MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 6 / 1983

Stochastische Geometrie, Geometrische Statistik, Stereologie

30.1. bis 5.2.1983

Die Tagung fand unter der Leitung von Herrn R.V.Ambartzumian (Erevan) und Herrn W.Weil (Karlsruhe) statt. Sie hatte 30 Teilnehmer, von denen 23 Vorträge hielten. Es war nach langer Zeit die zweite Tagung dieser Art; die erste war 1969. Die damals schon erkennbaren stochastischen Ansätze haben in der Zwischenzeit zu einer stürmischen Entwicklung des ehemals aus Integralgeometrie und Geometrischen Wahrscheinlichkeiten bestehenden Gebietes geführt. Die Vielfalt der Themen sowie der zunehmende Einsatz von zufälligen Mengen und Punktprozessen spiegeln diese Entwicklung wieder. Leider war die große Gruppe von Mathematikern, die in der DDR auf dem Gebiet der Stochastischen Geometrie tätig sind, nicht vertreten.

Die Themen der Vorträge lassen sich grob in einen statistischen und einen geometrischen Bereich unterteilen. Die große Zahl der statistischen Vorträge beinhaltete u.a. die Statistik von Punktprozessen, Statistik auf der Sphäre und Anwendungen in der Stereologie. Die mehr geometrischen Vorträge behandelten zufällige Mengen und Punktprozesse konvexer Körper, wobei besonders die Anwendungsmöglichkeiten bekannnter Resultate aus der Konvexgeometrie auffiel. In mehreren Vorträgen wurden Faktorisierungen von Maßen angesprochen bzw. ausgenutzt. Zufällige Lagerungs- und Überdeckungsprobleme waren Gegenstand je eines Vortrags.

Vortragsauszüge

R.V.AMBARTZUMIAN:

Factorization in Integral and Stochastic Geometry

Factorization appears in Stochastic Geometry in many ways, and the lecture reviews several faces of this vast subject. In particular, factorization in many cases can replace ergodic-limiting procedures. With this approach a stumbling block not reflected by the theory which existed before is revealed. It is connected with frequent presence of infinite measures in the space of shapes.

ZVI ARTSTEIN:

Random sets arising in optimization and convergence of sums of random sets

An optimization problem of the form

$$v_R(w)=\max\left\{\frac{1}{R}\Sigma f_i(x_i(w)) : \frac{1}{R} x_i(w)=q\in\mathbb{R}^n\right\}$$

arises in a model of Arrow and Radner for allocations under uncertainity. If $f_i(\cdot)$ are iid's a theorem of Arrow and Radner says that $v_R(w) \to$ constant a.s. We interpreted this SLLN as a consequence of the SLLN for the graphs of $f_i(\cdot)$ i.e. as a SLLN for random sets. The ideas for establishing SLLN, CLT, and Large Deviations estimates for random sets with the Minkowski addition are displayed, with emphasis on the distinctions between the compact-valued case and the unbounded case. The latter is the one needed in the optimization problem.

RODNEY COLEMAN:

Sampling specimens

Methods for reducing the variance of an unbiased estimator of area in the point sampling of a planar domain are appropriate in stereological sampling by line and plane sections when we describe the sections as points in a "stochastic geometry" domain and use random point sampling over this domain.

RICHARD COWAN:

The random packing of disks in the neighbourhood of a given disk

We present a local model for packing of disks near one disk as an approximation to the local features of an ensemble of packed disks. Disks are "parked" sequentially on the circumference of the given disk in a manner similar to the way that Renyi considered cars to be parked on a kerbside. This theme is generalised to allow "parking" disks to touch previously parked disks. Properties of contact numbers, gap and island statistics, local densities can be derived for a substantial class of models of this type.

LUIS-M. CRUZ-ORIVE:

Directional models for stereology

The current stereological estimators of length and surface area densities are unbiased for isotropically oriented structures cut by arbitrarily oriented uniform random probes which are very small relative to the specimen (so that edge effects can be ignored). If the structure of interest is anisotropic, however, the probes should be isotropic uniform random (IUR) hitting the specimen. This is the classical stereological approach. Unfortunately, to take IUR probes is often inefficient and inconvenient in practice.

If the structure exhibits a single axis of anisotropy, it is often easy to take probes making a specified angle with that axis. In this case, it is useful to regard the structure as a realization of a spatial stochastic process hit by a probe with a specified direction. Suitable models can be constructed using well known axial directional distributions, notably the Dimroth-Watson distribution on the hemisphere. The model, the estimation of the concentration parameter of the axial distribution, the estimation of length and surface densities, and the validation of the model, are discussed and illustrated with real data.

JÜRG HÜSLER:

Random coverage of the circle and asymptotic distributions

Place n arcs of equal length a_n uniformly at random on the circumference of a circle. We discuss the asymptotic behaviour of the numbers of gaps, the uncovered proportion of the circle, the lengths of the maximal and of the minimal gap, depending on the rate of convergence of $a_n \to 0$ as $n \to \infty$.

OLAV KALLENBERG:

On exchangeable random sets

A general problem in random set theory is to give a reasonable definition of the conditional distribution, given that a certain set hits a fixed point. In the present talk, we consider only a very special class of random sets in $[0,1]$ or $\mathbb{R}_+$ which we call exchangeable. First of all, several characterizations of such sets are provided. It is then demonstrated, under certain regularity conditions, how the Palm distribution based on the so called local time random measure agrees in a certain limiting sense with the intuitive notion of conditioning.

HANS G.KELLERER:

Minkowski functionals of Poisson processes

Let $\mathfrak{S}$ be the lattice of sets generated by the class of compact convex sets in $\mathbb{R}^n$ and $[S]$ denote the minimal number of convex components of $S \in \mathfrak{S}$. Then Crofton's formula and the inequality $|\chi(S)| \leq [S]^n$ (Eckhoff 1980) yield bounds for the (suitably normalized) Minkowski functionals $v_i(S)$ by $[S]^n \lambda(S^r)$ (λ: Lebesgue measure, S^r: r-neighborhood of S). Defining "Minkowski polynomials" V(S):

$$V(S): t \mapsto \sum_{0 \leq i \leq n} \frac{v_i(S)}{i!} t^i ,$$

the Blaschke-Santaló theorem reduces to a multiplication mod t^{n+1} ($\sim$).- Now a translation invariant Poisson process $\mathcal{N}$ on $\mathfrak{S}$ can be characterized by an intensity $\gamma > 0$ and a distribution ν on the subset $\mathfrak{S}^o$ of centered sets of . If ν is rotation invariant with $\int [S]^n \lambda(S^r) d\nu < \infty$ for one/all $r > 0$ and Q^k denotes the random set covered by $\mathcal{N}$ at least k times, the main result states:

<u>Th 1</u> $\mathbb{E}(V(S_o \cap Q^k)) \sim V(S_o) \sum_{l \geq k} e^{-\gamma\bar{V}(\nu)} \cdot \frac{1}{l!}(\gamma\bar{V}(\nu))^l$ for all $S_o \in \mathcal{S}$,

where $\bar{V}(\nu)$ is the ν-average of V(S). From this the expectations of $v_i(S_o \cap Q^k)$ follow by differentiation. Special applications concern the case ν-sup dimS<n and yield e.g. the expected number of intersection points. The results extend to a finite family of independent Poisson processes $\mathcal{N}_1,\ldots,\mathcal{N}_k$; for $Q_j := \bigcup \mathcal{N}_j$ e.g. one has the formula:

<u>Th 2</u> $\mathbb{E}(V(S_o \cap (Q_1 \cap \cdots \cap Q_k))) \sim V(S_o) \prod_{1 \leq j \leq k} (1-e^{-\gamma_j \bar{V}(\nu_j)})$,

where γ_j and ν_j correspond to $\mathcal{N}_j$.

WILFRID S.KENDALL:

<u>Stochastic differential geometry; Brownian motion and curvature</u>

Theorem: Suppose N is a complete 2-dimensional Riemannian manifold, sectional curvatures everywhere bounded above by $-H^2 < 0$. Then Brownian motion on N has an asymptotic limiting direction whose law has topological support the whole of the circle of limiting directions.

The case dimN=2 is particularly amenable because then geodesics dissect N into 2 parts.

The techniques of the proof also have application to probabilistic proofs of theorems in differential geometry.

KLAUS KRICKEBERG:

<u>Non-parametric and parametric statistical analysis of point processes</u>

Point processes arise in geometrical statistics as they do in survey sampling: to describe a sampling plan for the purpose of analising a given structure. They also arise to construct models for random geometrical structures, and are thus subject to statistical analysis. The talk concentrated to the latter aspect. The geometrical type of the underlying space allows some preliminary reduction by factorization, disintegration etc., in particular stationarity. For a fixed domain of observation and translation-invariant processes in $\mathbb{R}^d$, the natural unbiased estimators of reduced moment measures are given. The asymptotic theory starts with the 0∞-law of stochastic geometry, and treats consistency, Glivenko-Cantelli type theorems, asymptotic normality and convergence of empirical processes. For the corresponding estimates of the densities of reduced moment measures, upper and lower bounds for the speed of convergence are provided, following Jolivet. Parametric models are only mentioned briefly.

URSULA MOLTER:

About the distribution of sizes of corpuscules in a solid and the distribution in its sections by k-planes

We consider the case in n-dimensional euclidean space E_n, where a convex n-dimensional body, Q, contains randomly distributed h-dimensional corpuscules similar to K, and is intersected by linear manifolds E_k, with $h+k>n$. We found the integral equation which gives a relation between the number of corpuscules - per volume unit of Q - with similitude ratio λ, and the number of sections - per volume unit of $Q \cap E_k$ - with (h+k-n)-dimensional volume σ.

In the case of h-dimensional spherical corpuscules, we found the function which gives the probability that the volume of $K \cap E_k$ is between σ and $\sigma + d\sigma$, and solved the integral equation mentioned before.

JEAN-PAUL RASSON:

Estimation of convex domains in $\mathbb{R}^2$, $\mathbb{R}^3$,... from different information sources

We suggest a solution to the following problem raised by Prof. J.G.Kendall: "Let us suppose that a realisation of a Poisson Point Process in the plane has been mutilated in such a way that we can only observe the points inside an unknown convex domain. Try to make some inference about the shape, size, and location of the domain."

This can be regarded as the 2-dimensional version of the well-known problem of estimating the two unknown bounds of an interval on the real line, observing a fixed number of points inside this interval. We justify the classical estimations in this case by using the principles of equivariant estimation.

However, the move from 1 to 2 dimensions raises very interesting new features: here we are faced with the problem of estimating the shape of the domain, which is not a finite-dimensional parameter. It appears as a nuisance parameter for the estimation of the size and the position of the domain. We proceed to its elimination by relative maximum likelihood techniques applied to the maximal likelihood of this parameter and the size of the domain. We give the estimation of the size and location of the domain which correspond to the choices we made in the one-dimensional case. Our final solution will be a homothetic expansion of the convex hull from its centroid.

We consider also the dual problem (in the sense of Ziezold) where we observe n isotropic random lines cutting a given test region T but without hitting an internal unknown convex domain $D \subseteq T$. Then the sufficient statistics, H_n^d, is the smallest convex polygon containing D and it can be parametrized by its length l_n, its centroid g_n and its shape s_n. Using the same techniques of equivariant estimation and maximum relative likelihood elimination of a parameter, we find as final solution $g_n + cH_n^d$, a contraction of H_n^d from its centroid. c is choosen such that $E(c \cdot l_n) = 1$.

BRIAN D.RIPLEY:

Edge effects in spatial stochastic processes

Edge effects are important even asymptotically in two or more dimensions. Lattice processes provide a convenient class of examples to assess the scale of the problem, and have a recent simple edge correction proved by Guyon (1982). Point processes and random sets provide greater difficulties; the geometry becomes paramount. The relative merits of several approaches will be assessed.

Ripley (1982) "Edge effects in spatial stochastic processes" in "Statistics in Theory and Practice: Essays in honour of Bertel Matréon" ed. by B.Rannely, pp.247-62

Ripley (1983) "Analyses of nest spacings" in "Statistics in Ornithology" ed. by B.J.M.Morgan and P.M.North, Springer

HAROLD RUBEN:

Decomposition of probability measures for sets of isotropic random points

Under three different sets of circumstances for independent isotropic random points in $\mathbb{R}^n$ the probability measure of (r+1)-configurations (ordered sets of r+1 points) in $\mathbb{R}^n$ $(1 \leq r \leq n-1)$ can be decomposed into factors involving a probability measure on the space of r-flats in $\mathbb{R}^n$ (extended Grassmann manifold) invariant under the group of Euclidean motions in $\mathbb{R}^n$ and a probability measure of the corresponding "local" (r+1)-configurations in $\mathbb{R}^r$ after suitable homothetic mappings. This allows, for instance, the determination of the distribution of such quantities as the distance of the configurations from the origin and the r-dimensional volume of the corresponding simplicial convex hulls. Various limit Gaussian results can also be obtained.

ROLF SCHNEIDER:

Results on convex bodies applied to geometric probabiltites

Sometimes results from the theory of convex bodies can be applied to obtain information on certain random geometric objects or spatial processes. One example is Matheron's method of associating a certain zonoid with every Poisson process in the space of hyperplanes. We describe some related results and applications of classical inequalities. New results on extremal properties of certain random hyperplanes or Poisson fields of convex bodies are obtained via the application of Minkowski's theorem on the existence of a convex body with given area function.

BERNARD W.SILVERMAN:

Poisson limit theorems for U-statistics and their application to problems in spatial statistics

Many statistics of interest in spatial statistics are of the form

$$T_n = g_n(\xi_{i_1},\dots,\xi_{i_k})$$

where $\xi_1,\dots,\xi_n$ are observations in some space, g_n is a symmetric zero-one function of k arguments, and the sum is over all subsets $\{i_1,\dots,i_k\}$ of $\{1,\dots,n\}$. Under suitable conditions, the distribution of T_n is asymptotically Poisson. As corollaries, it can be shown that the j^{th} smallest interpoint squared distance in an i.i.d. sample in the plane has, when suitably rescaled, a χ^2_{2j} distribution and that the process of smallest distances converges weakly to a Poisson process. Another example of a statistic of this type arises when seeking collinearities in a plane data set.

MARIUS STOKA:

Le problème de l'aiguille de Buffon dans l'espace euclidien E_n et quelques considérations statistiques

Soit E_n l'espace euclidien à n dimensions de coordonnées $x_1,\dots,x_n$. La mesure élémentaire cinématique dans E_n, invariante par rapport au groupe de mouvements euclidiens, est $dK=dP\wedge dO_{n-1}\wedge\dots\wedge dO_1$, où $dP=dx_1\wedge\dots\wedge dx_n$ et dO_h est l'élément d'aire sur la sphère h-dimensionnelle unité dans l'espace E_n. Notons $\mathcal{R}_n$ le réseau déterminé par des parallélépipèdes de côtés parallèles aux axes de coordonnées et de longueurs respectives $a_1,\dots,a_n$. En utilisant la mesure cinématique, nous démontrons le théorème: La probabilité pour qu'un ségment aléatoire ω de longueur $\ell<\inf(a_1,\dots,a_n)$, uniformément distribué par rapport à la mesure cinématique, coupe le réseau $\mathcal{R}_n$ est

$$(1)\quad P = 1-\frac{2^{n-1}\Gamma(\frac{n}{2})}{\pi^{\frac{n}{2}}\cdot a_1\cdots a_n}\cdot\int_0^{\frac{\pi}{2}}\cdots\int_0^{\frac{\pi}{2}}(a_1-\ell\cos\varphi_1)(a_2-\ell\sin\varphi_1\cos\varphi_2)\cdots$$

$$(a_n-\ell\sin\varphi_1\cdots\sin\varphi_{n-1})\sin^{n-2}\varphi_1\cdots\sin\varphi_{n-2}d\varphi_1\cdots d\varphi_{n-1}.$$

A partir de ce résultat nous démontrons que les événements d'un certain système d'événements associé à la configuration $(\omega,\mathcal{R}_n)$ sont dépendents et après nous nous occupons de l'estimation de la probabilité (1).

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FRANZ STREIT:

Statistical Analysis of Point Processes (with Applications to Geometric Statististics)

Optimal and locally optimal tests are constructed and investigated which enable us to judge, whether the intensity function of a Poisson point process changes its 'standard behaviour' at an unknown time-point or on the border of a domain in the sample space.

RICHARD A.VITALE:

Determining conditions for random sets

The talk will discuss random elements valued in the compact convex subsets of $\mathbb{R}^n$ and how certain collections of functionals uniquely identify probability measures.

GEOFFREY S.WATSON:

Random Points on Hyperspheres

Unit vectors x in $\mathbb{R}^2$, or points on the hypersphere Ω_q, appear in many sciences. If law of x is uniform on Ω_q, then $x = \frac{z}{\|z\|}$ where $\mathcal{L}z = G_q(0,I_q)$. We begin by proving some correlation distributions under the weakest assumptions and discussing the limit as $q \to \infty$ with extensions from Ω_q to the Stiefel manifold $V_{m,q}$. The statistical theory for distributions with densities of the forms $f(\mu'x)$ and $f(\|x_{|V}\|)$ where $\|\mu\|=1$ and V is a p-dimensional subspace is then sketched as the sample size $n \to \infty$. Some models of processes leading to distributions on Ω_q provide further classes of distributions. The talk concludes with several problems of geological interest - the estimation of surfaces from measurements of their normals and smoothing and interpolation of vector (including unit vector) fields.

WOLFGANG WEIL:

Stereological formulas for stationary random sets

Since stationary random sets X in $\mathbb{R}^d$ are a.s. unbounded, the classical quermassintegrals have to be replaced by quermass densities $D_j(X)$, j=0,...,d. For those X for which $X \cap K$ is a.s. in the convex ring whenever K is convex and compact, different possibilities to define $D_j(X)$ are reported. One of them uses results of McMullen and Hadwiger on translation invariant valuations. Some density formulas for stationary X are presented which express the bias for natural estimators for the surface area density and the density of the Euler-Poincaré characteristic. For isotropic X, analogs to the principal kinematic formula and the Crofton formula hold. Under further assumptions, these are due to Davy (1978).

JOHN ANDRÉ WIEACKER:

Visibility and intersection in a Poisson field of convex bodies

Let $\mathfrak{K}$ be the space of all convex bodies of $\mathbb{R}^d$ topologized by the Hausdorff metric. We consider a Poisson process on $\mathfrak{K}$ with intensity ϑ satisfying $\vartheta(\{K\in\mathfrak{K} \mid K\cap A\neq\emptyset\})<\infty$ for all compact subsets A of $\mathbb{R}^d$. Under the assumption that for every hyperplane H the mean (d-1)-volume of the orthogonal projection on H of the bodies of the induced field is positive, we compute some mean values concerning intersection and visibility in the induced field, which can be expressed by geometric characteristics of the mean projection body or of its polar reciprocal body. Using geometric inequalities for convex bodies, we deduce relations between these mean values and some extremal properties of motion invariant Poisson processes.

MARIO WSCHEBOR:

The level sets of a random surface

Let $\{X(t) \mid t\in\mathbb{R}^d\}$ be a random surface, u a fixed level and C_u, A_u, B_u the sets $\{t \mid X(t)=u\}$, $\{t \mid X(t)<u\}$, $\{t \mid X(t)>u\}$. Our purpose is the study of the random variables $Q_T(A_u)$ and $Q_T(B_u)$, which are the relative perimeters of A_u and B_u respectively, with respect to the open set $T\subset\mathbb{R}^d$, and these will be an appropriate geometric measure of C_u if and only if the given random surface does not have local extrema on the level $\underline{u}$, with probability 1. Two kinds of results will be presented: (a) sufficient conditions - that in a sense, are necessary - to assure the non-existence of local extrema at the level $\underline{u}$ and (b) formulae (of the Rice-Ito type) for the moments of $Q_T(A_u)$.

Tagungsteilnehmer

R.V.Ambartzumian
Erevan
Barekamutian 22b
Institute of Mathematics
USSR

Richard Cowan
CSIRO Division of Mathematics
and Statistics
Box 218, Lindfield
NSW, Australia, 2070

Zvi Artstein
Department of Mathematics
The Weizmann Institute
Rehovot 76100
Israel

Luis Cruz-Orive
Anatomical Institute
Buelstr. 26
Postfach 139
CH-3000 Bern 9
Schweiz

H.-H. Bock
Institut für Statistik
RWTH Aachen
Wüllnerstr. 3
5100 Aachen
W-Germany

Goetz Heller
Mathematisches Institut II
Universität Karlsruhe
Englerstr. 2
7500 Karlsruhe
W-Germany

Rodney Coleman
Department of Mathematics
Imperial College
London SW 7 2 BZ
England

Jürg Hüsler
Institut für mathem. Statistik
Sidlerstr. 5
CH-3000 Bern
Schweiz

Olav Kallenberg
Department of Mathematics
CTH
S-412 96 Göteborg
Schweden

Hans Kellerer
Mathematisches Institut
Universität München
Theresienstr. 39
8000 München 2
W-Germany

Wilfrid Kendall
Department of mathem. Statistics
The University of Hull
Cottingham Road
Hull HU 6 7 RX
England

Klaus Krickeberg
U.E.R. de Mathêmatiques
Logique formelle et Informatique
Universitê de Paris V
12 rue Cujas
f-75005 Paris

Ursula Molter
Departamento de Matematica
Universidad de Buenos Aires
Facultad de Ciencias Ex. y Nat.
1428 Buenos Aires
Argentina

Frank Piefke
Institut für angew. Mathematik
Pockelstr. 14
3300 Braunschweig
W-Germany

Vincenzo Pipitone
Istituto di Matematica
Via Archirafi n° 34
I-90123 Palermo
Italy

Jean Paul Rasson
FNDP, Dêpartement de Mathêmatiques
8, Rempart de la Vierge
B-5000 Namur
Belgium

Brian Ripley
Department of Mathematics
Imperial College
London SW 7 2 BZ
England

Harold Ruben
Department of Mathematics
McGill University
Montreal
Quebec
Canada H3X 2Z6

Rolf Schneider
Mathematisches Institut
Albertstr. 23 b
7800 Freiburg
W-Germany

W.Schaal
Mathematisches Institut
Universität Marburg
Lahnberge
3550 Marburg

Bernard Silverman
School of Mathematics
University of Bath
Bath BA 2 7 AY
England

Geoffrey S.Watson
Department of Statistics
Fine Hall
Princeton University
Princeton N.J. 08544
USA

Marius Stoka
Istituto di Geometria
Via Principa Amedeo, 8
I-10123 Torino
Italy

Wolfgang Weil
Mathematisches Institut II
Universität Karlsruhe
Englerstr. 2
7500 Karlsruhe
W-Germany

Franz Streit
Section de Mathématiques
Université de Genève
2-4 Rue de Lièvre
CH-1211 Genève 24
Schweiz

John André Wieacker
Mathematisches Institut
Hebelstr. 29
7800 Freiburg
W-Germany

Corrado Tanasi
Istituto di Matematica
Via Archirafi n° 34
I-90123 Palermo
Italy

Mario Wschebor
147, rue de Tolbiac
F-75013 Paris
France

Richard A.Vitale
Mathematics
Claremont Graduate School
Claremont
California 91711
USA

Herbert Ziezold
FB 17 der GH Kassel
Heinrich-Plett-Str. 40
3500 Kassel
W-Germany

Berichterstatter: G. Heller