .$$

For this computer programs were developed and the convergence properties of the algorithms were discussed. Of special interest are the rates of convergence. It can be shown that the procedures converge quadratically (resp. linearly) under additional assumptions.

The programs were used to solve some numerical examples. One application concerning an inverse problem for differential equations is considered in detail. For that it becomes clear, how difficult it might be, to calculate derivatives in concrete problems.

R. Jeroslow:

Some Computational Results on Integer Modellings

In our earlier paper, we characterized integer and bounded-integer representability of sets and functions, and provided two broad classes of "sharp" modellings for representable objects. "Sharp" modellings are characterized by the fact that their linear relaxation, which is used in algorithms such as branch-and-bound, are "best possible".

We will show that earlier modellings in the literature, notably those for multiple-choice constraints, are not sharp. While our sharp modelling involves, both, more variables and constraints, we will provide computational results which yield significantly lower computer time, and number of nodes, for our sharp modelling. While all problems are "small" (three to forty-five integer variables), the advantage of the sharp modellings appear to improve with size.

We will also explore the issue of "modelling linkage" and show the computational advantages of newer modellings.

E. L. Johnson:

Binary Groups and the Chinese Postman Problem

Some results from the group problem are specialized to binary groups. In particular, the idea of homomorphic lifting is shown to give a way of lifting facets from minors of the associated binary matrix. Ideas of binary clutters are related to binary group problems. A dual binary group problem is shown to correspond to the blocking clutter of a given binary clutter. The postman problem and its dual are given as examples of pairs of binary group problems whose polyhedra are blocking polyhedra in Fulkersons sense. A pair of problems are introduced, the odd circuit problem and the co-postman problem and a conjecture is made that their associated polyhedra are blocking pairs of polyhedra when the graph has no K_5 minor.

B. Korte:

Greedoids

The principle of greediness plays a fundamental role both in the design of "continous" algorithm (where it is called the steepest descent or gradient method) and of discrete algorithms. The discrete structure most closely related to greediness is a matroid; in fact, matroids may be characterized axiomatically as those independence systems for which the greedy solution is optimal for certain optimization problems (e.g. linear objective, bottleneck). In this lecture we discuss combinatorial structures called "greedoids" which were introduced in earlier papers. Again, optimality of the greedy solution for a broad class of objective functions characterizes these structures (including breadth first search, shortest path, scheduling under precedence constraints and others). We also discuss some structural aspects of greedoids. The closure operator of greedoids is not monotone and has a relaxed Steinitz - Mc Lane exchange property. A weakened monotonicity property defines closure-feasible sets. The rank function of greedoids is only locally submodular. However, restricted to sets for which the rank is equal to its maximum intersection with a basis (rank-feasible sets), the rank function is submodular. These systems give raise to interesting structural properties. We also dicuss the relations of greedoids to geometries introduced by U. Faigle and we prove a Rado - Hüll theorem for greedoids.

C. Lemarêchal:

On Higher Order Algorithms in Convex Optimization

We study the problem of minimizing a real functional f . A Newton-like method requires first an approximation $D(d)$ of $f(x + d) - f(x)$ at the current iteratex, valid for d to an order higher than 1 , and consists in minimizing D . In our talk we will introduce a new concept of such an approximation without the classical differentiability assumptions. The connections with the classical concept based on Taylor development of a smooth function are exhibited. The material is used to study a conceptual algorithm. Some hints are given how one might implement these ideas.

(joint work with J. Zowe)

L. Lovasz:

Matroid Matching

The matroid matching problem is a common generalisation of non-bipartite matching and matroid intersection. Unfortunately, it can be shown (Korte and Lovasz) that no polynomial time algorithm exists to determine the maximum size of a matching in a 2-polymatroid. But there is a result valid for all 2-polymatroids which gives a good characterization of the matching number provided a certain configuration, named double flower, can be eliminated. This leads to a min-max formula and a polynomial time algorithm in the case of linear polymatroids. For the case of linear polymatroids, another algorithm using Monte Carlo methods can also be given.

O. L. Mangasarian:

Normal Solution of Linear Programs

The purpose of this talk is to describe methods for computing normal solutions of linear programs, that is solutions with least norm for some norm. The linear programming solution with least 2-norm can be obtained by either projecting a sufficiently large but finite multiple of the gradient of the objective function on the feasible region, or by optimizing an exterior penalty function for the dual linear program for sufficiently large but finite value of the penalty parameter. Successive overrelaxation methods are well suited for obtaining the least 2-norm solution of a linear program. The problem of minimizing the 2-norm of both the primal and the dual optimal variables and slacks can be reduced to an unconstrained minimization of a convex, parameter-free, globally differentiable, piecewise-quadratic function with a Lipschitz continous gradient. The problem of finding a solution with least 1-norm or ∞-norm can be reduced to a parametric linear programming problem or to the minimization of a piecewise-linear function on the nonnegative orthant.

L. McLinden:

Convex Programs Having Bounded Feasible Regions

Pairs of convex optimization problems mutually dual via the pertubational duality framework are analysed with respect to their possible recession behavior. As a main illustration, we prove the following existence result for the problem of minimizing a convex function subject to convex inequality constraints, together with its usual Lagrangian dual. If both problems are feasible, then either there exists a ray of primal feasible vectors on which each constraint function assumes arbitrarily large negative values or else there exists a ray of dual feasible vectors along which the dual (concave) objective function decays at no more than a linear rate. This generalizes results Clark and Duffin gave for the linear and convex cases, respectively. Since the proof technique is quite general, it might be used to develop similar results for other particular problem structures and in general settings.

R. R. Meyer:

Piecewise-linear Approximation Methods

Piecewise- linear approximation methods are developed for both separable and nonseparable convex optimization problems. By utilizing an implicit grid approach with suitable computational enhancements it is possible to attain robust methods with very fast linear convergence rates. In the separable case it is further shown that Lagrangian relaxation can be efficiently employed in conjunction with this approach to produce not only tight lower bounds on the optimal value but also computationally effective grid sizes for variables. Numerical results in the separable case with a variety of test problems, including models with more than 2000 variables, have established that guaranteed seven figure accuracy in the objective function is typically obtained in no more than seven major iterations.

J. J. Moré:

Estimation of Sparse Hessian Matrices and Graph Coloring Problems

Large scale optimization problems often require an approximation to the Hessian matrix. If the Hessian matrix is sparse then estimation by differences of gradients is attractive because the number of required differences is usually small compared to the dimension of the problem. The problem of estimating Hessian matrices by differences can be phrased as follows: Given the sparsity structure of a symmetric matrix A, determine A with the least possible number of evaluations of Ad. We approach this problem from a graph theoretic point of view and show that both direct and indirect approaches to this problem have a natural graph coloring interpretation. The complexity of the problem is analysed and efficient practical heuristic procedures are developed. Numerical results are used to illustrate the differences between the various approaches.

G. L. Nemhauser:

The Dominating Set Problem

A node of a graph is said to dominate itself and all other nodes of the graph that are adjacent to it. A dominating set is a subset of nodes that together dominate all nodes of the graph. The dominating set problem is to find a minimum cardinality dominating set. A 2-stable set is a subset of nodes with the property that if u and v are in the subset then u and v are not adjacent and there does not exist another node x that is adjacent to u and v. The 2-stable set problem is to find a maximum cardinality 2-stable set. It is easy to see that:
maximum cardinality of a 2-stable set $\leq$ minimum cardinality of a dominating set.
We study classes of graphs for which this inequality is an equality. Results are given in terms of excluded subgraphs. We also present polynomial-time algorithms for both problems on some classes of graphs for which the equality holds.

(This work was done jointly with G. Chang)

K. Neumann:

Cost and Time Optimization in Decision Activity Networks

Decision activity networks are acyclic activity networks where each node v is assigned a weight consisting of an "entrance characteristic" (which represents the number of incoming activities that have to be terminated in order to activate v) and an " exit characteristic" (representing the number of outgoing activities that are begun as soon as v has been activated). Each subset of the arc set whose corresponding activities are executed and which satisfies the "node weight condition" can be interpreted as a "network realisation".

Furthermore, each arc is assigned a weight representing the duration or the cost of executing the corresponding activity. We are then looking for such a network realization that the sink is activated (i.e., the project is completed successfully) and the total cost or the duration of the project is minimized. To solve those problems, Siedersleben has developed a branch-and-bound method. Lower bounds for the cost minimization are determined by solving min cost flow problems or, to obtain tighter bounds, min cost flow problems with multipliers. In case that the exit characteristic equals the outdegree for each node, the determination of lower bounds for the time minimization leads to min-max χ-matchings where χ is the entrance or exit characteristic of the respective node. Solving the general case for the time minimization is still an open question.

A. Prêkopa:

Optimal Daily Scheduling of the Electricity Production by Thermal Power Plants

Results of a project that has been going on for 8 years are presented. A large scale mixed variable (discrete - continous; dicrete = 0, 1) model is formulated for the optimal daily operation of the thermal power plants in the Hungarian electrical energy system. One day is subdivided into 27 periods and for given forcast concerning the electricity production of all blocks (generators) of all power plants is determined by solving this large scale mathematical programming problem. The most important feature of this model construction is that the network conditions are included into the constraints making quadratic some of the constraints while the others are linear. The solution goes via linearization + Benders decomposition + heuristics.

A. Recski:

On Strong Maps of Matroids with Applications

The partial order between two matroids on the same underlying set S is called a strong map if closed sets of M_1 correspond to closed sets of M_2. If M_1 and M_2 are representable over the same field by matrices with column set S then the strong map implies the possibility of a representation with the respective matrices A and $\left(\frac{A}{B}\right)$. This concept arises quite naturally in certain engineering applications since the rows of the representing matrices are then just the linear constraints posed by the actual physical models.

Contraction and truncation are the two best known examples of strong maps. In the present talk we mention two other strong maps with applications in electric engineering and in statics.

One of the applications leads to a new method of constructing unusual strong maps of graphic matroids. They are related to the existence of 1-factors in the corresponding graphs.

S. M. Robinson:

Nondegeneracy in Nonlinear Programming

In a previous paper we studied the local structure of the feasible set of a nonlinear programming problem under the assumption of constraint regularity. Here we introduce the stronger condition of nondegeneracy, and we show that if this condition holds at a feasible point then the portions near that point of the feasible sets of all problems close to the given problem are actually diffeomorphic to one another and to a fixed polyhedral convex set defined by the linearized problem. We present applications of this result to nonlinear programming theory (optimality conditions) and algorithms (convergence analysis).

J. B. Rosen:

Performance of Approximate Algorithms for Global Minimization

The performance of a class of algorithms for solving global minimization problems is analysed. Problems which may have a large number of variables appearing only linearly (in addition to the nonlinear variables) are considered. The analysis is based on finding an ε-approximate solution, in the sense that the function value found is known to be no more than $\varepsilon\Delta\Psi_1$ greater than the global minimum, where $\Delta\Psi_1$ is a known scale factor and the tolerance ε is specified. Each algorithm considered is characterized by two quantities α and ρ. A bound on the computing effort T_F required to obtain an ε-approximate solution is given in terms of the problem size, α, ρ and ε. In particular, it is shown that T_F increases no more than linearly with the number of linear variables, and that provided $\alpha\rho \leq 1$, the value of T_F is a linear (or sublinear) function of $(\frac{1}{\varepsilon})$.

R. B. Schnabel:

Tensor Methods for Nonlinear Equations and Unconstrained Minimization

Standard methods for nonlinear equations and unconstrained minimization are based on linear and quadratic models of the nonlinear function, respectively. Tensor methods augment these standard models with simple approximations to higher derivative terms, called tensors. They are intended to produce more efficient algorithms, without appreciably increasing the cost of forming, storing, or solving the model of the nonlinear function. They may be especially helpful for problems where the Jacobian or Hessian matrix at the solution is singular, because the extra derivative information can increase the linear convergence of Newton's method on such problems. We describe a tensor algorithm for nonlinear equations that uses analytic Jacobians, and present promising test results. We then discuss how tensor algorithms may be constructed for nonlinear equations without using analytic derivatives, and for unconstrained minimization.

A. Schrijver:

Packing and Covering of Crossing Families of Cuts

Let C be a crossing family of subsets of the finite set V (i.e., $\emptyset, V \notin C$, and if $S, T \in C$, $S \cap T \neq \emptyset$, $S \cup T \neq V$ then $S \cap T$, $S \cup T \in C$). If $D = (V, A)$ is a directed graph, a set A' of arcs of D is a cut (induced by C) if $A' = d_A^-(V')$ (:= set of arcs of D entering V') for some V' in C. A covering (for C) is a set of arcs entering each set in C (i.e., intersecting all cuts induced by C). Then the following are equivalent:

(1) for each directed graph $D = (V, A)$, the minimum size of a cut induced by C is equal to the maximum number of pairwise disjoint coverings for C;

(2) for each directed graph $D = (V, A)$ and each "length" function $\ell: A \to Z_+$, the minimum length of a covering is equal to the maximum number t of cuts $C_1,\ldots,C_t$ induced by C such that no arc a is in more than $\ell(a)$ of these cuts;

(3) there are no V_1,V_2,V_3,V_4,V_5 in C such that $V_1 \subseteq V_2 \cap V_3$, $V_2 \cup V_3 = V$, $V_3 \cap V_4 = \emptyset$, $V_3 \cup V_4 \subseteq V_5$.

This theorem has as corollaries theorems of Menger, Ford & Fulkerson, König-Egerváry, Gupta, Fulkerson, Edmonds, Feofiloff & Younger, Frank. Also, the optimization problems corresponding to (1) and (2) are polynomially solvable.

D. F. Shanno:

Computational Experience with Large Unconstrained Optimization Problems

The talk considers the problem of minimizing f(x) when x is an n vector with n large. The truncated Newton methods of Dembo and Steihaug are cosidered, and tested on a collection of test problems. An immediate conclusion is that the Dembo-Steihaug method of dealing with negative curvature appears completely satisfactory. Also, the inexact solution of the linear equations defining Newton's method appears useful, but not uniformly across all probelms. The method also appears to work well in general compared to other methods on penalty functions, but requires a great deal of work for functions with badly conditioned Hessians. Several suggestions are made as to direction for this work. A brief discussion of the comparison of truncated Newton and conjugate gradient algorithms with the algorithms of Buckley-Lenir and Toint-Griewank is included.

E. Spedicato:

More about a Class of Direct Methods for Linear Systems

The class of direct type algorithms for linear systems recently developed by Abaffy and Spedicato is further generalized by introducing additional parameters and dropping the full rank condition for the coefficient matrix. Various alternative formulations are considered it is also proved that LR and Choleski factorization are implicitly contained in the class. Some numerical experiments are discussed.

L. Teschke:

On the Relationship between the Objective Function and the Feasible Domain of a Nonlinear Optimization Problem

Starting from results of linear optimization, optimization problems are dealt with where the objective function is nonlinear and the set of feasible elements is neither polyhedric nor convex. A general theory is developed where a ternary relation in the set of feasible elements plays a central role reflecting the relationship between the objective function and the set of feasible elements of the problem. There is given a subset of the set of feasible elements where the objective function assumes their maximal or minimal function values. If the ternary relation is chosen in such a way that it is adapted to the special form of the optimization problem we can attain the subset containing as few elements as possible.

L. E. Trotter:

An Application of Matroid Polyhedral Theory to Unit-Execution Time, Tree-Precedence Job Scheduling

In 1961 Hu gave an integral min-max condition for the scheduling of identical jobs constrained by tree-precedence restrictions on identical processors. We show that this result may be deduced from an integral min-max result of Edmonds on covering the elements of a matroid by its bases and that this viewpoint suggests ways of extending the original scheduling result to a setting which allows due dates on the jobs.

M. Vlach:

Some Flow Shop Scheduling Problems with Parallel Machines

Mathematically speaking the probelm considered can be stated as follows: Given a real m by n matrix $A = [a(i,j)]$ and a real valued function f on R^m find an n-tupel $p = (p_1, p_2, \dots, p_n)$ of permutations of the set $\{1,2,\dots,m\}$ that will minimize the function

$$p \to f(c_1(p,A), c_2(p,A), \dots, c_m(p,A))$$

where

$$c_i(p,A) = \sum_{j=1}^{n} a(\bar{p}_j(i), j) , \quad i = 1,2,\dots,m .$$

V. Vujčić:

Methods for Semi-infinite Programming

This paper presents two methods for solving the following semi-infinite programming problem:

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$$\min_{x \in X} f(x) , \quad X = \{x \in R^n \mid \langle a(t), x \rangle \leq b(t) , t \in K\} ,$$

where $f: R^n \to R$, $K \subseteq R^m$ is compact set, $a: K \to R^n$, $b: K \to R$. In addition, it is assumed that $a(t)$ and $b(t)$ satisfy Lipschitz conditions on K and that $\text{int } X \neq \emptyset$. The idea of the methods is to approximate the feasible set X by a sequence of sets (X_k) such that $X_k \subset \text{int } X$ and each X_k is the intersection of finitely many half-spaces. In that way, solving the semi-infinite problem is replaced by solving a sequence of linearly constrained nonlinear programming problems. The basic difference between the two methods is that the first method uniformly approximates the feasible set, while the second method approximates the feasible set only in the neighbourhood of optimal points. Convergence results for the first method are proved under the assumption that the objective function $f(x)$ is continous. Similar convergence results are obtained for the second methods under the assumption that f is convex.

(joint work with M. Ašić)

D. de Werra:

Some Chromatic Characterizations of Perfect Graphs

Perfect graphs are characterized by the property of having so-called "canonical" colorings; a related concept of coloring will be defined for strongly perfect graphs; these will be called strongly canonical colorings.

One will then be able to consider extensions of strongly perfect graphs by defining some types of colorings (namely supercanonical colorings) which are intermediate between canonical and strongly canonical colorings. This will define a proper subclass of perfect graphs which strictly contains the strongly perfect graphs.

L. A. Wolsey:

Lot Sizing: The Convex Hull of Solutions and an Application to Multi-Item Capacitated Problems

It is shown that the convex hull of solutions for the uncapacitated single-item lot-sizing problem (the Wagner - Whitin model):

$$\min \{\Sigma c_i x_i + \Sigma f_i y_i : \sum_{i=1}^{\ell} x_i \geq d_{1\ell} , x_i \leq d_{iT} y_i , x_i \geq 0 , y_i \in \{0,1\}\} ,$$

$d_{i\ell} \equiv \sum_{t=i}^{\ell} d_t$, is obtained by adding the constraints $\sum_{i=1}^{\ell} \min \{x_i, d_{i\ell} y_i\} \geq d_{1\ell}$ $\ell = 1,\ldots,T$ which can be represented by an exponential family of inequalities. It is also shown that the problem can be formulated as a simple plant location problem whose LP solution is integer.

Results on the solution of multi-item capacitated lot sizing problems by mixed integer programming are reported, where a subset of the inequalities described above are added to the standard problem formulation.

(joint work with I. Barany, T. Van Roy)

K. Zimmermann:

Some Nonlinear Optimization Problems with Extremal Operations

Some Optimization problems over a special algebraic structure $E(\oplus , \otimes)$ with $E = R \cup \{-\infty\}$, $\alpha \oplus \beta = \max (\alpha, \beta)$, $\alpha \otimes \beta = \alpha + \beta$, $\forall\, \alpha,\beta \in E$ will be considered. Especially we investigate the problem of the form

$$\max_{1\leq i\leq m} |x_i - \bar{x}_i| \to \min$$

subject to

$$x = B \otimes u \text{ , } h \leq u \leq H \text{ ,}$$

where $x \in E^m$, $h,H \in E^n$ are given and B is a given matrix of the type $m \times n$ with elements $b_{ij} \in E$; the element $B \otimes u \in E^m$ is defined as follows: $(B \otimes u)_i = \sum_{j=1}^{n}{}^{\oplus} (b_{ij} \otimes u_j)$, $i = 1,\ldots,m$. A finite algorithm for solving this problem will be described and relations to some discrete-time machine-scheduling problems will be discussed. Possible extensions and generalizations will be also discussed.

U. Zimmermann:

One-parametric Bottleneck Transportation Problems

We consider bottleneck transportation problems

$$z(t) := \min_{x\in P_T} \max_{x_j > 0} c_{ij}(t) \qquad (t \in \mathbb{R}^k)$$

where P_T is the transportation polytope and c is a continous function satisfying some finiteness assumption. For the more general case of parametric bottleneck linear programs the stability region of any feasible solution is easily seen to be the finite union of certain closed convex cones in $\mathbb{R}^n$. A sufficient criterion for optimality reduces the number of changes in the optimal value function when passing from one of the regions to another. The extremal value function is continous and, if c is convex, then it is convex on the inverse image of these cones, but even in the linear case it is not concave on $\mathbb{R}^k$.

For the one-parametric case we tested the known primal methods for solving bottleneck transportation problems. The developed codes perform linear in $n\,m$ when the threshold totals heuristic of Finke and Smith is used. The best of these codes - based on the method due to Szwarc; Srinivasan and Thompson - was applied in a sequential method with increasing parameter t to solve the one-parametric bottleneck transportation problems. The observed computational complexity on problems with uniformly distributed c_{ij}^{μ} is subquadratic in $(n\,m)$ with increasing exponent when increasing the range of the coefficient ($c_{ij}(t)$ linear and quadratic).

Berichterstatter: U. Derigs

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