MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 11/1983

Mathematische Stochastik

6.3. bis 12.3.1983

Die Tagung fand unter der Leitung von Ch. Preston (Bielefeld) und P. Clifford (Oxford/England) statt. Die Mehrheit der Teilnehmer bestand aus relativ jungen Wissenschaftlern aus der Bundesrepublik.

In den Vorträgen ergaben sich folgende Schwerpunkte:
(i) Perkolationstheorie, (ii) Wachstums- und verwandte Prozesse, (iii) Theorie und Anwendung von Diffusionsprozessen, (iv) Stochastische Felder und ihre Anwendungen in der Physik.

Neben den zu diesen Hauptgebieten gehörigen Vorträgen gab es auch Berichte über neuere Ergebnisse aus anderen Teilgebieten der Wahrscheinlichkeitstheorie und Mathematischen Statistik sowie wertvolle informelle Diskussionen über Querverbindungen und Weiterentwicklungen in der Stochastik.

Vortragsauszüge

Th. BARTH

Infinite-dimensional diffusions

From the Itô point of view a diffusion ist the solution of a stochastic differential equation. In order to understand a process with values in R^I (where I is a countable index set) as a diffusion, we have to define the stochastic integral of an $I\times I$-matrix valued process with respect to a system $W = (W_i)_{i\in I}$ of independent one-dimensional Brownian motions. For this purpose R^I is considered as the union of spaces

$$E_\alpha = \{ x \in R^I : |x|_\alpha = (\Sigma_I \alpha_i x_i^2)^{1/2} < \infty \}$$

with weight function $\alpha = (\alpha_i)_{i\in I}$, $\alpha_i > 0$. The stochastic integral of an α-integrable process is continuous and takes values in E_α. Moreover, it is the same in any bigger space E_β. There is also an Itô-calculus linked to the spaces E_α. The notion of a stochastic differential equation

$$X(t) = C + \int_0^t f(s,X(s))\, ds + \int_0^t G(s,X(s))\, dW(s) \quad .$$

and its solution are independent of the spaces E_α once the coefficients are integrable for some α, and thus are notions of R^I. An existence and uniqueness theorem is proved for random coefficients f, G satisfying some appropriate conditions. This framework is sufficiently large to include interesting examples from the theory of infinite interacting particle systems, for example the continuous spin model of Doss and Royer (Z. für Wahrsch. 1978).

J. VAN DEN BERG

On the continuity of the percolation probability function

The lecture is based on a recent paper by J. van den Berg and M. Keane. Let G be a countably infinite, connected, locally finite graph and let s be a designated vertex of G. Denote by $\theta(p)$ the percolation probability function for bond percolation on the pointed graph (G, s), and let p_H be the critical probability. We show that if p is strictly larger than p_H and if for p there is a unique p-open component then θ is continuous at p. We also show the following theorem from which the above result can be derived as a corollary:

$$\theta(p) - \theta(p^-) = \Pr\{ s \text{ belongs to an infinite p-open connected component of } G \text{, which itself has critical probability } 1 \}.$$

E. BOLTHAUSEN

On Hoeffding's combinatorial central limit theorem

A combination of a method of Charles Stein with an idea of Bergström leads to a very simple elementary proof of the classical Barry-Esseen theorem. The same method gives an estimate of the remainder in Hoeffding's central limit theorem stating that $\Sigma_i\ a_{i,\pi(i)}$ is approximately normally distributed, where $(a_{i,j})$ is a $n \times n$ matrix and π is a uniformly distributed random permutation. It appears likely that the method could be pushed further to lead to Edgeworth expansions.

D. DÜRR

Stochastic acceleration in two dimension

We consider a random potential $U(\underline{x},\omega)$ created by a uniform distribution of "scatterer" potentials $V(\underline{x})$ centered at the points of a Poisson point process (Ω,F,P) , $\omega = (\underline{r}_1, \underline{r}_2, \ldots)$, and hence $U(\underline{x},\omega) = \sum_{r_i \in \omega} V(\underline{x} - \underline{r}_i)$.

We assume that $V(\underline{x}) \in C^3$ is spherically symmetric and supp $V(\underline{x}) < \infty$. Introduce now for $\varepsilon > 0$ the scaled Poisson field (Ω,F,P^ε) with intensity $\frac{1}{\varepsilon}$ 2d and the scaled scatterer $V^\varepsilon(\underline{x}) = \varepsilon V(\frac{\underline{x}}{\varepsilon^2})$ in d dimensions. The equations of motion for a particle in this potential field are

$$\frac{d\underline{x}^\varepsilon(t)}{dt} = \underline{v}^\varepsilon(t) , \quad \underline{x}^\varepsilon(0) = \underline{x}_0 \in R^d , \quad \frac{d\underline{v}^\varepsilon(t)}{dt} = - \sum_{\underline{r}_i \in \omega} \nabla V^\varepsilon\left(\frac{\underline{x} - \underline{r}_i}{\varepsilon^2}\right) , \quad \underline{v}^\varepsilon(0) =$$

$\underline{v}_0 \in R^d$. $(\underline{x}_t^\varepsilon,\underline{v}_t^\varepsilon)$ is a stochastic process on (Ω,F,P^ε) . We show that for $d \geqq 2$ $\underline{v}_t^\varepsilon$ converges in distribution to $\underline{v}_t$ as ε tends to zero, where $\underline{v}_t$ is a Wiener process on the sphere with radius $|\underline{v}_0|$. This was done jointly with S. Goldstein and J.L. Lebowitz. For $d \geq 3$ this result was previously obtained in a more general situation by Kesten and Papanicolou.

H. FÖLLMER

Markov fields and n-parameter martingales

Let $X = (X_t)_{t \in Z_+^n}$ be a bounded n-parameter martingale with respect to the σ-algebras generated by an n-dimensional Markov random field. Modifying an example of Dubins and Pitman, we see that X may fail to converge almost surely even if the Markov field is uniquely determined by its conditional probabilities (absence of phase transition). On the other hand we show that bounded n-parameter martingales do converge almost surely if the Markov field satisfies Dobrushin's uniqueness condition.

P. GÄNSSLER

Weak convergence of non-borel measures on a metric space with applications to empirical processes

Concerning weak convergence results for random elements in the space $D \equiv D[0,1]$, the necessary methodical background is rather extensive mainly in connection with introducing the Skorokhod J_1-topology in D to cope with measurability problems and to get a separable (Polish) space on which the laws of the random elements under consideration can be viewed as Borel measures. If, instead of the J_1-topology, D is equipped with the much simpler structure of the uniform topology, D becomes a non-separable metric space and random elements of interest need not be Borel-measurable (inducing measures defined an a σ-algebra smaller than the Borel σ-algebra). But still a suitable theory of weak convergence of non-Borel measures on arbitrary (possibly non-separable) metric spaces is available through the work of Dudley (1966) and Wichura (1968) for which the usual limit theorems can be proved once the weak convergence concept has been suitably defined. A slightly modified approach of the latter theory is presented. The advantage of this approach is that it applies e. g. simultaneously to more general situations such as multidimensional empirical processes or empirical processes indexed by classes of sets or functions in general sample spaces.

F. GÖTZE

Expansions for von Mises functionals

Berry-Esseen estimates for the speed of convergence and expansions hold for the distribution function of a sum of von Mises functionals up to the order $r \geq 2$. Under moment and variance conditions (without continuity assumptions on the distribution of the i.i.d. observations) the maximal order of approximation by expansions is shown to be $O_\varepsilon(n^{-r/2 + \varepsilon})$, $\varepsilon > 0$. The results apply to goodness-of-fit statistics, as well as to the central limit theorem in L^{2p}, $p \geq 2$, the rate of convergence being $O(n^{-1})$ for centered balls, provided a fourth moment exists.

G. GRIMMETT

A recreational problem in discrete probability

The following elementary problem has two attractive properties: it may be explained to the (mathematical) non-mathematician, but its solution poses difficulties.

Question Let $Z = \{\ldots,-1,0,1,\ldots\}$ and let $\{p_i : i \geq 1\}$ be a given sequence of numbers satisfying $0 \leq p_i \leq 1$. For each ordered pair (i,j), where $i < j$, with probability p_{j-i} we join i and j with an edge, independently of all other pairs. Under what conditions is the ensuing random graph G connnected?

It is easy to show that G is a.s. disconnected if $\Sigma p_i < \infty$. It is not so easy to show (Grimmett/Marstrand) that if $\Sigma p_i = \infty$ and $p_i \geq p_{i+1}$ for all i, then G is a.s. connected.

After the lecture, M. Keane showed how these results may be adapted to show that if $\Sigma p_i = \infty$ and $p_i > 0$ then G is a.s. connected.

M. KEANE

Percolation and Ergodic Theory

Given an ergodic Z^d-action on a probability space and a measurable subset A of the space, call a point $x \in A$ a percolation point of A if there exists a path $\underline{0} = z_0, z_1, z_2, \ldots$ from $\underline{0}$ to infinity in Z^d such that for each $i \geq 0$, $z_i(x) \in A$ (where $z_i(x)$ denotes the point obtained by the action on x of $z_i \in Z$). Denote by A^* the set of percolation points of A, and say that A percolates if $\Pr\{A^*\}>0$. Examples and problems in this framework, which generalizes the classical theory of Bernoulli percolation, are discussed.

G. KELLER

Asymptotic behaviour of controlled Galton-Watson processes

For stochastic processes of the type $X_{n+1} = X_n + g(X_n)(1+\xi_{n+1})$, where (ξ_n, F_n) is a martingale difference sequence and $E(\xi_n^2) < \infty$, we show that under some regularity assumptions on $g(t)$ and $\sigma^2(t)$ ($\sigma^2(X_n) := E[\xi_{n+1}^2 | X_n]$) and additional moment assumptions on ξ_n which depend on the growth-rate of g ,

(1) $X_n/y(n) \to x_{\{X_n \to \infty\}}$ in probability ,

(2) $(X_n - y(n))/g(y(n)) = (\sum_{k=1}^{n} \xi_k)(1 + o_p(1))$ on $\{X_n \to \infty\}$

provided that

(3) $g(t) = o(t/\log t)$ and $g(t) = o(t/\int_1^t \sigma^2(u)\frac{g'(u)}{g(u)}du)$ as $t \to \infty$,

where $y(t)$ is the solution of $\dot{y} = g(y)$, $y(0) = 1$. Under some more assumptions on the ξ_n (1) - (3) are equivalent.

Examples are controlled Galton-Watson processes $X_{n+1} = X_n + \sum_{k=1}^{X_n} Z_k^{n+1}$, $Z_k^{n+1} \in \{-1,0,1,\ldots\}$. If $E[Z_k^{n+1}|X_n] \sim X_n^\alpha$, $var[Z_k^{n+1}|X_n] \sim X_n^\beta$, then the above results imply that (1) holds if $\alpha < 0$ and $\beta < 1+\alpha$, whereas for (2) we have to assume additionally that $\beta \geqq 0$ or $\beta > 1+3\alpha$ since otherwise discretization effects may dominate the stochastic fluctuations. (These additional assumptions are far from being necessary, however.) These results are joint work with G. Kersting (Frankfurt) and U. Rösler (Göttingen).

G. KERSTING

On stochastic growth: A diffusion approach

Topic of the talk was the discussion of the asymptotic behaviour of the solution of the stochastic differential equation $dX_t = g(X_t)dt + \sigma(X_t)dW_t$ on the event $\{X_t \to \infty$ as $t \to \infty\}$. Essential assumptions are $g > 0$, $\int_1^\infty g(s)^{-1}ds = \infty$, which excludes explosions. It is convenient to analyse in a first step the behaviour of $G(X_t)$, ($G(t) = \int_1^t g(s)^{-1} ds$), for which a.s. approximation can be given. In a second step this result is used to obtain approximations to X_t (if possible). It turns out that a whole lot of questions concerning the behaviour of X_t are equivalent (e.g. if X_t is asymptotically deterministic, or if it obeys a normal law after renormalisation). These results could be useful in the study of growth-phenomena.

H. KESTEN

Large deviation estimates in first-passage percolation

Let $\{t(e): e \text{ an edge of } Z^d\}$ be an i.i.d. family of nonnegative random variables with distribution F. For a path r on Z^d the passage time of r is $t(r) = \sum_{e_i \text{ belongs to } r} t(e_i)$.

We consider the following passage times:

$a_{0n} = \inf\{t(r): r \text{ a path from } (0,0,\ldots,0) \text{ to } (n,0,\ldots,0)\}$,

$b_{0n} = \inf\{t(r): r \text{ a path from } (0,0,\ldots,0) \text{ to some point } (n,m_2, \ldots, m_d)\}$,

$\ell_n(k) = \inf\{t(r): r$ a path from some point $(0,m_2, \ldots, m_d)$ to some point $(n, m'_2, \ldots, m'_d)$ such that r minus its endpoints is contained in $(0,n) \times [-k,+k]^{d-1}\}$.

It is well known that if $E\, t^2(e) < \infty$, then there exists a constant $\mu = \mu(F)$ such that $\frac{1}{n} a_{0n} \to \mu$ and $\frac{1}{n} b_{0n} \to \mu$ a.e. and in L^1. We show that also $\frac{1}{n} \ell_n(n) \to \mu$, and give estimates for large deviations. For example

THEOREM Assume $E\, t^2(e) < \infty$ and $\mu(F) > 0$. Let $0 < \varepsilon < \beta := \sup\{x: F(\mu-x) > 0\}$ (i.e., $\mu-\beta$ is the left endpoint of supp (F)). Then there exist constants $0 < A,B,C < \infty$ such that

$$P\{\theta_n < n(\mu-\varepsilon)\} \leq A \exp - Bn$$

for $\theta_n = a_{0n}$, b_{0n} or $\ell_n(n)$. Moreover

$$P\{\theta_n < n(\mu\ \varepsilon)\} = \exp - (Cn + o(n))$$

for $\theta_n = a_{0n}$ or $\ell_n(n)$. Analogous estimates hold for $P\{\theta_n > n(\mu+\varepsilon)\}$. Some applications to flows of randomly capacitated networks and resistances of random electrical networks will be discussed.

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C. KIPNIS

Hydrodynamical picture of interacting systems

We consider an infinite number of particles on Z^d interacting by simple exclusion, and call x_t the position of a tagged particle located at time zero at the origin. We prove that if the other particles (the "bath") are initially distributed according to a Bernoulli distribution with parameter p then $\varepsilon x_{t\varepsilon^{-2}}$ converges weakly to a Brownian motion. This result is used to show how it implies diffusion of colour in the hydrodynamical limit when two types of mechanically identical coloured particles are mixed. This shows that on the macroscopical level we have the equation $\frac{\partial p}{\partial t} = D(p) \frac{\partial^2 p}{\partial x^2}$.

W. KLONECKI

On the structure of linear admissible estimators

In 1966 A. Cohen noted that if $Y \in R^n$ has a normal distribution $N(\mu,\sigma^2 I)$, then L Y, where L is an nxn real matrix, is a linear admissible estimator of μ with respect to the quadratic risk function if and only if L is symmetric and has all its eigenvalues in the closed interval $[0,1]$. The subject of my talk was to present extensions of this result.

Using a technique originated by Olson, Seely and Birkes (1976) and developed by myself (1979) and La Motte (1982) I have established (one step) necessary conditions of Rao's type for a linear estimator to be admissible in the class of linear estimators within a general linear model. These conditions become sufficient for the regression model with a non-negative covariance matrix and for the model with the mean lying in a subspace and the covariance operators varying through the set of all non-negative definite symmetric matrices. Applications of these general results to the problem of characterising the invariant quadratic estimators for the variance components in a variance-co-variance mixed linear model have been indicated.

P. KOSTER

Asymptotic behaviour of state-dependent Markov branching processes

The almost sure limiting behaviour of divergent time-homogeneous Markov processes on the non-negative integers which generalize Markov branching processes is studied. The span of time between two jumps is exponentially distributed with the exponent again depending linearly on the state. The offspring distributions are now allowed to be state-dependent with expectations which are an non-increasing function of the state. Most of the results for ordinary supercritical Markov branching processes are generalized, including a necessary and sufficient condition for divergence (with natural rates) similar to the (x log x)-condition.

R. LERCHE

Approximations to the first exit time distribution of Brownian motion for curved boundaries and the shape of sequential Bayes tests

Several results about the tangent approximation for the density of the first exit time distribution of Brownian motion for a curved boundary were reviewed. Let $W(t)$ denote the standard Brownian motion and let $\psi(t)$ be a positive smooth boundary. Then the density $f(t)$ of the distribution of

$$T = \inf \{ t > 0 \mid W(t) \geq \psi(t) \}$$

can be approximated by the "tangent approximation" $g(t)$, where

$g(t) = \frac{\Lambda(t)}{t^{3/2}} \varphi(\frac{\psi(t)}{t^{1/2}})$ with $\Lambda(t) = \psi(t) - t\psi'(t)$. $g(t)$ is the hitting density at the point t to the tangent $(\Lambda(t)+u\psi(t))$ of the curve ψ at the point t . The formula for $g(t)$ is calculated from a well-known formula due to Levy. The approximation results hold for a large class of boundaries (cf. Genner/Lerche Z. für Wahrsch. 1981), and refinements provide excellent approximations to the true exit densities. Statistical applications of these results to the theory of sequential testing of composite hypotheses were mentioned.

T. LYONS

Criteria for recurrence and transience

If $\underline{a} : X \times X \to R$ is a positive symmetric function and for each $i \in X$ $\sum_{j \in X} a_{ij} = \pi_i$ is finite then $(X, \underline{a}, Y_n)$ is a reversible Markov chain where Y_n is a random walk with transition probability $p_{ij} = a_{ij}/\pi_i$. Let $\varphi : X \to R$; we may define $\nabla\varphi : X \times X \to R$ by $(\nabla\varphi)_{ij} = a_{ij}(\varphi_i - \varphi_j)$, and if $\underline{u} : X \times X \to R$ is antisymmetric we may define $(\text{div } u)_{ij} = \sum_j u_{ij}$. It is a theorem that Y_n is transient if and only if there is an antisymmetric function $\underline{u}$ with $(\text{div } \underline{u})_i = \delta_{i_0 i}$ and $\sum u_{ij}^2 / a_{ij} < \infty$. As div does not depend on $\underline{a}$ it follows that the a_{ij} may be changed significantly without affecting transience. The proof depends greatly on the identity $\langle \Delta\varphi, \psi \rangle = -\langle \nabla\varphi, \nabla\psi \rangle$ for suitable choices of inner product, and has an extension to complete Riemannian manifolds. It would be interesting to know how much one can change the a_{ij} and still preserve such properties as having no bounded non-constant harmonic functions.

P. MAJOR

Critical phenomena in statistical physics; Dyson's hierarchical model

We are interested in the behaviour of equilibrium states in statistical physics. In interesting cases there is a critical temperature where the behaviour is essentially different from that at other temperatures. Unfortunately we cannot rigorously prove these results in the general case. On the other hand Dyson's hierarchical model can be rigorously investigated and it can explain the behaviour of general models. We present some important results about this model. The case of multi-dimensional spins ist also considered. In this case a limit theorem holds with an unorthodox normalization for all low temperatures. This is in great contrast to the scalar case where such a normalization can occur only at the critical temperature.

I. MEILIJSON

Cheating income tax - the carrot and the stick

A taxpayer with an income y who declares an income z pays a tax $t(z)$. A proportion p of taxpayers are audited. If audited, and $z < y$, then the taxpayer must pay a fine of $f(y,z)$, where $\inf\{t(z)+f(y,z) : z < y\} > t(y)$. The revenue of the tax authority is thus $\int \min(t(y),\inf\{t(z)+pf(y,z) : z<y\})dG(y)$, where G is the income distribution.

Without modifying any of t, p and f, is it possible to increase revenues? The following scheme is studied: An individual who paid less than the required amount of tax the last time he was audited is audited with probability p_2, the remaining taxpayers with probability p_1, where $p_1 < p_2$ are chosen so that stationary proportion (under optimal cheating) remains p. Let R_{p_1,p_2} be the revenue of the authorities, and let R_p be the revenue under the non-memory fixed sample proportion p (equal to the global proportion under p_1, p_2, if comparison is made). The main result is that if $\limsup_{x\downarrow 0} E(t(Y)|t(Y)\leq x)/x = \alpha > 0$ then $\limsup_{p_2\downarrow 0} \lim_{p_1\downarrow 0} (R_{p_1,p_2}/R_p) \geq (1-\alpha)^{-1}$. For example, if G has a continuous positive density at zero and t has a positive derivative at zero, then $\alpha = 1/2$ and dynamic sampling improves over fixed sampling by at least 100% for small values of p. The second result is that if the individuals are risk averters then the improvement is even more pronounced, even if the (concave) utility function varies from individual to individual and is unknown to the authority. This is a joint work with Michael Landsberger (Haifa). It appears as "Incentive generating state dependent penalty system" in Journal of Public Economics, 19, (1982), 333-352.

Gefördert durch DFG Deutsche Forschungsgemeinschaft

P. A. MEYER

Quelques Résultats sur les capacités et les maisons de jeu

Les travaux de Dellacherie permettent de rattacher avec facilité à la theorié des capacités un bon nombre de résultats de théorie descriptive des ensembles autrefois considérés comme très fins (comme le théorème de separation de Novikov) et de démontrer beaucoup de résultats nouveaux. Les méthodes nouvelles sont de trois types:

1) remplacement des arguments ensembles par des arguments fonctions,

2) utilisation de capacités plusieurs arguments (ou une infinité dénombrable d'arguments),

3) utilisation de noyaux capacitaires.

Ces méthodes permettent de démontrer par example que dans toute maison de jeu (au sens de Dubins et Savage) analytique, il existe "beaucoup" de fonctions excessives boreliennes, et si l'operateur de réduite est capacitaire, beaucoup de fonctions excessives continues.

M. NAGASAWA

A microscopic description of Monkey distributions

Given a distribution density φ (spatial) of a population, we define $K(x) = \frac{1}{2}j(x)^2$, where $j(x) = \frac{1}{2}\varphi^{-1}(x)\varphi'(x)$, and $Q(x) = \frac{1}{2}\varphi^{-1}(x)(-\frac{1}{2}\varphi'(x))$. If

(1) $$Q(x) + K(x) + V(x) = \lambda \qquad \text{for all } x$$

then the distribution φ is stationary, where $V(x)$ is a macroscopic containing potential. Assume $\varphi = u^2$, then the equation (1) is equivalent to

(2) $$\frac{1}{2}u'' + (\lambda - V(x)) = 0 .$$

According to Kolmogorov a diffusion process $dx_t = dW_t + b(x_t)dt$ has an invariant distribution density $\varphi(x)$ if and only if $b = \frac{1}{2}\varphi^{-1}\varphi'$. Thus we have a diffusion process behind the eigenvalue problem (2). We call the diffusion process x_t "a typical particle" of a population in an equilibrium state which has the distribution density $\varphi(x)$. Let us consider a microscopic description of the population under consideration; an N-particle system is described by

$$dx^i = dW^i + (a(x^i) + \int B(x^i-\xi)\ \mu_N(d\xi))dt ,$$

where $\mu_N = \frac{1}{N} \sum_{j=1}^{N} \delta_{x^j}$. Assuming the law of large numbers: $\mu_N \to \infty$ (as $N \to \infty$) we obtain

$$dx = dW + (U(x) + \int B(x-\xi)\mu(d\xi))\ dt .$$

We interprete this diffusion process as the "typical particle". Therefore we get $B(x) = U(x) + \int B(x-\xi)\varphi(\xi)\ d\xi$. Since $b = \frac{1}{2}\varphi^{-1}\varphi'$, the microscopic drift field $U(x)$ and the mutual interaction $B(x-y)$ are determined by

$$\frac{1}{2}\varphi^{-1}\varphi' = U(x) + \int B(x-\xi)\varphi(\xi)\ d\xi .$$

As an example, taking $V(x) = x^2$, several cases are discussed.

S. RICHARDSON

Speed of convergence of the central limit theorem for m-dependent processes on Z^d

We consider random variables $\{ X_j , j \in Z^d \}$, centered with $\sup_j E|X_j|^{2+\delta} < \infty$, $0 < \delta \leq 1$, and an increasing sequence $\{A_n\}_{n\geq 1}$ of subsets of Z^d . Let $S_n = \frac{1}{\sigma_n} \sum_{j\in A_n} X_j$, where $\sigma_n^2 = \mathrm{Var}(\sum_{j\in A_n} X_j)$. The order of the convergence $\Delta_n = \sup_x |P(S_n \leq x) - \Phi(x)|$ (where Φ is the standard normal distribution function) is investigated and the following theorem is proved:

If the variables $\{ X_j , j \in Z^d \}$ are m-dependent (i.e. $\sigma\{ X_i , i \in I \}$ and $\sigma\{ X_j , j \in J \}$ are independent whenever $d(I,J) \geq m$) and if $\inf \sigma_n^2 / |A_n| > 0$ ($|A|$ denotes the cardinality of A) then

for $\delta = 1$ $\quad \Delta_n = 0\left((\log \sigma_n)^{\frac{d-1}{2}} / \sigma_n \right)$,

$\delta < 1$ $\quad \Delta_n = 0\left(\sigma_n^{-\delta} \right)$.

The proof is based on a technique developed by Tickhomirov for stationary processes on Z.

H. RIEDER

A contamination game in robust asymptotic interval estimation of a simple regression parameter

Bounded influence regression (in the sense of Hampel-Mallows-Krasker-Welsch) has been conjectured by Huber ("Minimax aspects of bounded influence regression"; to appear in J.A.S.A. 1983) to be pessimistic in the following sense: It is the minimax strategy of the Statistician in a contamination game in which Nature knows the regressors and may distribute contamination among them subject only to some overall average contamination. This conjecture is not correct in the theory of Fisher information and minimax asymptotic variance. The present paper shows that the conjectured saddle point is realized in the framework of asymptotic interval estimation based on infinitesimal neighborhoods, which would rather seem to be in favour of the H-K-W-estimate.

M. RÖCKNER

Markov Properties of Gaussian Generalized Fields

Let (F_e,E) be a regular extended Dirichlet space with reference measure m on X in the sense of M. Fukushima, where X is a locally compact, second countable space. Define the space M_E of measures of bounded energy by

$$M_E := \{ \mu : \mu \text{ a Radon measure on } X \text{ with } (\int|u|d|\mu|)^2 \leqq \text{const. } E(u,u) \text{ for all } u \in F_e \cap C_0(X) \} .$$

Given $\nu \in M_E^+$ let $U\nu \in F_e$ denote the associated potential.

Consider a family $(X_\mu)_{\mu \in M_E}$ of real, Gaussian, mean zero random variables on a (complete) probability space (Ω,A,P) such that

$$\int X_\mu X_\nu dP = E\,(U\mu^+ - U\mu^- ,\ U\nu^+ - U\nu^-) \quad , \mu,\nu \in M_E .$$

For $A \subset X$ denote by $\sigma(A)$ the σ-field generated by X_μ, $\mu \in M_E^+$ with $\mu \subset A$, and the P-zero sets.

Our main result is the following:

Theorem The following assertions are equivalent:

(i) $(X_\mu)_{\mu \in M_E}$ is Markovian with respect to any $A \subset X$; i.e., $\sigma(\bar{A})$ is conditionally independent of $\sigma(\overline{X \setminus A})$ given $\sigma(\partial A)$.

(ii) $(X_\mu)_{\mu \in M_E}$ is Markovian with respect to any open subset of X. ("Global Markov property")

(iii) $(X_\mu)_{\mu \in M_E}$ is Markovian with respect to any relatively compact, open subset of X. ("Local Markov property")

(iv) (F_e, E) has the local property.

Applications to Nelson's free Euclidean field of quantum field theory and to Rozanov's generalized random functions are given.

U. RÖSLER

Duality of entrance and exit laws

For diffusions on R^n a duality is known, although not quite understood, which arises also for infinite particle systems. This duality is given by $E_x[f(X(t),y)] = E_y[f(x,X^*(t))]$, where $f(x,y) = 1_{x \le y}$. This relation provides also some relation between excessive measures and excessive functions. Especially, for a diffusion on R the duality is given by interchanging the scale and speed function. The bounded exit laws (if they exist) are lattice isomorphic to bounded Borel functions and the entrance laws are lattice isomorphic to monotone exit laws, which again are lattice isomorphic to monotone functions (provided non-trivial exit laws exist).

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L. ROSCHENDORF

On the minimum discrimination information theorem

The minimum discrimination information theorem allows us to calculate the projection of a probability measure on a set of distributions determined by linear constraints. A partial converse of this result is due to Csiszar. We give a generalization of these results to the case of minimizing φ-divergence distances, including x^{α}-distances and total variation. Extensions are possible to related distances as e.g. the Hellinger distance. Some applications are given to problems of entropy maximization and problems of measures with given marginals.

W. STUTE

Parameter Estimation in Smooth Empirical Processes

In this paper we derive almost sure representation theorems and limit distribution results for the statistical solution of a general parametric equation of integral type evaluated at the empirical distribution function. In particular, these may be applied to R- and (scale-invariant) M-estimates, CvM minimum distance estimates and estimates derived from U-statistics.

G. WINKLER

On the projective limit of simplices

Call a subset K of a locally convex space E measure-convex if any tight probability measure on K has its barycenter in K. If p and q are two such measures then $p < q$ will mean that $\int\varphi\, dp \leq \int\varphi\, dq$ for every continuous bounded convex function φ on K. In the spirit of the Choquet-Meyer theorem we call K a simplex if it is measure-convex and if every $x \in K$ has a unique representing probability measure on K which is maximal with respect to $<$.

<u>Theorem</u> Let $(I, \leq)$ be a set having a countable cofinal subset. For $i \in I$ let E_i be a locally convex space and $K_i \subset E_i$ a simplex. For $i \leq j$ let $\varphi_{ij} : K_j \to K_i$ be affine and continuous and such that (a) $\varphi_{ii} = id_{K_i}$, (b) if $i \leq j \leq k$ then $\varphi_{ki} = \varphi_{ji} \circ \varphi_{kj}$. Then the projective limit

$$K = \varprojlim K_i := \{(x_i)_{i \in I} : x_i = \varphi_{ji}(x_j) \text{ whenever } i \leq j\} \text{ is a simplex.}$$

This theorem generalizes to the non-compact case results of Davies, Vincent-Smith and Edwards. An application to the projective limit of substochastic kernels defined on standard Borel spaces was given, which in the case $I = R$ gives a construction for the entrance-boundary.

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