MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 9/1983

Matrizentheorie und numerische lineare Algebra

20.2. - 26.2.1983

Die Tagung wurde geleitet von L. Elsner (Bielefeld), W. Niethammer (Karlsruhe) und R.S. Varga (Cleveland). Es nahmen 38 Personen teil, darunter Wissenschaftler aus China, Frankreich, Israel, Kanada, Niederlande, Schweden, Tschechoslowakei und USA.

Die Tagung brachte Forscher aus dem Gebiet der numerischen linearen Algebra und der Matrizentheorie zusammen. Sie bot die Gelegenheit, über neue Ergebnisse zu berichten und damit wechselseitig Anregungen zu geben und zu empfangen. Auf der einen Seite wurde über anwendungsorientierte Fragen aus der Matrizentheorie (Nichtnegative Matrizen und M-matrizen, indefinite Probleme, Stabilität, Normen) berichtet. Auf der anderen Seite wurde über Ergebnisse auf dem Gebiet der iterativen Behandlung linearer Gleichungssysteme und Eigenwertaufgaben vorgetragen. Dazu kamen einige Vorträge über

Intervallmathematik in der linearen Algebra.

In den Vorträgen, den anschließenden Diskussionen und persönlichen Gesprächen konnten wertvolle wissenschaftliche Kontakte geknüpft und vertieft werden. Dazu trug vor allem auch die bekannt entspannte und persönliche Atmosphäre des Instituts bei; hierbei sei an dieser Stelle der Institutsleitung im Namen aller Teilnehmer herzlich gedankt.

Den Teilnehmern wird die Gelegenheit geboten, ihren Beitrag für die Veröffentlichung in einem Sonderband der Zeitschrift "Linear Algebra and its Applications" einzureichen.

Vortragsauszüge:

O. AXELSSON:

Numerical linear algebra aspects in the solution of finite element problems

There exist extensive finite differences codes for the numerical solution of diffusion problems $-\underset{\sim}{\nabla} \cdot (k \underset{\sim}{\nabla} n) = f$ with appropriate boundary conditions. We address the problem of devising methods for use of these codes in a finite element context. We show that this can be done by use of spectral equivalence between linear finite element matrices and higher order finite element matrices.

We are then left with solving a matrix problem for the linear finite element matrix. Under reasonable assumptions about the mesh, these latter matrices are diagonally dominant M-matrices, i.e. of difference type.

The first part of the talk discusses how incomplete factorization may be applied in an efficient way on such M-matrices, in order to reduce the spectral condition number from $O(h^{-2})$ to $O(h^{-1})$, where h is the stepsize parameter, without any loss of sparsity. In the second part we prove the above mentioned spectral equivalence results.

Reference: O. Axelsson, A.V. Barker. Finite element solution of boundary value problems. Theory and Computations. Academic Press, to appear.

A. BERMAN, D. HERSHKOWITZ:

Characterization of acyclic D-stable matrices

For a graph $G = (V(G), E(G))$:

$$\Omega(G) = \{\omega \subseteq V(G);\ \forall i \in V(G),\ |\{i\} \cup N(i) \setminus \omega| \neq 1\}$$

where $N(i)$ is the set of neighbours of the vertex i.

For an $n \times n$ matrix A:

$V(G(A)) = \{1,\dots,n\}$

$E(G(A)) = \{(i,j);\ a_{ij} \neq 0 \text{ or } a_{ji} \neq 0\}$

$H(A) = \{(i,j) \in E(G(A));\ a_{ij}a_{ji} \geq 0\}$

$S(A) = E(G(A)) \setminus H(A)$

$G_S(A) = (V(G(A)), S(A))$

$\Omega(A) = \{\omega \in \Omega(G_S(A));\ i \in \omega,\ (i,j) \in H(A) \Rightarrow j \in \omega\}$

$P(A) = \{i;\ \det A[(i)_H] > 0\}$

where $(i)_H$ is the set of vertices which includes i and those vertices j for which there is a path of edges in $H(A)$ between i und j.

Theorem. An $n \times n$ irreducible matrix A such that $G(A)$ contain no cycles in D-stable if and only if $A \in P_0^+$ and the smallest set in $\Omega(A)$ which contains $P(A)$ is $\{1,\dots,n\}$.

E. BOHL:

On a boundary layer system

The system is defined as a finite analogue of a linearization of

$$-\varepsilon x'' + k(t)x' = f(x),\ x(0) = x(1) = 0$$

$$\varepsilon > 0,\ 0 < \bar{k} \leq k(t) \leq 1,\ f'(x) \leq 0,\ 0 \leq f(x).$$

It is shown that this system can be cut into two parts each of which may be represented by an extremly small system. A process of adaptation yields a representation of the solution of the full system which is necessarilly large if ε is very small.

D. BRAESS:

Multigrid Methods and the Solution of Large Linear Systems

Typical for the classical iterations when applied to linear systems of equations arising from finite element equations is their smoothing property. After a few iterations terms with large frequences are substantially reduced. Therefore one gets a powerful iteration when combining them with coarse grid corrections. The smoothing effect can be treated by algebraic methods specifically in the framework of discrete norms while the approximation property for the correction is more easily treated in the framework of Scales of Sobolev spaces. The transition between both scales can be done via the duality technique of Aubin-Nitsche.

A. BUNSE-GERSTNER:

An algorithm for the symmetric generalized matrix eigenvalue problem

A new method is presented for the solution of the matrix eigenvalue problem $Ax = \lambda Bx$, where A and B are real symmetric square matrices and B is positive semidefinite. In an initial step B is reduced to diagonal form $D = X^T BX$. Then starting with $A_0 = X^T AX$ and $D_0 = D$ two sequences $A_{i+1} = M_i^T A_i M_i$ and $D_{i+1} = M_i^T D_i M_i$ are computed such that all D_i are diagonal and the matrices A_i tend to diagonal form.

F. CHATELIN-LABORDE:

Aggregation/disaggregation methods

To deal with complex systems, aggregation/disaggregation are presented which model the two-way flow of information in a multilevel hierarchical management system. A review of the use of such methods in economics and Markov decision processes is presented. Then a mathematical analysis is given for the application of such methods to the solution of $Ax = bx$ oder $Ax = \lambda x$ where A is a large matrix. A comparison with multigrid methods is made.

W. DAHMEN:

Factorization of totally positive Block Töplitz matrices

Let $A = (A_{i-j})_{i,j\in\mathbb{Z}}$ be a totally positive biinfinite block Töplitz matrix generated by a sequence $\{A_j\}_{j\in\mathbb{Z}}$ of $N\times N$ matrices and let $A(z) = \sum_{j\in\mathbb{Z}} A_j z^{-j}$ denote its symbol. The existence of an L-U factorization of A (in all nontrivial cases) into lower, upper triangular totally positive block Töplitz matrices allows to confine the discussion to the lower triangular case. The following product representation of $A(z)$ is established:

$$A(z) = \overset{\infty}{\pi}(I+a_j(z))C(z)(\overset{\infty}{\pi}(I-b_j(z)))^{-1},\ \Sigma(a_j(1)+b_j(1)) < \infty.$$

Here $a_j(z), b_j(z)$ denote the symbol of a simple nonnegative block Töplitz matrix whose non-zero entries occur only in the first subdiagonal. Moreover, it is shown that $G(z), G(z)^{-1}$ are entire while $\det G(z) \equiv 1$. As a consequence the infinite system $Ax = b$ has for any $b \in \ell_\infty(\mathbb{Z})$ a unique solution $x \in \ell_\infty(\mathbb{Z})$ iff $\det A((-1)^N) \neq 0$.

M. EIERMANN:

On the Construction of Asymptotically Optimal Semiiterative Methods

Using the concept of "uniformly distributed" nodes from Approximation Theory, we investigate semiiterative methods for solving linear systems of equations. For a given compact set U we construct a one-step non-stationary method which is asymptotically optimal for all operators T having their eigenvalues in U.

L. ELSNER:

Optimal block-diagonal scaling

Let $\mathbb{C}^n = X_1 + X_2 + \ldots + X_k, X_i$ linear subspaces and $\mathcal{T} = \{T = (T_1, \ldots, T_k);\ Im(T_i) = X_i,\ i = 1, \ldots, k\}$. We consider the question of determining $Y \in \mathcal{T}$ such that the condition number with respect to the Frobenius matrix norm $K_F(Y) = \|Y\|_F \|Y^{-1}\|_F$ is minimal among all matrices in $\mathcal{T}$. This problem arises in connection with the numerical treatment of linear equations and of eigenvalue problems.

$\mathrm{Min}\{K_F(T),\ T \in \mathcal{T}\}$ can be determined in terms of the canonical angles between X_i and $Y_i = \bigcap_{j \neq i} X_j^{\perp}$. The set of all matrices $Y \in \mathcal{T}$, for which the minimum is attained, can be described. The solution for $k = n$ was given previously by R.A. Smith (1967).

M. FIEDLER:

Hankel and Loewner matrices

Mutual relations between the (finite dimensional) Hankel, Loewner (i.e. matrices of the form $\left(\frac{c_i - d_j}{y_i - z_j}\right)$ where y_i, z_j are fixed mutually distinct complex numbers), Bêzout and Töplitz matrices as well as further relations to rational interpolation, reciprocal differences and algebraic-geometrical situations (apolarity of a dual quadric and a rational normal curve in a complex projective (n-1)-space) are investigated. In particular, the case of nonsingular (Hankel etc.) matrices corresponding in each case to a pencil of relatively prime polynomials is discussed and formulae for the inversion (unexceptional) of a Hankel matrix using the solutions of two linear systems with this Hankel matrix and special right-hand sides and for the inverse of a Loewner matrix in the form of the product of a "dual" transpose Loewner matrix and two diagonal matrices are presented.

R. FREUND:

Über zwei cg-Verfahren zur Lösung von Gleichungssystemen mit Matrizen der Form $A = I - N$, $N = -N^T$

Seien $x_o, b \in \mathbb{R}^n$ und A eine reelle $n \times n$-Matrix der Gestalt $A = I - N$ mit schiefsymmetrischen $N = -N^T$. Für solche speziellen A lassen sich mittels einer einfachen Lanczos-Rekursion ausgehend von $p_o := r_o := b - Ax_o$ Vektoren p_j, $j \leq 0$, erzeugen, die im Sinne von

$$p_j^T A A^T p_l = 0, \; j \neq l,$$

orthogonal sind, und welche Basisvektoren für die Krylov-Unterräume

$$S_k := [r_o, Ar_o, A^2 r_o, \ldots, A^{k-1} r_o], \; k \geq 1,$$

darstellen. Verwendung der p_j bzw. der $A^T p_j$ als Suchrichtungen führt zu zwei cg-ähnlichen Algorithmen, die durch

$$x_k = \arg \min_{x \in x_o + S_k} \| b - Ax \|_2 \quad \text{bzw.} \quad x_k = \arg \min_{x \in x_o + A^T S_k} \| x - \bar{x} \|_2$$

definierte Näherungen x_k, $k \geq 1$, für die Lösung $\bar{x}$ des Gleichungssystems $Ax = b$ liefern. Im Zusammenhang mit Fehlerschranken stößt man bei einem der beiden Verfahren auf ein interessantes Approximationsproblem.

S. FRIEDLAND:

Stable Norms

A vector norm $|\cdot|$ on nxn square matrices M_n is called stable if $|A^p| \leq K|A|^p$ for all $A \in M_n$. Stable norms are important in stability theory of numerical methods for partial differential equations. Conjecture (C. Johnson) Spectral dominant norm on M_n is stable. We shall verify this conjecture in some special cases. In particular, we prove the conjecture for invariant norm $|UAU^{-1}| = |A|$, where U runs on the set of unitary matrices. Our proof yields the optimality of the Lax-Wendroff condition. Other results and problems will be also mentioned.

J. GARLOFF:

Intervals of P-matrices

Intervals of matrices with respect to the usual entrywise partial ordering are considered. Sufficient and necessary conditions for an interval of matrices to contain only P-matrices, diagonally stable, positive definite, and totally nonnegative matrices are given.

W. HACKBUSCH:

Multi-grid convergence for a special singular perturbation problem

The known general results about convergence of multi-grid iterations do not apply to singular perturbation problems. In this situation one has to study model problems. A model problem that can be analysed is, e.g. $-\varepsilon\Delta u + \vec{c}\,\vec{\nabla} u = f$ in a rectangle with periodic boundary conditions. Unfortunately, in that case quite other eigenfunctions appear as in the case of Dirichlet boundary conditions. This fact motivates the analysis of a singular perturbation problem with Dirichlet boundary conditions. In the lecture the one-dimensional problem $-\varepsilon u'' + u' = f$ in $(-1,+1)$ subject to $u(\pm 1) = \emptyset$ serves as model problem. The analysis confirms the practical results in the sense that the change from periodic to Dirichlet boundary conditions does not deteriorate the convergence of the multi-grid iteration severely.

K.-P. HADELER:

On copositive matrices

A real symmetric matrix is called copositive, if the corresponding quadratic form is nonnegative on the cone of nonnegative vectors (Motzkin 1952). A recursive criterion for copositivity in terms of eigenvalues or determinents is shown, from which one can derive the criteric of Cottle, Habetler, Lemke, and Garsia. For order three all copositive matrices are explicitly given. This result can be used to exclude the existence of periodic orbits in algebraic differential equations in the plane.

C.R. JOHNSON:

Positive Definite Completions of Partial Hermitian Matrices

We consider partial Hermitian matrices, i.e. Hermitian matrices, some of whose entries are specified and some of whose entries are unspecified. We assume the diagonal is specified and positive. A positive definite completion of a partial Hermitian matrix is a specification of the unspecified entries which results in a positive definite (Hermitian) matrix. If all specified principal minors of a partial Hermitian matrix are positive, does it have a positive definite completion?

a) In general the answer is "no" , but if the undirected graph of the specified entries is chordal (every circuit of the length ≥ 4 has a "chord"), then the answer is "yes" and this condition on the graph is best possible.

b) If there are completions, there will be a unique one which maximizes the determinant, and

c) the determinant maximizing one is the unique completion whose inverse has zeros in the unspecified positions of the original partial Hermitian matrix.

P. LANCASTER:

Matrices and indefinite scalar products

This lecture concerns joint work with I. Gohberg and L. Rodman of Tel Aviv University. Non-degenerate indefinite scalar products ob $\mathbb{C}^n$ can be identified with nonsingular hermitian matrices $H \in \mathbb{C}^{n\times n}$. The ideas of $n \times n$ matrices which are selfadjoint, unitary, etc., in such a scalar product are useful and natural. A systematic development of such a theory is indicated with some emphasis on the canonical form for a pair (A,H) where A is H-selfadjoint under transformations $(A,H) \to (T^{-1}AT,\ T^*HT)$. As an example the matrix polynomial $L(\lambda) = \lambda^2 I + L_1\lambda + L_0$ is introduced $(L_1^* = L_1,\ L_0^* = L_0)$. The linearization $\begin{bmatrix} 0 & I \\ -L_0 & -L_1 \end{bmatrix}$ is $\begin{bmatrix} L_1 & I \\ I & 0 \end{bmatrix}$-selfadjoint and leads to the construction of canonical matrices $J \in \mathbb{C}^{2n\times 2n}$ in Jordan form and $X \in \mathbb{C}^{n\times 2n}$ containing all information on the eigenvalues, and right eigenvectors, of $L(\lambda)$ respectively. Unsolved problems concerning

(i) the inverse problem (when is a pair (X,J) a Jordan pair for some hermitian matrix polynomial?), and

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(ii) the extension problem, are presented. The latter has considerable physical significance for the construction of oscillatory systems with partial information given on eigenvalues and modes of vibration.

R. LOEWY:

On matrices having equal corresponding principal minors

Let A,B be n×n matrices with entries in a field F. We say A and B satisfy property D if B or B^t is diagonally similar to A. It is clear that if A and B satisfy property D, then they have equal corresponding principal minors, of all orders. The question is to what extent the converse is true. There are examples which show the converse is not always true. We modify slightly the problem and give conditions on a matrix A which guarantee that if B is any matrix which has the same principal minors as A then A and B will satisfy property D. These conditions on A are formulated in terms of ranks of certain submatrices of A and the concept of irreducibility. The proofs are algebraic and combinatorial and involve perturbing some of the elements of A by indeterminates. As a by product of our approach we obtain a generalization of the Frobenius-König theorem.

I. MAREK:

<u>Some inequalities for the spectral radius of certain cone preserving linear maps</u>

Let Y be a Banach space generated by a normal cone $K, Y = K - K$. Let $L(Y)$ denote the space of bounded linear maps of Y into Y.

1. For certain class of cone preserving maps $T \in L(Y), TK \subset K$, the following inclusion result is shown:
   Two $n \times n$ matrices L and U with nonnegative real elements are constructed such that the corresponding spectral radii are related as follows
   $$r(L) \leqq r(T) \leqq r(U).$$

2. If S and T both in $L(Y), S \leqq T$, are K-irreducible and $V \in L(Y)$ commutes either with S or T, then under some conditions upon S and T the difference of the spectral radii $r(T)-r(S)$ is bounded both below and above by values of V on suitable elements of the cone K.

3. Let Y be a Hilbert space. If $T \in L(Y)$ is K-irreducible and its dual T^* leaves K invariant and u, v both in K are Perron eigenvectors of T and T^* respectively, then
   $$(Tv, U) \geqq (v, Tu).$$

An abstract basis of this inequality is derived and some applications are shown, e.g. Fiedler-Levinger theorem on convex combinations of a nonnegative matrix and its transposed is generalized.

G. MAYER:

Über die Konvergenz von Intervall-Matrizenpotenzen

Nach einer kurzen Einführung in die Intervallrechnung wird ein notwendiges und hinreichendes Kriterium zur Konvergenz der Folge von Intervall-Matrizenpotenzen $\{A^k\}$ angegeben. Ferner werden weitere hinreichende Konvergenzkriterien aufgestellt.

V. MEHRMANN:

On classes of matrices containing M-matrices, totally nonnegative and hermitian positive semidefinite matrices

We consider several classes of matrices that are generalizations of the M-matrices, totally nonnegative and hermitian positive semidefinite matrices. We show that most of these classes can be characterized by the same determinantal inequality, which we call a generalized Fan-inequality. We introduce a new class of matrices, the V-matrices and show that with respect to a certain partial order, the V-matrices are the minimal subclass of the τ-matrices containing M-Matrices and hermitian positive semidefinite matrices, and the maximal diagonally invariant subclass of the τ-matrices. We furthermore give an interlacing property for the minimal real eigenvalues of principal submatrices of a V-matrices.

N.NEUMAIER:

Interval Arithmetic, Sublinear Maps and Matrix Inversion

In order to investigate the amount of overestimation of interval algorithms for linear equations two new tools are introduced: the concept of a normal sublinear map, generalizing the left multiplication by an interval matrix, and the systematic use of Ostrowski's comparison operator which maps $A=(a_{ik})$ to $<A> = (a'_{ik})$ where $a'_{ii} = \inf|a_{ii}|, a'_{ik} = -\sup|a_{ik}| (i \neq k)$. The fixpoint of Gauss-Seidel iteration, and the "solution" from Gauss elimination define normal sublinear maps A^F and A^G which enclose the (sublinear) interval hull map A^H. From $|A^F| = <A>^{-1}$, $|A^G| \leq <A>^{-1}$ for H-matrices, the amount of overestimation can be analyzed. If A is not an H-matrix, it is considered when preconditioning by an approximate inverse leads to acceptable bounds. In particular, this is the case when the midpoint is inverse positive. The proof of the last statement involves the Prager-Oettli bounds for inaccurate linear systems and is related to "relative spectral radii" with respect to the midpoint.

E. DEUTSCH, M. NEUMANN:

Derivatives of the Perron root and dominant eigenvalue convexity

Expressions are given for the second order partial derivatives of the Perron root at an essentially nonnegative and irreducible matrix. These expressions involve positive diagonal scalings of the group inverse of an associated M-matrix. The signs of the second partial derivatives with respect to the (i,j)-th entry are thus determined by the corresponding signs of the entries of the group inverse of the associated M-matrix. The Hessian of the Perron root as a function of the diagonal entries is also obtained and discussed.

W. NIETHAMMER:

On the construction of semiiterative methods by interpolation

Given a nonsingular linear system $Ax = b$, a splitting $A = M-N$ leads to the one-step iteration (1) $x_m = Tx_{m-1} + c$ with $T := M^{-1}N$ and $c := M^{-1}b$. We investigate semiiterative methods with respect to (1) under the assumption that the eigenvalues of T are contained in some compact set $U \subset \mathbb{C}$, $1 \notin U$, using results about "maximal convergence" of polynomials from Approximation Theory we describe semiiterative methods which are asymptotically optimal with respect to U.

G. OPFER:

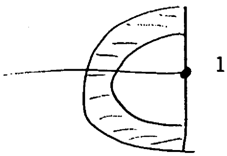
Richardson-iteration for nonsymmetric matrices

The Richardson-iteration under investigation is an iteration scheme $x_{j+1} = x_j - \alpha_j(Ax_j - a)$, $j = 1,2,\dots,n$ ($n \in \mathbb{N}$ given) for solving linear equations $Ax = a$. The main problem is to determine the α_j's in an optimal fashion and to investigate the convergence behavior of the x_j's. It turns out that for an arbitrary nonsingular (real or complex) matrix the α_j's can be deduced from the polynomial $p(z) = 1 - \sum_{j=1}^{n} \beta_j z^j$ which is minimal in the uniform norm on a compact set S which contains the eigenvalues of A. For some cases the mentioned minimal polynomial will be given explicitly. An inverse problem is introduced which is related to a stability problem. Numerical results are exhibited. Collaboration with Glenn Schober, Indiana University, is acknowledged.

J. de PILLIS:

Many iterative methods in solving $Ax = (1-B)x = b$ require $\sigma(B)$, the spectrum of $n \times n$ matrices B lies to the left of the line $L = \{z : Re(z) = 1\} \subset \mathbb{C}_1$, e.g., Chebyshev semi-iterative, Gauss-Seidel (I-B is pos. def.), SOR, Jacobi w. diag dominant A).

Reminiscent of the 2nd order Richardson type (Chebyshev semi-iterative) method, we consider the stationary
$y_{n+2} = (a_1 B+a_2)y_{n+1}+(a_3 B+a_4)y_n+(a_1+a_3)b$ for any y_o, y_1.
We are able to follow a 4-step strategy to produce from the geometry of $\sigma(B)$, the optimal quartet a_1, a_2, a_3, a_4.
This scheme may require complex arithmetic, as it does for hermitian (not pd) $A = A^*$. In fact,
$y_{n+2} = -\frac{iA}{\rho(A)}(y_{n+1} - \frac{1}{2}y_n) + y_{n+1} + \frac{ib}{2\rho(A)}$ gives (albeit slow) convergence of $y_n \to x$, $Ax = b$. This scheme handles any invertible $I-B$ where $\sigma(B)$ is a "horseshoe" shape



V. PTÅK:

To each polynomial p of degree n we may assign the Schur-Cohn quadratic form $H(p)$; if it happens to be nonsingular, then the number of positive (negative) squares in its diagonal form gives the number of roots of p inside (outside) the unit disc. In two joint papers with N.J. Young it is shown that this theorem as well as a substantial generalization there of is a consequence of a simple algebraic identity. We obtain a whole family of forms which may be used the same way as the classical one and, which is perhaps more important, we may write down explicitly the convergence which gives H its diagonal form. We then show that the only way the form H may become singular is when the polynomial either has a root of modulus one on a pair of roots α, β with $\alpha\bar{\beta} = 1$. In this case it is possible to use a similar quadratic form (essentially the H corresponding to the derivative of a certain factor of p) to give a complete description of the roots.

A. RUHE:

Rational Krylov sequence methods for eigenvalue computation

Algorithms for large sparse eigenvalue problems are considered. The direct iterative methods as Lanczos, Arnoldi, all find approximations from the Krylov sequence

$$K_j(A,x) = \{x_1, Ax_1, \ldots, A^{j-1}x_1\}$$

and compute polynomials of the matrix by taking linear combinations. Now we consider instead inverse iterative methods which use

$$R_j(A,x) = \{x_j\}_{j=1}^{j}, \quad x_{j+1} = (A-\mu_j I)^{-1} x_j$$

and compute rational functions of the matrix. Zeros of the rational function yield approximative eigenvalues and can be computed as eigenvalues of a $j \times j$ linear matrix pencil. Fast convergence can be excepted if the poles μ_j get close to some of the eigenvalues.

H. SCHNEIDER:

Theoretical Results on regular splittings of a singular M-matrix which depend on graph structure

A matrix $A = D-P$, where $D \doteq \mathrm{diag}(A) \geq 0$ and $P \geq 0$ is a (singular) M-matrix if the spectral radius $\rho(D^{-1}P) \leq 1$ ($\rho(D^{-1}P) = 1$). The graph (support) of A will be denoted by $\Gamma(A)$, its transitive closure by $\overline{\Gamma(A)}$. A splitting $A = M-N$ will be called regular if $M^{-1} \geq 0$ and $N \geq 0$, graph compatible if $\Gamma(M) \subseteq \overline{\Gamma(A)}$, and an M-splitting if M is a non-singular M-matrix. A graph compatible, regular M-splitting is called perfect.

Theorem 1: Let $A = M-N$ be a perfect splitting of the singular M-matrix A. Then

(i) $\rho(M^{-1}N) = 1$,

(ii) $\mathrm{mult}_1(M^{-1}N) = \mathrm{mult}_1(D^{-1}P)$,

(iii) $\mathrm{ind}_1(M^{-1}N) = \mathrm{ind}_1(D^{-1}P)$.

If $P \geq 0$, then the cycle index $c(P)$ is the g.c.d. of the lengths of circuits of $\Gamma(P)$. If P is irreducible, the number of eigenvalues on the spectral circle is $c(P)$.

Theorem 2: Let $A = M-N$ be a regular M-splitting of the irreducible M-matrix A. For each circuit α of $\Gamma(A)$ let ν_α (resp. μ_α) be the number of arcs which belong to $\Gamma(N) \setminus \Gamma(M)$ (resp. $\Gamma(N)$). Then

$$c(M^{-1}N) = \gcd\{\nu_\alpha, \nu_\alpha + 1, \ldots, \mu_\alpha : \alpha \text{ is a circuit of } \Gamma(A)\}$$

and $c(M^{-1}N)$ equals the number of eigenvalues on the spectral circle of $M^{-1}N$.

Ji-guang SUN:

On the stability of generalized singular values

We study perturbation bounds for generalized singular values of two matrices having the same number of columns. A Weyl type theorem and a Hoffman/Wielandt type theorem are obtained.

R.S. VARGA:

Recent Applications of SOR and SSOR

In same joint work with W. Niethammer and D.-Y Cai, the following analog (of the Young/Varga functional equations $(\lambda+\omega-1)^p = \lambda^{p-1}\omega^p\mu^p$) is derived.

$$\text{Let } A = \begin{bmatrix} A_{1,1} & A_{1,2} & & 0 \\ & \ddots & \ddots & \\ & & \ddots & A_{p-1,p} \\ A_{p,1} & & & A_{p,p} \end{bmatrix} \text{ be in } \mathbb{C}^{n,n}, \text{ with}$$

Nonsingular $A_{i,i}$, $1 \leq i \leq p$. Then, the relation relating the eigenvalues λ of the symmetric successive overrelaxation (SSOR) method, to the eigenvalues μ of the block Jacobi matrix associated with A is

$$[\lambda-(1-\omega)^2]^p = \lambda[\lambda+1-\omega]^{p-2}(2-\omega)^2\omega^p\mu^p.$$

This is then used to obtain new results for convergence and divergence domain for SSOR applied to H-matrices. Then, in some joint work with W. Niethammer and J. de Pillis, very large overdetermined systems of least-squares problems are treated via block SOR methods applied to consistently ordered weakly-cyclic of index 3 matrices. Exact convergence and divergence domains in ω for such applications are then given, which extend and correct results in the literature.

H. WIMMER:

Die algebraische Riccati-Gleichung

Die Matrizengleichung

$$XDX + XA + A^{*}X - C = 0 \qquad (R)$$

wird diskutiert. Alle Matrizen in (R) sind komplexe $n \times n$ Matrizen, D, C und die Lösung X sind hermitesch, D ist positiv-semidefinit. - $\chi(G)$ bezeichne das charakteristische Polynom einer Matrix G, $\delta(V)$ den kleinsten gemeinsamen Nenner aller Unterdeterminanten einer rationalen Matrix V. Der Unterraum $C(A,D) := \mathrm{Im}\,(D, AD, \ldots, A^{n-1}D)$ ist invariant unter A, durch $\chi(A) = h\chi(A|_{C(A,D)})$ sei das Polynom h definiert. Im folgenden sei $q \in \mathbb{C}[z]$ stets ein Polynom vom Grad n, bei dem für $\lambda \notin i\mathbb{R}$ nicht zugleich mit λ auch $-\bar{\lambda}$ eine Wurzel ist.

$M = \begin{pmatrix} A & D \\ C & -A^{*} \end{pmatrix}$ sei die zu (R) gehörige hamiltonsche Matrix.

<u>Satz:</u> Die folgenden Aussagen sind gleichwertig.

(a) Es gibt eine eindeutig bestimmte Lösung X von (R) mit $\chi(A + DX) = q$.

(b) $(h(z), \bar{q}(-z)) = 1$, $\chi(M) = (-1)^n q(z)\bar{q}(-z)$ und alle Elementarteiler, die zu rein imaginären Eigenwerten von M gehören, haben einen geraden Grad.

(c) $(h(z), \bar{q}(-z)) = 1$ und

$$(I0)(M-zI)^{-1}\binom{0}{I} = V(z)V^*(-z),\ \delta(V) = \chi(A|_{C(A,D)}),$$

Zeilenrang $_{\mathbb{C}}V = \dim C(A,D)$.

Ch. ZENGER:

On dual operator norms and the spectrum of matrices

Let $|\cdot|$ be a norm on $\mathbb{C}^n$ and $\|A\| = \max\limits_{\substack{x\in\mathbb{C}^n\\|x|\neq 0}} \frac{|Ax|}{|x|}\ \forall A \in \mathbb{C}^{n,n}$

the corresponding operator norm. Then it is shown that if the dual operator norm

$$\|B\|^D = \max_{\substack{A\in\mathbb{C}^{n,n}\\ \|A\|\neq 0}} \frac{|\mathrm{tr}\, BA|}{\|A\|}\quad \forall B \in \mathbb{C}^{n,n}$$

has the property

$$\|B\|^D \geq \sum_{i=1}^{n} |\lambda_i(B)|\quad \forall B \in \mathbb{C}^{n,n}$$

where $\lambda_i(B)$ an the eigenvalues of B, then $|\cdot|$ has to be a Euclidian norm.

Berichterstatter: V. Mehrmann

Tagungsteilnehmer

Prof. Dr. Götz Alefeld
Institut für Angewandte Mathematik
Universität Karlsruhe
Englerstraße 2
D-75oo Karlsruhe

Dr. Angelika Bunse-Gerstner
Fakultät für Mathematik
Universität Bielefeld
Universitätstraße 1
D-48oo Bielefeld 1

Professor O. Axelsson
Department of Mathematics
University of Nijmegen,
Toernooiveld, 6525 ED
Nijmegen

Professor Francoise Chatelin-Laborde
Department de Mathematique
Universitē de Grēnoble
F-38o41 Grēnoble, Isere, France

Professor Abraham Berman
Technion-Israel Institute of Technology
Haifa 32ooo, Israel

Prof. Dr. Wolfgang Dahmen
Fakultät für Mathematik
Universität Bielefeld
Universitätsstraße 1
48oo Bielefeld 1

Prof. Dr. E. Bohl
Fakultät für Mathematik
Universität Konstanz
Postfach 556o
D-775o Konstanz

Professor Emeric Deutsch
Department of Mathematics
Polytechnic Inst. of New York
333 Jay Street
Brooklyn, N.Y. 112o2

Dipl.-Math. Heinrich Bolz
Institut für praktische Mathematik
Universität Karlsruhe
Englerstraße 2
75oo Karlsruhe

Dr. Michael Eiermann
Institut für praktische Mathematik
Universität Karlsruhe
Englerstraße 2
D-75oo Karlsruhe

Prof. Dr. D. Braess
Abteilung für Mathematik
Universität Bochum
Universitätsstraße 15o
D-463o Bochum-Querenburg

Prof. Dr. Ludwig Elsner
Fakultät für Mathematik der
Universität Bielefeld
Universitätstraße 1
48oo Bielefeld 1

Gefördert durch
DFG Deutsche Forschungsgemeinschaft

Professor Miroslav Fiedler
Ceskoslowenskā Akademie Ved
Mathematický Ustav
Praha 1, Zitnā Ulice 25

Professor Peter Lancaster
Department of Mathematics
University of Calgary
Calgary, T2N1N4, Alberta, Canada

Dipl.-Math. R. Freund
Institut für Angewandte Mathematik
Universität Würzburg
Am Hubland
87oo Würzburg

Professor Raphael Loewy
Technion-Israel Institute
of Technology
Haifa 32ooo, Israel

Professor Smuel Friedland
Institute of Mathematics
Hebrew Univ. at Jerusalem
Jerusalem, Israel

Professor Ivo Marek
Mathematicko-Fysikalni Fakulte
Universita Karlovy
Praha 1

Dr. J. Garloff
Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Straße 1o
D- 78oo Freiburg i.Br.

Dr. Günter Mayer
Institut für Angewandte Mathematik
Universität Karlsruhe
Englerstraße 2
75oo Karlsruhe

Prof. Dr. W. Hackbusch
Institut für Informatik und
praktische Mathematik
Universität Kiel
Olshausener Str. 4o-6o
D-23oo Kiel 1

Dr. Volker Mehrmann
Fakultät für Mathematik
Universität Bielefeld
Universitätsstraße 1
48oo Bielefeld 1

Prof. Dr. K.-P. Hadeler
Fakultät für Biologie
Universität Tübingen
Auf der Morgenstelle 1o
D-74oo Tübingen

Dr. N. Neumaier
Institut für Angewandte Mathematik
Universität Freiburg
Hermann-Herder-Straße 1o
D-78oo Freiburg i.Br.

Dr. C.R. Johnson
University of Maryland
Inst. f. Physical Science and
Techn.
College Park, MD 2o742

Professor Michael Neumann
Department of Mathematics
University of South Carolina
Columbia, SC 292o8

Prof. Dr. W. Niethammer
Institut für praktische Mathematik
Universität Karlsruhe
Englerstraße 2
D-75oo Karlsruhe

Prof. Dr. H.R. Schwarz
Seminar für Angewandte Mathematik
Universität Zürich
CH-8o32 Zürich

Prof. Dr. Gerhard Opfer
Inst. für Angewandte Mathematik
Universität Hamburg
Bundesstraße 55
D-2ooo Hamburg 13

Prof. Dr. Josef Stoer
Institut für Angewandte Mathematik
und Statistik
Universität Würzburg
Am Hubland
D-87oo Würzburg

Prof. Dr. John de Pillis
Department of Mathematics
University of California
Riverside, CA 92521

Dr. Ji-guang Sun
Computing Center
Academia Sinica
Peking, P.R. China

Professor Vlastimil Ptăk
Ceskoslowenskă Akademie Ved
Mathematický Ustav
Praha 1, Zitnă Ulice 25

Prof. Dr. R.S. Varga
Institute for Comp. Math.
Kent State University
Kent, Ohio 44242, USA

Professor Axel Ruhe
Dep. of Inf. Proc.
Inst. of Mathematics and
Statistic
University of Umeå
S-9o181 Umeå, Sweden

Prof. Dr. Harald Wimmer
Mathematisches Institut
der Universität Würzburg
Am Hubland
D-87oo Würzburg

Professor Hans Schneider
Department of Mathematics
University of Wisconsin
Madison, WI 537o6

Prof. Dr. Christoph Zenger
Leibniz-Rechenzentrum
der Bayerischen Akademie
Barerstraße 21
D-8ooo München 2