MATHEMATISCHES FORSCHUNGSINTITUT OBERWOLFACH

Tagungsbericht 5/1983

Noetherian Rings

23.1. bis 29.1 1983

The conference was organized by W. Borho (Wuppertal), A. Rosenberg (Ithaca) and L. Small (La Jolla). After annual conferences on ring theory in the period 1966-1977, this was the first conference since 1977 to deal relatively broadly with rings. Since many important results have been obtained recently in the application of general ring theory to other areas of algebra, it was decided to devote only two days to general noetherian ring theory and the remaining three to the following areas of applications: Rings of differential operators, Enveloping algebras of Lie algebras and Group rings and their relation with group theory.

The purpose, obviously, was to bring together the producers as well as the users of ring theory so that each group could learn from the other. Indeed, many mathematical discussions, took place between representations of various specialities and a very real cross fertilization could be seen occuring.

The conference was structured around six expository lectures of 1 1/2 - 2 hours duration with the following titles:

1. Goldie rank and applications.

2. Survey of recent progress in the theory of rings with polynomial identities.

3. Gabber's proof of the integrability of the characteristic variety and some applications to the theory of D-modules.

4. Primitive ideal structure of the enveloping algebra of a semi simple Lie algebra.

5. Primitive ideal structure of the enveloping algebra of a general Lie algebra.

6. Noetherian group rings

In addition to these, there were shorter talks in the areas of the expository lectures, given, as much as possible, on the same day as these.

The topics that were presented included the quotient ring techniques leading to additivity principles and their applications to enveloping algebras, PI and group rings, new results in "non-commutative algebraic geometry", and the recent work of Gabber on the equidimensionality of the characteristic variety. In addition various dimensionality results were discussed as were regularity questions in rings. In connection with PI rings, there were talks on generic matrices and the radical of such rings. Some exploration of extending the notion of regularity from the commutative case were also presented. The equivalence of categories that occurs when dealing with rings of differential operators was also dealt with as were various questions arising from the action of a group of automorphisms on a ring. Below are abstracts of the talks arranged in the order the talks were given.

J.T. STAFFORD: GOLDIE-RANK AND APPLICATIONS TO NOETHERIAN RINGS

The aim of this survey talk was to discuss various results concerned with the Goldie rank, Grk(M), of a module M over an arbitrary, prime, Noetherian ring R. Hopefully these results will be useful in applications of ring theory to other areas, and certainly they provide useful techniques for avoiding localization.

The first topic was the Joseph-Small additivity principle. A number of versions of this result now exist, but the one we discussed was a rather general version due to Warfield. This gives a relationship between the Goldie ranks of prime ideals in a Noetherian ring R and those in an arbitrary Noether overring S. Warfield's proof rather neatly avoids the localization questions that appear in most of the other versions.

The second topic concerned the minimal number of generators, g(M), of a module M. It can be shown that $\hat{g}(M,P) = Grk_{R/P}(M/MP)/Grk(R/P)$ is a satisfactory interpretation of the "local number of generators of M at the prime ideal P". This enables one to obtain an upper bound for g(M) for a module M over a Noetherian ring in terms of the $\hat{g}(M,P)$. It also gives one a relationship between the Goldie rank of a prime Noetherian ring R and of its prime factor rings.

The final topic was the notion of reduced rank of a module M over an arbitrary Noetherian ring R, which reduces to Grk(M) when

R is prime. Reduced rank has been used by Chatters-Goldie-Hajainavis-Lenagan to provide easy proofs of some well-known results as well as to generalize Jategaonkar's principal ideal theorem.

R. RESCO: DIMENSION INEQUALITY FOR NOETHERIAN RINGS

If R is a commuative Noetherian ring of finite global dimension, then it is an easy consequence of the Auslander-Buchsbaum-Serre theorem that K dim R = gl.dim R. While examples of Fields and Smith show that this equality need not hold for general Noetherian rings, the following question remains open: If R is a two-sided Noetherian ring of finite global dimension, is K dim R $\leq$ gl.dim R?

In this talk we give a brief survey of a few important instances where an affirmative answer to this question has been obtained:

1) A filtered algebra whose associated graded ring is a commutative Gorenstein ring (Roos, 1972).

2) A fully bounded Noetherian ring with enough clans (Brown-Hajarnavis-MacEachern, 1982).

3) A semiprime Noetherian PI-ring (Resco-Small-Stafford, 1982).

K.R. GOODEARL: KRULL DIMENSION AND HEIGHT

A descending chain $P_0 > P_1 > \ldots > P_n$ of prime ideals in a commutative noetherian ring R may be viewed module-theoretically as a sequence $R/P_0, R/P_1, \ldots, R/P_n$ of critical R-modules such that each module in the list is a proper factor of the next one. Such sequences of critical modules also arise in studying noncommutative Krull dimension, which leads to the idea of using such sequences to define a notion of height for critical modules. Some modifications of the most obvious definition are necessary, due to the existence of incompressible critical modules, and the existence of critical modules that do not remain critical when tensored with natural ring extensions such as differential operator rings or skew-Laurent rings. Some possible definitions of height will be discussed, and the uses of height in computing the Krull dimensions of differential operator rings and skew-Laurent rings will be sketched.

A. GOLDIE: STRONGLY REGULAR ELEMENTS.

In a noetherian ring R an element s is *strongly regular* if $Tsx = 0 \Longrightarrow Tx = 0$ $(x \in R)$ for all ideals $T \triangleleft R$, including $T = R$. The set of strongly regular elements is left-right symmetric. Thus $ysV = 0 \Longrightarrow yV = 0$ for $(y \in R,\ V \triangleleft R)$.

Denote the set of strongly regular elements of R by $\mathcal{S}_R(0)$.

Thm. $\mathcal{S}_R(0) = \bigcap_{\mu(R)} \mathcal{C}(M)$ where M is any middle annihilator ideal (the set of these is $\mu(R)$).

$= \bigcap_{\pi(R)} \mathcal{C}(P)$ where $P \in \pi(R)$ = the set of prime middle annihilator ideals.

(Note $M = \langle x \in R \mid A\,x\,B = 0$ for some ideals, $A, B \triangleleft R$ and $AB \neq 0\rangle$.

Thm. $\mathcal{S}_R(0)$ contains the unique maximal $\ell(r)$ Ore set of regular elements of R and so if R has a full quotient ring then any regular element of R is strongly regular.

Finally an example due to Small and Stafford (Proc.L.M.S.(3) 44 (1982), pp.385-404) is used to show that $\mathcal{S}_R(0)$ may be a proper subset of the set of regular elements (and yet not the units of R) and this example fails to have a full quotient ring but has a quotient ring relative to $\mathcal{S}_R(0)$.

C.R. HAJARNAVIS: HOMOLOGICALLY HOMOGENEOUS RINGS

A ring R with central subring C is called homologically homogeneous (over C) if (i) R is right Noetherian, (ii) R is integral over C, (iii) rt.gℓ.dim R $< \infty$ and (iv) whenever $\mathrm{ann}_C(V) = \mathrm{ann}_C(W)$ for simple rt.R-modules we have $\mathrm{pd}_R(V) = \mathrm{pd}_R(W)$.

Main examples are (i) Commutative Noetherian rings of finite global dimension, (ii) Right Noetherian local rings of finite rt. global dimension and integral over their centers, (iii) (Bernstein) Certain group algebras occurring in the representation theory of p-adic groups. Main theorems are

Thm.1: R is C-Macaulay. Thm.2: Localization of R at a semi-prime ideal of the center is again hom.hom. Consequently:

(i) K.dim R = cl K dim R = gl.dim R.

(ii) R is a direct sum of prime hom.hom.rings.

(iii) If $x_1,\dots,x_n$ is a C-sequence in R ($x_i \in C$) then the ring $R/\sum x_i R$ has a quasi-Frobenius quotient ring.

M.F. HERVÉ: ADDIVITY PRINCIPLES FOR RANKS

If A is a ring which does not necessarily have a unit element, we denote $\mathrm{nrk}(A) = \sup\{n \in \mathbb{N}^*, x^n = 0,\ x^{n-1} \neq 0\}$; we denote grk(A) the left Goldie rank of A. We remark that, if A is a left Goldie prime ring, nrk(A) = grk(A) = nrk(I) for every non-zero two-sided ideal I of A.

Now let A be a ring with unit element, M a unitary left A-module of finite length, the annihilators of the factors in a composition series are left primitive ideals $\{H_i\}_{i \in I}$. Under conditions (C1) $\forall i \neq j,\ H_i \not\subset H_j$

(C2) each A/H_i is left Goldie,

we give an additivity principle: $nrk(M) = \sum_i z_i grk(A/H_i)$, $z_i \in \{0,1\}$; this is obtained by using suitable filtrations of M.

This additivity principle is related to, but different from, Joseph-Small's.

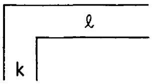
C. JENSEN: DECIDABILITY QUESTIONS FOR NOETHERIAN RINGS

Various classes of decidable and undecidable rings were described: e.g., any free associative algebra and any Weyl algebra over an arbitrary field is undecidable. For each t, $0 \leq t \leq \infty$ there exists a decidable noetherian ring of global dimension t. For commutative rings we have: Let R be a commutative, decidable, noetherian ring, then gl. dim. R = 0,1, or ∞. If R is a noetherian $\mathbb{Q}$-algebra with k dim $R \geq 2$, then R is undecidable.

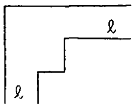
S. AMITSUR: RECENT PI-RING RESULTS

A survey of recent results on two out of the four major topics of PI-theory was given. The four directions of research are: 1) A quantitative-qualitative description of the identities I(R) of a PI-ring R. 2) The varieties of associative algebras and their identities. 3) Structure theory of affine and noetherian PI rings. 4) Extension of the theory of commutative rings to PI-rings-non commutative algebraic geometry of matrices. Highlights of the first part of the talk were the finite generation of the T-ideal (ideal of identities) of the 2×2 matrix ring. Kemer's result that the universal algebra $F(\mathfrak{M})$ of a variety $\mathfrak{M}$ contains a maximal T-nilpotent ideal which is the intersection of a finite number of prime T-ideals. The description of the variety of the prime T-ideals was given with the aid of the infinite Grassman algebra. Finally after the history of the non commutative Hilbert Nullstellensatz for commutative and PI-rings was presented, we brought A. Braun's result that the Jacobson radical of affine PI-rings is nilpotent and presented the counterexample of Beidar (after Small) of a prime affine ring over a countable field without nil ideal whose Jacobson radical is non zero.

The second hour was devoted to the work of Regev-Berle-Drensky-Kemer-Amitsur on the relation between the symmetric group representation and the set of identities of PI-ring. By identifying the multilinear homogeneous polynomials P_n with the group ring kS_n of the symmetric group, the identities $I_n(R) \subseteq P_n$ is a left ideal, and $kS_n/I_n(R)$ is a left S_n-module. Its dimension $c_n(R)$ and its (c_0) character (*) $\chi_n(R) = \sum_{a_D \neq 0} a_D \chi_D$ are studied. The new results are that $(\sum a_D) = O(n^k)$ for some integer k(Berele-Drensky-Regev), and all Young diagrams D lie in a hook



with $\frac{k\ell}{l+k} \geq$ const. If $c = \underline{\lim}\, c_n(R)^{1/n} \leq \overline{\lim}\, c_n(R)^{1/n} = G$, then for $\ell > \tilde{c}$ the identities corresponding D of (*) lie in a hook of the form



Finally, we have $c \leq C \leq 2(c^2-1)$.

W. SCHELTER: SMOOTHNESS IN AFFINE PI-RINGS

Let R be an affine PI ring satisfying the identities of $n \times n$ matrices. The smallest such n is the PI degree of R.

We say Spec R is n-smooth if for any ring S of PI degreee n and any surjective ring map $S \to R$, with nilpotent kernel, there exists a splitting as a ring map. If R is affine of PI deg n, we would say it is smooth if it is n-smooth. Our main result so far is that if $k = \bar{k}$ (the coefficient field = k), then hereditary prime rings are smooth. It is conjectured that they are the only smooth 1-dimensional affine prime PI rings. Actually we verify $H^2(R,M) = 0$ so that all surjections onto R with nilpotent kernel split. This is similar to Wedderburn's principal theorem.

C. PROCESI: GENERIC TWO BY TWO MATRICES

We study the ring of m generic matrices and related objects as a representation of the general linear group GL(m) acting linearly on the variables. For 2×2 matrices and characteristic not 2 or 3 we have complete results.

A. BRAUN: THE NILPOTENCY OF THE RADICAL IN FINITELY GENERATED PI RINGS

We shall discuss the proof of the following result, answering affirmatively a long-standing open problem.

"Let R be a finitely generated PI ring over a central noetherian subring. Then, N(R), the nil radical of R is nilpotent."

This has an important application to the structure question of finitely generated PI algebras. Some further applications are also discussed.

M.P. MALLIAVIN: MINIMAL INJECTIVE RESOLUTIONS OF ENVELOPING ALGEBRAS OF SOLVABLE LIE ALGEBRAS

Let G be a solvable Lie algebra finite dimensional over a field k of characteristic 0, $A = U(G)$. As a left module over itself, A possesses a minimal injective resolution:

$$0 \to A \to E_0 \to E_1 \to \dots \to E_n \to 0$$

where $E_0 = K$, the field of fractions of A and n is the dimension of G. For $i \geq 1$, $E_i = E_i^I \oplus E_i^{II}$ where $E_i^I = \coprod_{\substack{P \in \mathrm{Spec}(A) \\ \mathrm{ht}P = i}} E(A/P)$ ($E(-)$ denotes injective envelope) and $E_i^{II} = \coprod E(A/I)$, I being a n-irreducible left ideal of A, A/I being critical and $K.\dim(A/I) \underset{\neq}{<} K.\dim(A/Q)$ $Q = \mathrm{Ann}_A(A/I)$.

<u>Proposition 1</u>. $E_n^{II} = (0)$ and E_n^I is isomorphic to each of the following modules:

i) $\varinjlim_I \mathrm{Ext}_A^n(A/I, A)$, where I runs over the set of 2-sided ideals of A of finite codimension.

ii) $V(G) = \{f : A \to k,\ f$ k-linear map s.t. $\exists\ I \triangleleft A$, $[A/I : k] < \infty$, s.t. $f(I) = 0\}$.

If $k = \mathbb{C}$ let $\mathcal{S}$ be denote the Ore set defined by K. Brown relatively to a prime ideal P of A: $\mathcal{S} = \bigcap_{\lambda \in O(P)} \mathcal{C}(\tau_\lambda(P))$ where $O(P)$ is some subgroup of $(G/[G,G])^*$ depending on P and τ_λ is the winding automorphism of A relative to λ.

<u>Proposition 2</u>. The ring $\mathcal{S}^{-1}A$ is the ring of fractions of A with respect to the torsion theory cogenerated by $\coprod_{\lambda \in O(P)} E(A/\tau_\lambda(P))$.

J. BJÖRK: INVOLUTIVENESS OF CHARACTERISTIC VARIETIES

If R is a filtered ring and its associated graded ring $gr(R)$ is commutative then a Poisson product can be defined on $gr(R)$ as follows: Let $\xi = \bar{x}$ and $\eta = \bar{y}$ be two homogeneous elements in $gr(R)$. So here $x \in \Sigma_k$ and $y \in \Sigma_\ell$ are elements in R. Since $gr(R)$ is commutative $\Rightarrow xy-yx \in \Sigma_{k+\ell-1}$ and its image in $\Sigma_{k+\ell-1}/\Sigma_{k+\ell-2}$ depends only on ξ and η and is denoted by $\{\xi,\eta\}$. Following O.Gabber's article "The integrability of the characteristic variety" in Amer. Journ. of Math. Vol.103, 445-468 (1981) we prove the following

Theorem. If $gr(R)$ is a commutative Noetherian ring which in addition is an algebra over the field Q of rational numbers then the radical ideals $\sqrt{\mathcal{B}(L)}$ are closed under the Poisson product for every left ideal L in R.

Remark. If $L \subset R$ then $\mathcal{B}(L)$ = the ideal in $gr(R)$ generated by all principal symbols of elements in L.

Applications. Given $L \subset R$ we find $\mathrm{Kr.dim}_{gr(D)}(gr(R)/\sqrt{\mathcal{B}(L)})$ and denote this integer by $d(R/L)$. It is called the d-dimension of the cyclic left R-module R/L. Clearly $0 \leq d(R/L) \leq \mathrm{Kr.dim}(gr(R))$ but if $\{\ ,\ \}$ is "non-degenerated" then the Theorem above gives a lower bound for $d(R/L)$.

Example. Let G be a complex (or compact) Lie group. (It is given abstractly as a non-singular affine algebraic variety). The enveloping algebra $u(\mathfrak{g})$ is then a subalgebra of $\Gamma(G,\mathcal{D}_G)$ where $\mathcal{D}_G$ = the sheaf of differential operators on the complex analytic manifold G. If $\mathcal{M}$ is any coherent $\mathcal{D}_G$-module (say for simplicity $\mathcal{M} = \mathcal{D}_G/\mathcal{L}$ where $\mathcal{L}$ is a coherent sheaf of left ideals in $\mathcal{D}_G$, then $\mathcal{B}(\mathcal{L})^{-1}(0)$ appears as an involutive conic subvariety of the symplectic cotangentbundle $T^*(G)$. We put $\mathcal{B}(\mathcal{L})^{-1}(0) = SS(\mathcal{D}_{G/\mathcal{L}})$ and it is called the <u>characteristic variety</u> of the left $\mathcal{D}_G$-module $\mathcal{D}_{G/\mathcal{L}}$. In particular one finds that every component of $SS(\mathcal{D}_{G/\mathcal{L}})$ has (complex) dimension $\geq \dim_C(G) = \dim_C(\mathfrak{g})$.

<u>A special case</u>. If L is a left ideal in $U(\mathfrak{g})$ and $U(\mathfrak{g})$ is mapped into $\Gamma(G,\mathcal{D}_G)$ (i.e., any $\delta \in \mathfrak{g}$ extends to a globally defined half invariant vector field on the group G and so on), then $\mathcal{L} = \mathcal{D}_G L$ is a coherent sheaf of left ideals in $\mathcal{D}_G$.

Here we find $SS(\mathcal{D}_{G/\mathcal{L}}) \subset T^*(G)$ and if $\pi: T^*(G) \to G$ is the projection to the base manifold and e is the identity in $G \Longrightarrow \pi^{-1}(e) \cap SS(\mathcal{D}_{G/\mathcal{L}}) = SS(U(\mathfrak{g})/L)$ = the usual associated variety of the $U(\mathfrak{g})$-module $U(\mathfrak{g})/L$.

This scheme makes it possible to apply the theory of the sheaf $\mathcal{D}_X$ of differential operators with holomorphic coefficients to various questions about left ideals in $U(\mathfrak{g})$. We offer two such applications.

1. <u>Equidimensionality of</u> $SS(U(\mathfrak{g})/L)$. If $U(\mathfrak{g})/L = M$ is a left $U(\mathfrak{g})$-module of pure dimension, i.e., $\text{Kr.dim}_{U(\mathfrak{g})}(M_0) = k$ for some fixed integer k where we also assume that k equals $\dim(SS(U(\mathfrak{g})/L) \Longrightarrow SS(U(\mathfrak{g})/L)$ has pure dimension k.

2. <u>Cutting dimensions</u>. Let $M = U(\mathfrak{g})u$ be a cyclic $U(\mathfrak{g})$-module and let $\phi \in \text{Hom}_{U(\mathfrak{g})}(M,M)$. Can put $\phi(u) = Pu$ for some P in $U(\mathfrak{g})$. If we <u>assume</u> that $\mathcal{B}(P)$ does not vanish identically on any irreducible component of $SS(M)$ then $SS(M/\phi(M))$ equals $SS(M) \cap \mathcal{B}(P)^{-1}(0)$.

<u>Final Remarks</u>. As said above one can use $\mathcal{D}$-theory to prove 1) and 2) where the general results were found by <u>M.Sato</u> et al. (see in particular the article on Prehomogeneous Vector Spaces [Inventiones

Math. Vol.62 (117-179), 1979] whose Appendix contains a proof of (2) in a general setting. To be precise: In this article (2) above is only announced and proved when M has simple characteristics (i.e., $\mathcal{B}(L) = \sqrt{\mathcal{B}(L)}$.) However, using the combination of the methods from Gabber's and Sato's work I, have verified (2) in the general case. So Proposition A.4 on page 175 in Sato's article is valid without assuming that the $\mathcal{E}_X$-module $\mathcal{M}$ has simple characteristics.

J. ROOS: RELATIONS BETWEEN COMMUTATIVE AND NON-COMMUTATIVE NOETHERIAN RINGS

Let A be a filtered (not necessarily commutative) ring, such that gr(A) (= the graded associated ring) is a commutative noetherian ring. Suppose that it is known that gr(A) belongs to one of the most frequently studied classes of commutative noetherian rings: Gorenstein rings, regular rings, complete intersections, Cohen-Macauley rings, etc.

Problem: Determine the corresponding properties of the ring A.

We will solve this problem in some cases. Applications include e.g. A = $U(\mathfrak{g})$/primitive ideal ($\mathfrak{g}$ = finite-dimensional Lie algebra), where $U(\mathfrak{g})$ is equipped with the natural increasing filtration.

J. JANTZEN: A SURVEY OF RESULTS ON PRIMITIVE IDEALS OF $U(\mathfrak{g})$, $\mathfrak{g}$ SEMISIMPLE

Let $\mathfrak{g}$ be a semi-simple finite dimensional complex Lie algebra, $U(\mathfrak{g})$, its enveloping algebra, and X the set of primitive ideals of $U(\mathfrak{g})$. This talk gave a survey over the classification of X and some results known about the structure of $U(\mathfrak{g})/I$ for $I \in X$. It discussed the relations of these problems with the representation theory of the Weyl group of $\mathfrak{g}$ and the theory of Harish-Chandra modules for $\mathfrak{g} \times \mathfrak{g}$, especially those (denoted by $\mathcal{L}(M,N)$) of $\mathfrak{g}$-finite maps from one $\mathfrak{g}$-module M to another one N. Among other things the ring structures of $U(\mathfrak{g})/\mathrm{Ann}(M)$ and of $\mathcal{L}(M,M)$ for a $\mathfrak{g}$-module M of finite length were compared.

A. JOSEPH: ASSOCIATED VARIETIES OF PRIMITIVE IDEALS

Let $\underline{g}$ be a semisimple Lie algebra over an algebraically closed field of characteristic zero.

Theorem. For each $I \in \mathrm{Prim}\, U(\underline{g})$ the variety $\underline{V}(I)$ of zeros of gr I in g^* is irreducible.

The proof has four main ingredients. One, the Gabber-Kashiwara equidimensionality theorem which impies that $\underline{V}(I)$ has no lower dimensional components. Two, a characterization proposed by the author and established recently by Hotta, of the Springer correspondence between nilpotent orbits and Weyl group representations. Three, the correspondence between clans of primitive ideals and Weyl group representations defined by the Goldie ranks of primitive quotients. Four, the Casselman functor η and the Gabber separation theorem for solvable Lie algebras. Gabber's theorem (and we note here that none of the other authors' special cases suffice) allows one to translate information on the formal character of $\eta(U(\underline{g})/I)$ determined by the Goldie rank polynomial of I, to the variety $\underline{V}(I) \cap \underline{n}^+$ which is then shown via Hotta's theorem to have all its top dimensional components lying in a single G orbit closure.

R. RENTSCHLER: A SURVEY OF RESULTS ON PRIMITIVE IDEALS OF U(G), G ARBITRARY

Let $U(\underline{g})$ be the enveloping algebra of a $\mathbb{C}$-Lie-algebra $\underline{g}$, $\dim \underline{g} < \infty$.

1) A prime ideal P of $U(\underline{g})$ is primitive

$\iff$ P is locally closed in Spec $U(\underline{g})$ (Moeglin)

2) Definition and properties of the Dixmier-Duflo map from the set of linear forms on $\underline{g}$ having solvable polarizations into the space Prim $U(\underline{g})$ of primitive ideals of $U(g)$.

From now, let $\underline{g} = \mathrm{Lie}(G)$, G a connected linear algebraic group.

3) Let $\underline{u}$ be an ideal of $\underline{g}$, $I \in$ Prim $U(\underline{g})$. Then (Moeglin-Rentschler):

i) $\exists Q \in$ Prim $U(\underline{u})$ such that $I \cap U(\underline{u}) = \bigcap_{\nu \in G} \nu Q$

Any two such Q's are conjugate under G.

ii) Let $H = \{\nu \in G \mid \nu Q = Q\}$ and $\underline{h} = \mathrm{Lie}(H)$.

$\exists P \in$ Prim $U(\underline{h})$ such that $P \cap U(\underline{u}) = Q$ and $\mathrm{Ind}(P, \underline{h} \uparrow \underline{g}) = I$

Any two such P's are conjugate under H.

4) Duflo description (1981) of the primitive ideals of $U(\underline{g})$ by a canonical surjective map:

$$\coprod_{f \in \Sigma} X(f) \twoheadrightarrow \text{Prim } U(g) \quad \text{where}$$

Σ = set of linear forms on $\underline{g}$ of unipotent type

$X(f) = \{Q \in \text{Prim } U(\underline{g}(f)) \mid x - f(x) \in Q \text{ if } x \in \text{unipotent part of } \underline{g}(f)\}$

$\underline{g}(f) = \{x \in \underline{g} \mid f([x,g]) = 0\}$

5) The factored map $(\coprod_{f \in \Sigma} X(f))/G \to \text{Prim } U(g)$ is bijective (consequence of 3)).

K. BROWN: GABBER'S WORK ON EQUIDIMENSIONALITY

An account was given of the following theorem of Gabber: Let A be a ring with an associated graded ring B which is commutative Noetherian of finite global dimension ω, and such that B_m has global dimension ω for all maximal ideals m of B. Let M be a finitely generated A-module. Define the dimension $d_A(M)$ of M to be $\sup\{d_m(\operatorname{gr} M) : m \text{ a maximal ideal of } B\}$, where $d_m(\operatorname{gr} M) = \dim(B_m \otimes_B \operatorname{gr} M)$; and the characteristic variety $\operatorname{ch} M$ of M to be the subset $V(\operatorname{Ann}_B(\operatorname{gr} M))$ of $\operatorname{spec}(B)$. If $s \geq 0$ and $d_A(N) \geq s$ for all non-zero submodules N of M, then every irreducible component of $\operatorname{ch} M$ has dimension at least s.

P. PANTER: WROBEL'S WORK ON THE HAUSER DIMENSION

Continuing T. Tiger's project of finding a bound on the composition series' length of non-noetherian modules over associative Lie algebras,

I. Wrobel has recently shown that the injective resolutions of such modules form a submanifold in the 27th Grassmanian. This has enabled Wrobel to complete Hauser's work and to give an explicit formula for the chain length of such resolutions.

S. SMITH: RING THEORY AND BEILINSON-BERNSTEIN EQUIVALENCE

We consider the following: G is connected complex semisimple Lie group, with Borel subgroup B containing a maximal torus T; the associated Lie algebras are denoted $\underset{\sim}{g}$, $\underset{\sim}{b}$, $\underset{\sim}{h}$ and $U = U(\underset{\sim}{g})$ is the enveloping algebra of $\underset{\sim}{g}$; for each $\lambda \in \underset{\sim}{h}^*$ Beilinson and Bernstein have constructed a sheaf of twisted differential operators

$\mathcal{D}_\lambda$ on $X = G/B$ such that the global sections $\Gamma(X,\mathcal{D}_\lambda)$ is the ring $D_\lambda = U/\text{ann } M(\lambda)$ where $M(\lambda)$ is the Verma module of highest weight $\lambda-\rho$ (ρ is the half-sum of positive roots). They prove that if λ is dominant, regular there is an equivalence between the category D_λ-Mod of left D_λ-modules and the category $\mathcal{M}(\mathcal{D}_\lambda)$ of sheaves of left D_λ-modules which are quasi-coherent as sheaves of $\mathcal{O}$-modules ($\mathcal{O}$ is the structure sheaf of G/B). Let (U_α) be a finite open affine cover of X (say each U_α is a translate of the large Bruhat cell), put $A_\alpha = \Gamma(U_\alpha,\mathcal{D}_\alpha)$ (so each $A_\alpha \cong A_n$ a Weyl algebra, $n = \dim X$). Consider the diagonal embedding $D_\lambda \to \oplus_\alpha A_\alpha$ obtained from the restriction maps. We discuss the following theorem and its consequences:

Theorem (Hodges, Smith). The embedding $D_\lambda \to \oplus A_\alpha$ makes $\oplus A_\alpha$ a faithfully flat right D_λ-module if and only if the categories D_λ-Mod and $\mathcal{M}(\mathcal{D}_\lambda)$ are equivalent.

D. FARKAS: NOETHERIAN GROUP RINGS

The foundations of the theory of polycyclic group rings are reviewed, with one eye toward the dominant noetherian themes and the other eye toward the inevitability of such a theory for the group theorist.

I begin with applications of Artin-Rees properties and various Nullstellensatz-like results. This leads to the notion of plinth which, in turn, becomes the starting point for Roseblade's Prime Controller.

The state of the art in localization is illustrated with a solution to one case of the Burnside Problem. Finally, I discuss new techniques of abelian and nonabelian valuation theory.

S. MONTGOMERY: GROUP GRADED RINGS, SMASH PRODUCTS, AND GROUP ACTIONS.

If G is a finite group of automorphisms of a k-algebra R, then (in the terminology of Hopf algebras) R is a "module-algebra" for the group algebra $k[G]$. Dually, a k-algebra A is graded by $G \iff A$ is a "module-algebra" for $k[G]^*$. One may then form the smash product $A\#k[G]^*$; it plays a role similar to that played by the skew group ring $R*G$ in the case of group actions. Using this notation, elementary algebraic proofs are given of the "Duality theorems for actions and co-actions", known results for von Neumann algebras (the theorems say that $(R*G)\#k[G]^* \cong M_n(R)$ and $(A\#k[G]^*)*G \cong M_n(A)$, where $n = |G|$). As applications, we prove incomparability of primes between A and A_1, the identity component; this generalizes Lorenz and Passman's incomparability theorem for crossed products. We also answer a question of Bergman on graded Jacobson radicals.

V. HARCHENKO: NONCOMMUTATIVE INVARIANTS OF FINITE GROUPS AND NOETHERIAN VARIETIES

Let a finite group G act on a finite-dimensional space V over a field F. The action of G can be extended to the canonical action on the tensor algebra $F\langle V\rangle$ of the space V. If $\mathfrak{M}$ is a variety of F-algebras, then its T-ideal of identities is invariant under the actions of G, and the action of G is induced upon the free algebra $F_{\mathfrak{M}}(V)$ of the variety $\mathfrak{M}$ freely generated by a basis of V.

Definition. An $\mathfrak{M}$-invariant of the group G is an element of $F_{\mathfrak{M}}\langle V\rangle$ which is not moved by the action of G. The set of all

$\mathfrak{M}$-invariants of the group G constitutes a subalgebra $Inv^V_{\mathfrak{M}}(G)$ of the algebra $F_{\mathfrak{M}}\langle V\rangle$.

Theorem 1. Let F be a field of characteristic zero. The algebra of $\mathfrak{M}$-invariants of every finite group is finitely generated iff $\mathfrak{M}$ is locally weak Noetherian.

Theorem 2. Let G be a finite subgroup in GL(V). Then the algebra $Inv^V(G)$ of noncommutative invariants is finitely generated iff G is generated by a scalar matrix.

This theorem was independently proved by Formanek and Diks

M. LORENZ: GROUP RINGS AND DIVISION RINGS

This talk described some applications of non-commutative valuations and the so-called Hilbert-Neumann construction to the study of certain division algebras. Specifically, the division algebras D considered were those which are generated by some fin. gen. nilpotent group $G \leq D$. One striking feature of this class of division algebras is that D determines the group G to a large extent. For example, the factor G/Δ, Δ = FC-center of G, is determined by D (up to isomorphism). Moreover, D is not isomorphic to any division algebra E generated by some finite-dimensional non-commutative Lie algebra $\underline{g} \subseteq E_{[,]}$ (at least if the base field is algebraically closed). Finally, we indicated how the fact that these division algebras D embed into a suitable Hilbert-Neumann division algebra can be used to compute the Gelfand-Kirillov transcendence degree of D in some cases. Besides non-commutative valuations the proofs use work of Zalesskii on the structure of prime ideals in group algebras of fin.gen. nilpotent groups.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

R. SNIDER: SIMPLE ARTIN IMAGES OF GROUP RINGS

Let G be a solvable group and k a field. Let S be a simple Artin image of the group ring k[G]. Suppose that G is embedded in S by the homomorphism of k[G] onto S. Then G is abelian-by-locally finite. The proof uses the theory of annihilator free ideals as developed by Zalesskii. The theory of locally finite linear groups over division rings is also used (this theory is also due to Zalesskii).

B e r i c h t e r s t a t t e r: A. Rosenberg

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Gefördert durch
DFG Deutsche Forschungsgemeinschaft