MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 12 /1983

Spezielle Funktionen und Gruppentheorie

13.3. bis 19.3.1983

Die Tagung fand unter der Leitung der Herren Richard A. Askey (Madison, Wisconsin), Tom H. Koornwinder (Amsterdam) und Walter Schempp (Siegen) statt. Im Mittelpunkt des Interesses standen gruppentheoretische Methoden in der Theorie der speziellen Funktionen. Da die gruppentheoretischen Techniken in den letzten Jahren zunehmend an theoretischer und praktischer Bedeutung gewonnen haben, wurde bei der Planung der Tagung ein ausgewogenes Verhältnis zwischen Themen aus der Theorie der speziellen Funktionen und ihren Anwendungen angestrebt. Deshalb wurden neben Mathematikern auch Physiker eingeladen, die in Übersichtsvorträgen und kürzeren Vorträgen über die erzielten Fortschritte berichteten.

Die behandelten Themen lassen sich etwa wie folgt gruppieren:

- Spezielle Funktionen, Orthogonalpolynome

- Jacobi-Funktionen und halbeinfache Lie-Gruppen

- Spezielle Funktionen und Heisenberg-Gruppen

Gefördert durch DFG Deutsche Forschungsgemeinschaft

- Spezielle Funktionen und Kombinatorik
- Symmetrie und Trennung der Variablen
- Anwendungen in der Physik
- Anwendungen in der Elektrotechnik, Medizin und Ozeanographie

Mit 39 (zum überwiegenden Teil ausländischen) Teilnehmern aus Belgien, Deutschland, Frankreich, Groß-Britannien, Italien, Japan, Kanada, den Niederlanden, der Schweiz, Tunesien und den Vereinigten Staaten von Amerika war die Kapazität des Instituts so ausgeschöpft, daß eine Reihe weiterer Interessenten leider nicht mehr berücksichtigt werden konnte.

Das wissenschaftliche Niveau der insgesamt 36 Vorträge bestätigte die Erwartungen an die zum ersten Mal in Oberwolfach über dieses Thema abgehaltene Tagung in vollem Umfang. Es bestätigte zugleich die Absicht der Tagungsleitung, eine Auswahl der gehaltenen Übersichtsvorträge zu veröffentlichen. Der Tagungsband wird unter dem Titel

"New trends in special functions:
Group theoretical aspects and applications"

in der Reihe "Mathematics and Its Applications" des Verlages D. Reidel, Dordrecht-Boston-London, erscheinen.

Dem Direktor des Mathematischen Forschungsinstituts, Herrn Professor Dr. M. Barner, und seinen Mitarbeitern sei für die freundliche Aufnahme und zuvorkommende Hilfe sehr herzlich gedankt.

Vortragsauszüge

R. ASKEY:

Orthogonal Polynomials

A summary of the classical orthogonal polynomials that are hypergeometric series was given. Some extensions were described including the continuous q-ultraspherical polynomials of Rogers and the associated Laguerre and Hermite polynomials.

L. AUSLANDER:

Fast Fourier Transforms and Finite Heisenberg Groups

Let A be a separable locally compact Abelian group with dual group $\hat{A}$. Assume that $\Delta \neq 0$ is a discrete subgroup of A such that A/Δ is compact. Let $\mathfrak{F}$ denote the Fourier transform from $L^2(A)$ to $L^2(\hat{A})$. We introduce a nilpotent group N structure on $A \times \hat{A} \times T$, where T is the image of the dual pairing in the circle group, by defining multiplication by $(a,\hat{a},t)(b,\hat{b},s) = (a+b,\hat{a}+\hat{b},ts\langle a,\hat{b}\rangle)$. Using the theory of intertwining operators for irreducible unitary representations of N, we obtain a factorization of $\mathfrak{F} = F_2MF_1$. If we take A a finite Abelian group we obtain the Cooley-Tukey algorithm and for other groups results of A. Weil.

E. BANNAI:

Toward the Classification of (P and Q)-Polynomial Association Schemes

The current situation of the classification problems of association schemes which have multiple P and/or Q structures was discussed. D. Leonard (SIAM J. Math. Anal. 1982) proved that the orthogonal polynomials which have the <u>(P and Q)-property</u> coincide with Askey-Wilson polynomials ${}_4\Phi_3$ (including certain limiting cases). Our aim is to obtain the classification of <u>(P and Q)-association schemes</u>, starting from Leonard's theorem.

So far, we proved the following:

<u>Theorem</u> (E. Bannai and T. Ito). If diameter d is large, then all the eigenvalues of (P and Q)-association schemes (which are not ordinary n-gons) are integers.

We believe that this result will be useful for our aim, in particular to prove the following conjectures: (i) We can choose the base q of Askey-Wilson polynomial of such an association scheme as an integer (moreover as a prime power). (ii) Only ${}_3\Phi_2$ will appear (instead of general ${}_4\Phi_3$) for such association schemes.

F. CALOGERO:

<u>Matrices, Differential Operators and Polynomials</u>

Various applications have been outlined of the connection that exists between the two operators x. resp. d/dx and the two n × n matrices $\underline{X}$ resp. $\underline{Z}$ defined in terms of n arbitrary (different) numbers x_j by the formulae

$$\underline{X} = \mathrm{diag}(x_j), \quad Z_{jk} = (x_j - x_k)^{-1} \text{ if } j \neq k, \quad Z_{jk} = \sum_{l=1}^{n}{}' (x_j - x_l)^{-1}$$

if j=k. These applications include: (i) the construction of explicit matrices with known eigenvalues, eigenvectors and inverses, (ii) identities, (iii) properties of the zeros of the classical polynomials, (iv) singular integral equations, (v) a technique for the numerical solution of regular Sturm-Liouville problems.

F.-J. DELVOS:

A Periodic Taylor Formula

M. Tasche (J. Integral Equations 4 (1982), 55-75) has presented a unified treatment of several interpolation methods by means of the Taylor formula of operational calculus. We apply Tasche's approach to develop a Taylor formula involving the right invertible operator

$$Af = Df + (2\pi)^{-1}\int_0^{2\pi} f(s)ds$$

and the normalized jump functional

$$s(f) = (f(0) - f(2\pi))/(2\pi).$$

As an application Krylov's method for accelerating the convergence of Fourier series is obtained.

C.F. DUNKL:

Orthogonal Polynomials with a Symmetry of Order Three

For $\alpha > -1$ let $d\mu_\alpha := (v_1 v_2(1-v_1-v_2))^\alpha dv_1 dv_2$ be a measure on the triangle $E := \{(v_1,v_2): v_1, v_2 \geq 0;\ v_1+v_2 \leq 1\}$. It is desired to find an orthogonal basis of polynomials for $L^2(E,\mu_\alpha)$ which diagonalizes the period-three isometry $Uf(v_1,v_2) := f(1-v_1-v_2, v_1)$. A self-adjoint third-order differential operator D_α which commutes with U is constructed, and it is shown that D_α has simple spectrum on each subspace of polynomials of degree $\leq$ n and orthogonal to all polynomials of lower degree (n = 1,2,...). This depends on an explicit tridiagonal representation of D_α with respect to an orthogonal basis consisting of products of Jacobi polynomials. The theory is related to an orthogonality structure for $(Z_2)^3$-invariant polynomials on the surface of the sphere in $\mathbb{R}^3$.

L. DURAND:

Group Theory and Gegenbauer Functions of the Second Kind

It was shown several years ago that the Gegenbauer functions of the second kind $D_\nu^\alpha(\cosh\theta)$ satisfy product and addition formulas of the same form as those satisfied by the Gegenbauer polynomials $C_n^\alpha(\cosh\theta)$. The polynomials appear as spherical functions or matrix elements of rotations on the spheres $S^n = SO(n+1)/SO(n)$ for $\alpha = (n-1)/2$. In this setting, the product formula expresses the fact that $C_n^\alpha(\cos\theta)/C_n^\alpha(1)$ is a spherical function, while the addition formula expresses the group composition property for rotations.

We show that the functions of the second kind have a similar interpretation on the hyperbolic space (hyperboloid of one sheet) $H_-^n = SO(1,n)/SO(1,n-1)$ with respect to a semigroup of hyperbolic rotations with elements $g = h\, a_\theta h$, $h \in SO(1,n-1)$, where a_θ is a positive hyperbolic rotation in the 0,n plane, $\theta>0$. The functions D_ν^α for $\alpha = (n-1)/2$ appear as matrix elements in a multiplier representation of a_θ on $H_+^{n-1} = SO(1,n-1)/SO(n-1)$. By using the composition formula for hyperbolic rotations $a_{\theta_1} h_\phi a_{\theta_2} = h_{\phi_1} a_\theta h_{\phi_2}$, $\cosh\theta = \cosh\theta_1 \cosh\theta_2 + \sinh\theta_1 \sinh\theta_2 \cosh\phi$, and the known representation theory for rotations on H_+^{n-1}, we show that $D_\nu^\alpha(\cosh\theta)$ satisfies the addition formula

$$D_\nu^\alpha(\cosh\theta) = \int_{-i\infty-\alpha+\frac{1}{2}}^{+i\infty-\alpha+\frac{1}{2}} dl\; a(l,\alpha)\; e^{-2\pi i(\alpha+l)}(\sinh\theta_1 \sinh\theta_2)^l \cdot D_{\nu-l}^{\alpha+l}(\cosh\theta_1)\, D_{\nu-l}^{\alpha+l}(\cosh\theta_2)\, D_l^{\alpha-\frac{1}{2}}(\cosh\phi),$$

where $a(l,\alpha)$ is the same coefficient as appears in the addition formula for $C_\nu^\alpha(\cos\theta)$,

$$a(l,\alpha) = \Gamma(2\alpha-1)(\Gamma(\alpha))^{-2}(2l+2\alpha-1)4^l\Gamma(\nu-l+1)(\Gamma(\alpha+l))^2(\Gamma(\nu+2\alpha+l))^{-1}.$$

Gefördert durch DFG Deutsche Forschungsgemeinschaft

The product formula for the D's is then obtained by inverting this expression, with the result

$$(\sinh\theta_1 \ \sinh\theta_2)^l \ D^{\alpha+l}_{\nu-l}(\cosh\theta_1) \ D^{\alpha+l}_{\nu-l}(\cosh\theta_2)$$

$$= 2^{-2\alpha-2l+1}\Gamma(2\alpha-1)\Gamma(l+1)\Gamma(\nu+2\alpha+1)(\Gamma(\alpha+1))^{-2}(\Gamma(2\alpha+l-1)\Gamma(\nu-l+1))^{-1}$$

$$\cdot e^{i\pi(\alpha+2l)} \int_0^\infty D^\alpha_\nu(\cosh\theta) C_l^{\alpha-\frac{1}{2}}(\cosh\phi)(\sinh\phi)^{2\alpha-1} \, d\phi.$$

The special case $l = 0$ was obtained by Mizony as the product formula for "spherical functions" on H^n_-. The results can be generalized to Jacobi polynomials of the second kind $Q_\nu^{(\alpha,\beta)}(\cosh\theta)$ for which product and addition formulas are also known.

D. FOATA:

Combinatorial Methods in the Derivation of Special Function Identities

Several special functions, especially the hypergeometric orthogonal polynomials, can be given combinatorial interpretations. Those interpretations are next used to prove the classical identities involving the functions. As an illustration the generating function for the Jacobi polynomials

$$\sum P_n^{(\alpha,\beta)}(x)u^n = 2^{\alpha+\beta}R^{-1}(1-u+R)^{-\alpha}(1+u+R)^{-\beta}$$

with $R = (1-2xu+u^2)^{1/2}$, is derived by means of combinatorial methods (see Proc. Amer. Math. Soc. 87 (1983), 47-53).

G. GASPER:

Product Formulas for Special Functions and Their Applications

Let $\{p_n(x)\}$ denote a sequence of orthogonal polynomials.

Via the example of Jacobi polynomials $R_n^{(\alpha,\beta)}(x)$ normalized so that $R_n^{(\alpha,\beta)}(1) = 1$, it is pointed out that product formulas of the "linearization" type

$$(1)\qquad p_n(x)p_m(x) = \sum_k g(k,m,n)\, p_k(x),\quad g(k,m,n) \geq 0,$$

and its dual

$$(2)\qquad p_n(x)p_n(y) = \int p_n(z)\, d\mu_{x,y}(z),\quad d\mu_{x,y}(z) \geq 0,$$

(when they hold for suitable x,y) lead to convolution structures (Banach algebras), positivity of generalized translation operators, maximum principles for partial differential (or difference) equations, positive definite sequences, homogeneous stochastic processes, multipliers, heat and diffusion equations, inequalities, Laplace integrals, etc... Connections with addition formulas, Laplace type integral representations and product formulas of Watson, Bailey, and of Bateman types are discussed. It is also pointed out when product formulas of the types (1) and (2) are known (due to joint work with Mizan Rahman) to hold for the q-Racah polynomials

$${}_4\phi_3\left[\begin{matrix} q^{-n}, abq^{n+1}, q^{-x}, cq^{x-N} \\ aq, q^{-N}, bcq \end{matrix}\, ; q, q\right]$$ (a basic hypergeometric series).

P. GREINER:

Complex Analysis, the Heisenberg Group and Special Functions

I discussed the (i) symbolic calculus for left invariant convolution operators on the Heisenberg group, and (ii) the spherical harmonics which occur in the study of the Dirichlet problem in the unit ball on H_1.

(i) Let F and G induce singular integral convolution operators on H_1. Then

$$\hat{\mathcal{L}}([F *_H G]^\wedge) = \hat{\mathcal{L}}(\hat{F})\, \hat{\mathcal{L}}(\hat{G}),$$

where $\hat{\mathcal{L}}(\hat{F})$ denotes the Laguerre matrix for F.

(ii) The analogue of the Gegenbauer polynomials on H_1,

$$H_k^{(\alpha,n)}(e^{i\varphi}),$$

– for L_α – is given by

$$(1-\rho e^{i\varphi})^{-(|n|+n)/2\; - \;(1+\alpha)/2}(1-\rho e^{-i\varphi})^{-(|n|-n)/2+(\alpha-1)/2}$$
$$= \sum_{k=0} \rho^k\, H_k^{(\alpha,n)}(e^{i\varphi}).$$

F.A. GRÜNBAUM:

<u>Band-time Limiting, Recursion Relations and Nonlinear Evolution Equations</u>

The problem of recovering a function supported in $[-T,T]$ from its Fourier transform in $[-\Omega,\Omega]$ leads to the finite convolution operator with kernel $\sin(\Omega\xi)/\xi$. It happens that this operator commutes with the differential operator of the "prolate spheroidal wave functions". This poorly understood accident leads to the problem of determining the potentials $V(x)$ for $-d^2/dx^2 + V(x)$ admitting eigenfunctions $\phi(x,\lambda)$ that satisfy a differential equation in the spectral parameter λ, or a recursion relation in this spectral index. We find many examples of this situation, the simplest nontrivial one is $V(x,t) = -2(\log(x^3+t))''$ which solve the Korteweg-de Vries equation. Other nonlinear equations – some of them integro-differential – enter the picture too.

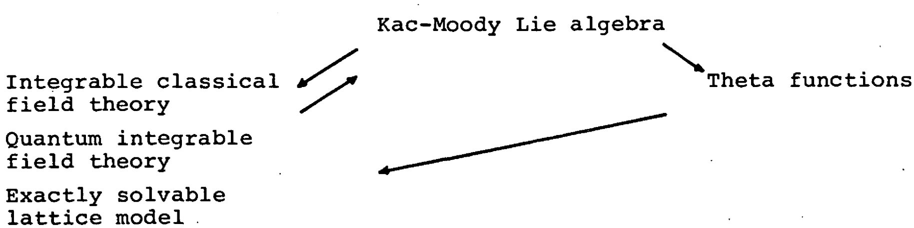
M. HAZEWINKEL:

Theta Functions and Integrable Systems

Given a Kac-Moody algebra one has V. Kac's character formula which can be rewritten in terms of classical theta functions and in the case of affine Lie algebras (Euclidean Lie algebras, Loop algebras, current algebras) this leads to identities like the Rogers-Ramanujan ones.

On the other hand it is by now well known that expressions like $-2(d^2/dx^2)(\ln(\vartheta(xU+tV+W)))$ solve KdV type equations and classical and quantum field theories and exactly solvable lattice models are closely related. Also the Rogers-Ramanujan identities are precisely the identities Baxter needed to solve the hard hexagon model.

Finally there are (several) constructions which associate an integrable system to a classical semi-simple Lie group (Toda lattices) and more generally to a Kac-Moody Lie algebra. Also the "symmetry algebra" of an integrable system has in many cases turned out to be a Kac-Moody Lie algebra (more or less). This leads to a meta-mathematical diagram



which hopefully will be commutative in some sense. The work on vertex representations, basic representations etc., notably that done by the Japanese school of Michio Sato strongly supports this view, as do a number of other observations which were presented in the lecture.

R. HERMANN:

Special Functions, Linear System Theory, and the Mikusinski Algebra

Make the continuous functions $f : [0,\infty[\rightarrow \mathbb{C}$ into an algebra as follows:

$$(f_1 * f_2)(t) = \int_0^t f_1(t-\tau) f_2(\tau) d\tau.$$

It is also a differential algebra:

$$(\delta f)(t) = tf(t).$$

"Special functions" have much to do with the Galois theory of this differential algebra. Methods are developed for relating this structure to the realization of functions as matrix elements of group representations.

B. HOOGENBOOM:

Intertwining Functions on Compact Lie Groups

We consider a generalization, called intertwining functions, of spherical functions on a compact Lie group G. These are functions on G, left -K-, and right -H- invariant (here K and H are the fixed point groups of two commuting involutions on G) which belong to some irreducible representation of G. We shall discuss some elementary properties of intertwining functions, such as equivalent defining conditions, and the action of the Weyl group on intertwining functions. Finally we give an outline of the proof that the intertwining functions on G can be considered as orthogonal polynomials with respect to a positive weight function, defined on a region in $\mathbb{R}^1$. This generalizes results of Vretare in the spherical case.

K.W.J. KADELL:

Andrews' q-Dyson Conjecture: $n = 4$

I.J. Good has given an elegant proof of a conjecture of F. Dyson for the constant term in the expansion of a function related to the root system A_{n-1}. Good's proof uses a functional equation which follows from the Vandermond determinant, which is the Weyl denominator function for A_{n-1}.

G. Andrews has conjectured a q-analog of the Dyson result. It has been proven for $n = 3$ by Andrews and when $a_i \equiv 1,2$ or ∞ for all n by I.G. Macdonald. We use a q-Vandermond to give a q-analog of Good's functional equation which contains an error term. For $n \leqq 5$, the error term symmetrizes to 0 and Andrews's conjecture reduces to showing that the constant term is symmetric in the a_i. We use brute force to establish some generalizations of a classical result on Weyl polynomials. Using 6 of these, we prove Andrew's q-Dyson conjecture for $n = 4$. We give some new constant term conjectures which are also related to the root system A_{n-1}.

T.H. KOORNWINDER:

The Generalized Abel Transform for $SL(2,\mathbb{C})$

Let G be a noncompact connected real semisimple Lie group with Iwasawa decomposition $G = K\,A\,N$ and with M being the centralizer of A in K. Suppose that each irreducible unitary representation of K restricted to M is multiplicity free. Then the algebra $I^{\infty}_{c,\delta}$ of K-central C^{∞}-functions on G with compact support which belong to a fixed K-type δ is commutative. Define the generalized Abel transform $f \mapsto F_f$ on $I^{\infty}_{c,\delta}$ by

$$F_f(m,a) := e^{\rho(\log a)} \int_N f(m\,a\,n)\, dn, \quad m \in M,\ a \in A,$$

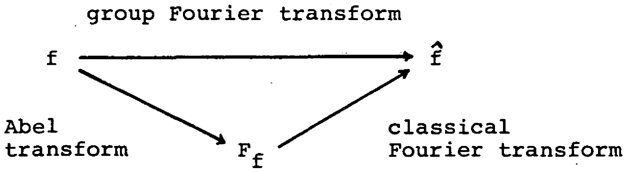
ρ being half the sum of the positive roots. Then F_f has the form

$$F_f(m,a) = d_\delta \sum_{\xi \in \hat{M}} F_{f,\xi}(a)\, \chi_\xi(m^{-1})$$

(d_δ = degree of δ, χ_ξ = character of ξ),

$$F_{f_1 * f_2, \xi} = F_{f_1,\xi} * F_{f_2,\xi},$$

$f \mapsto F_f$ is injective and we have the commutative diagram



A major problem is to rewrite $f \mapsto F_f$ as a "classical" integral transform, to find its inversion formula and to characterize its image space. In the lecture we present the solution for $G = SL(2,\mathbb{C})$: we obtain a double integral transform with kernel U_n (Chebyshev polynomial of second kind) depending on a complicated argument (joint work with R. Brummelhuis).

P. KRAMER:

<u>Coherent States Associated with a Lie Group and Some Applications in Physics</u>

Coherent states provide a continuous and often analytic basis for a unitary irreducible representation D of a Lie group G and its Lie algebra $\mathcal{L}$. A map is constructed from D to a symplectic manifold M with a Poisson bracket and a Poisson action of G (1). The map associates to D a co-adjoint orbit on the dual $\mathcal{L}^*$ of $\mathcal{L}$. If the elements of G are interpreted as observables in a quantum system, this map associates to the quantum system a classical system with phase space M. Examples and applications in physics include the Weyl, the unitary and the symplectic group and corresponding quantum systems (2).

---

(1) V.I. Arnold, Mathematical Methods of Classical Mechanics, Springer Berlin 1978

(2) P. Kramer and M. Saraceno, Lecture Notes in Physics 140, Springer Berlin 1981

R. LASSER:

Orthogonale Polynome und Hypergruppen

Es wird ein natürlicher Zusammenhang zwischen gewissen orthogonalen Polynomfolgen $(P_n(x))$ und Hypergruppenstrukturen auf $\mathbb{N} \cup \{0\}$ erläutert. Dabei spielen die Koeffizienten in der Linearisierung von den Produkten $P_n(x)P_m(x)$ eine entscheidende Rolle. Man findet so, daß für Jacobi-Polynome, q-ultrasphärische Polynome, assoziierte Legendre-Polynome und viele andere mehr der Begriff der Hypergruppe einen passenden abstrakten Hintergrund bildet.

G. LETAC:

Special Functions for the Homogeneous Tree

L'arbre homogène étant considéré comme un couple de Gelfand, on construit ses fonctions sphèriques (appelées polynômes de Dunan) $(P_n(x|q))_{n=0}^{\infty}$ définies par $P_0(x|q) = 1$, $P_1(x|q) = q$ et $(q+1)xP_n(x|q) = qP_{n+1}(x|q) + P_{n-1}(x|q)$, et sa mesure de Plancherel $\nu(dx) = \mathbb{1}_{[-\rho_q,\rho_q]}(x)(\rho_q^2-x^2)^{-1/2}(1-x^2)^{-1}dx$.

On donne alors la formule de multiplication

$$P_n(x|q)P_n(y|q) = \int_{-1}^{+1} P_n(z|q)\mu(x,y,dz)$$

en calculant explicitement μ. On met en évidence le fait commun aux groupes non-moyennables de rang 1, à savoir que $\mu(x,y,dz)$ est formée d'une atome éventuel plus une partie absolument continue par rapport à la mesure de Plancherel. L'exposé en fait en mettant constamment en parallèle le cas facile $SL(2,\mathbb{C})/SU(2)$.

J.D. LOUCK:

Canonical SU(3) Wigner Coefficients and Special Functions

We describe how an interesting class of polynomial functions in three parameters and in three barycentric coordinates occur in the description of the SU(3) invariant norm of canonically defined Wigner operators. The symmetries, reduction formulas, and zeros of these polynomials are developed in some detail. The zeros are particularly important because of their relation to the characteristic null space of canonically defined Wigner operators. It is shown that the generic polynomial in question, G_q^t, has value zero at each point in the weight space of irreducible representation $[q-t,0,-t+1]$ of the unitary group U(3), and moreover, that each zero occurs with a multiplicity at least as great (and probably equal) to the multiplicity of the weight. The role of a generalization of the Gauss hypergeometric series ${}_2F_1$ to t variables, and a generalization of the Saalschütz formula that results in the determination of these zeros is given in detail.

A.K. LOUIS:

Orthogonal Polynomials and the Radon Transform

The Radon transform of a real-valued function in $\mathbb{R}^N$ is defined as its integrals over all (N-1) - dimensional hyperplanes. This transform has found a spectacular application in medical imaging, in two dimensions with transaxial x-ray tomography, and in three dimensions with nuclear resonance (NMR) zeugmatography.

With the help of orthogonal polynomials on the unit ball in $\mathbb{R}^N$ we compute a singular value decomposition of the Radon transform and give a simple proof of the consistency conditions of Helgason and Ludwig, which describe the range of the transform.

This enables us to study the nonuniqueness problem occuring in computerized tomography. Also efficient algorithms can be based on the use of orthogonal polynomials.

I.G. MACDONALD:

Selberg's Integral and Root Systems

A brief account of some conjectures involving root systems and finite reflection groups, generalizing conjectures of

Dyson and Mehta. Selberg's integral can be used to resolve these conjectures for root systems of classical type. One conjecture (generalizing that of Dyson) asserts that if R is a finite reduced root system and k is a positive integer, the constant term in the Laurent polynomial $\prod_{\alpha \in R} (1-e^{\alpha})^k$ should be $\prod \binom{kd_i}{k}$, where the d_i are the degrees of the fundamental polynomial invariants of the Weyl group W of R. Likewise, a conjecture generalizing that of Mehta asserts that if W is a finite group of isometries of $\mathbb{R}^n$ generated by reflections, and if $l_i(x) = 0$ $(1 \leq i \leq N)$ are the (suitably normalized) equations of all the "mirrors" of reflections belonging to W, then the value of the integral

$$\int_{\mathbb{R}^n} \prod_{i=1}^{N} l_i(x)^{2k} \, d\gamma(x)$$

should be

$$\prod ((kd_i)!/k!),$$

where the d_i are as above and $d\gamma(x) = \exp(-|x|^2/2)dx(2\pi)^{-n/2}$ is Gaussian measure on $\mathbb{R}^n$. For more details, see SIAM J. Math. Analysis (13) 988-1007 (1983).

M.L. MEHTA:

Matrices with Quaternion Elements and Certain Multiple Integrals

Pfaffians of 2n x 2n antisymmetric matrices with complex numbers as elements are related to the "determinant" of n x n self-dual matrices with quaternions as elements. These determinants have a few little known wonderful properties. They can be exploited to evaluate certain multiple integrals often encountered in physics. The integrals which can be evaluated are of the form

$$\int \cdots \int p(x_1, \ldots, x_N)\, d\mu(x_{n+1}) \ldots d\mu(x_N), \quad 0 \leq n \leq N,$$

where

$$p(x_1, \ldots, x_N) = \prod_{1 \leq i < j \leq N} |x_i - x_j|^\beta, \quad \beta = 1, 2 \text{ or } 4,$$

or

$$p(x_1, \ldots, x_N) = \prod_{1 \leq i < j \leq N} (x_i - x_j)\ \mathrm{Pf}(f(x_i - x_j))$$

with f(x) any anti-symmetric function.

W. MILLER, JR:

The Symmetry Characterization of Variable Separation

Many special functions arise in the process of separating variables in one of the partial differential equations of mathematical physics. We discuss the general definition of variable separation and show, for R-separation of the Schrödinger equation $\Delta_n \Psi = E\Psi$ on a pseudo-Riemannian manifold, the relation between separation and the Lie symmetry algebra of the manifold. In particular we have the

Theorem (Kalnins, Miller): Necessary and sufficient conditions for the existence of an orthogonal R-separable coordinate system $\{x^j\}$ for $\Delta_n \Psi = E\Psi$ are that there exist n 2nd order differential operators

$A_i = \sum_{j,k} a_i^{jk}\, \partial_{y^j y^k}$ + lower order terms, where $A_1 = \Delta_n$ and

1) $[A_i, A_j] = 0$,

2) The matrices $\mathfrak{A}_i = (\mathfrak{A}_i^{jk})$, $i=1,\dots,n$ are linearly independent.

3) There is a basis $\{w^{(j)} = \lambda_l^{(j)} dy^l,\ j=1,\dots,n\}$ of simultaneous eigenforms for the matrices $\mathfrak{A}_i$.

If these conditions are satisfied then there exist functions $f^j(x)$ such that $w^{(j)} = f^j\, dx^j$ where dx^j is the differential of a separable coordinate.

We close with a simple graphical procedure for constructing all separable coordinate systems for the Schrödinger equations on the n-sphere and for Euclidean n-space.

S.C. MILNE:

An Umbral Calculus for Polynomials Characterizing U(n) Tensor Operators

After the change of variables $\Delta_i = \gamma_i - \delta_i$ and $x_{i,i+1} = \delta_i - \delta_{i+1}$ we show that the invariant polynomials ${}_uG_q^{(n)}(\ ,\Delta_i,;,x_{i,i+1},)$ characterizing U(n) tensor operators $\langle p,q,\dots,q,0,\dots,0\rangle$ becomes an integral linear combination of Schur functions $S_\lambda(\gamma-\delta)$ in the symbol $\gamma-\delta$, where $\gamma-\delta$ denotes the difference of the two sets of variables $\{\gamma_1,\dots,\gamma_n\}$ and $\{\delta_1,\dots,\delta_n\}$. We obtain a similar result for the yet more general bisymmetric polynomials ${}^m_uG_q^{(n)}(\gamma_1,\dots,\gamma_n;\ \delta_1,\dots,\delta_m)$. Making use of properties of skew Schur functions $S_{\gamma/1}$ and $S_\lambda(\gamma-\delta)$ we put together an umbral calculus for ${}^m_uG_q^{(n)}(\gamma;\delta)$. That is, working entirely with polynomials, we uniquely determine ${}^m_uG_q^{(n)}(\gamma;\delta)$ from ${}^m_uG_{q-1}^{(n)}(\gamma;\delta)$ and combinatorial rules involving Ferrers diagrams (i.e., partitions). As an application of writing ${}^m_uG_q^{(n)}(\gamma;\delta)$ as an integral linear combination of $S_\lambda(\gamma-\delta)$ we deduce "conjugation" symmetry for ${}^{m}_{u}G^{(n)}_{q}(\gamma;\delta)$ from the "transposition" symmetry by showing that these two symmetries are equivalent.

M. MIZONY:

Semi-Groupes de Causalité et Fonctions de Jacobi de 2ème Espèce

Nous definissons des représentations isométriques hilbertiennes d'un sous-semi-groupe de $SO_o(1,n)$, à partir d'un noyau de Poisson de 2ème espèce, lié aux fonctions de Jacobi de 2ème espèce.
Par contraction ces représentations approximent des représentations unitaires irréductibles du groupe des déplacements pseudo-euclidien: $\mathbb{R}^n \odot SO_o(1,n-1)$. En particulier nous retrouvons une formule du type Mehler-Heine pour les fonctions de Jacobi de 2ème espèce et les fonctions de Hankel et nous montrons que les semi-groupes considérés sont les semi-groupes de causalité globale de l'univers lorsque n = 4.

E. ONOFRI:

Lattice-Gauge Theory and Rogers-Szegö Polynomials

The talk is about the calculation of the following integral over the group of N x N unitary matrices

$$W_n(q,N) := (N\,Z(q,N))^{-1} \int_{U(N)} [dU]\,\mathrm{Tr}(U^n)\det[\vartheta_3(U|q)]$$

where [dU] is the Haar measure, $\vartheta_3(z|q) = \sum_{-\infty}^{+\infty} q^{n^2} z^n$ is Jacobi's θ-function, Tr means "trace" and Z(q,N) is a normalizing factor such that $W_o \equiv 1$. The integral arises in two-dimensional lattice gauge theory. The result involves basic hypergeometric functions (q-Jacobi polynomials)

$$W_n(q,N) = -(-q)^n (N(1-q^{2n}))^{-1} {}_2\phi_1 \left[\begin{matrix} q^{2n}, q^{-2n}; q^2; q^{2N+2} \\ q^2 \end{matrix} \right]$$

and was proved independently by G.E. Andrews and by me using different techniques. Details can be found in
E. Onofri, Nuovo Cimento 66A (1981) 293, G.E. Andrews and E. Onofri, "L.G.T., orthogonal polynomials and q-hypergeometric functions", Penn-State U. report # 82003 (unpublished).

M. RAHMAN:

An Integral of Products of Ultraspherical Functions and a q-Extension

Let $P_n(x)$ and $Q_n(x)$ denote the Legendre polynomial of degree n and the usual second solution to the differential equation, respectively. Din showed that $\int_{-1}^{1} Q_n(x)P_m(x)P_l(x)dx$ vanishes when $|l-m| < n < l+m$, and Askey evaluated the integral for arbitrary integral values of l, m and n. We extend this to the evaluation of $\int_{-1}^{1} D_n^{\lambda}(x)C_m^{\lambda}(x)C_l^{\lambda}(x)(1-x^2)^{2\lambda-1}dx$ where $C_n^{\lambda}(x)$ is the ultraspherical polynomial and $D_n^{\lambda}(x)$ is the appropriate second solution to the ultraspherical differential equation. Again the integral vanishes when $|l-m| < n < l+m$. A q-extension is found using the continuous q-ultraspherical polynomials of Rogers. A related integral of the product of three Bessel functions is also evaluated.

M. REIMANN:

Vector Fields on Hyperbolic space

The invariant differential operator for vector fields

$\Delta_H = \rho^{-n-2} S^{*} \rho^{n} S$, introduced by Ahlfors, is given a group theoretic interpretation. For this purpose, the invariant differential operators on the homogeneous space $X = SO_o(1,n)/SO(n-1)$ are classified. X is isomorphic to the cotangent bundle of the hyperbolic space $B = SO_o(1,n)/SO(n)$ and the non-commutative algebra of differential operators is generated by two operators D and D_Σ. The vector fields are characterized as eigenfunctions of D_Σ. From the Fourier analysis on B, as developed by Helgason, and from Ahlfors' solution of the Dirichlet problem for Δ_H, explicit expressions for the eigenvector fields of Δ_H can be derived.

W. SCHEMPP:

Radar Ambiguity Functions, the Heisenberg Group, and Holomorphic Theta Series

The basic group which stands at the crossroads of quantum mechanics and radar analysis is the real Heisenberg nilpotent group $\tilde{A}(\mathbb{R})$. It is the purpose of the lecture to indicate that a comprehensive theory of radar ambiguity functions can be developed by means of nilpotent harmonic analysis. As an application, a Poisson-Plancherel identity for radially symmetric radar autoambiguity functions is established. It implies an identity of Poisson-Plancherel type for Laguerre-Weber functions of different orders.

J.J. SEIDEL:

Harmonics and Combinatorics

The geometry of the sphere in $\mathbb{R}^d$, which provides a setting for various combinatorial configurations, is governed by spherical harmonics. Bounds for the cardinality of such configuration may be derived by use of a linear programming method. This applies to equiangular lines, root systems, and Newton numbers. The methods can be extended to the hyperbolic case, as well as to the discrete case. The present paper aims to survey these harmonic methods and some of their results.

I. SPRINGKHUIZEN-KUYPER:

Appell's Hypergeometric Function F_4 and a Generalization of the Hypergeometric Function of 2 x 2 Matrix Argument

Consider the solution of the differential equations for Appell's function F_4 which is regular in $(x,y) = (0,1)$. This solution can be represented as:

$$F_4^*(a,b,c,c';XY,(1-X)(1-Y))$$

$$= \sum_{m=0}^{\infty} \sum_{l=0}^{m} \frac{(a)_m (a-\gamma-\frac{1}{2})_l (b)_m (b-\gamma-\frac{1}{2})_l}{(c+\gamma+\frac{1}{2})_m (c)_l (\gamma+\frac{3}{2})_m l!} \frac{(\gamma+\frac{3}{2})_{m-l}}{(m-l)!} z^{\gamma}_{m,l}(X+Y,XY). \qquad (*)$$

In this formula:

$$\gamma = a+b-c-c'+1/2,$$

$$z^{\gamma}_{m,l}(X+Y,XY) = \frac{(2\gamma+1)_{m-l}}{(\gamma+\frac{1}{2})_{m-l}} (XY)^{(m+l)/2} R^{(\gamma,\gamma)}_{m-l}((X+Y)(XY)^{-1/2}/2),$$

$R^{(\gamma,\gamma)}_n(.)$ is a Gegenbauer polynomial, $R^{(\gamma,\gamma)}_n(1) = 1$.

For $\gamma = 0$ the right hand side of $(*)$ is the hypergeometric function of 2x2 matrix argument

$$_2F_1\left(a,b;c+\tfrac{1}{2};\begin{bmatrix} x & 0 \\ 0 & y \end{bmatrix}\right)$$

expressed as a series in "zonal polynomials", which are spherical functions on $GL(2,\mathbb{R})/O(2)$ (Herz, James, Constantine). Joint work with T.H. Koornwinder.

D. STANTON:

Variations of the Macdonald Identities

The Macdonald identities are derived by functional equations. They are reduced to multi-colored signed graphs, one color for each generator of the Weyl group. This allows variations which Macdonald did not consider.

R. TAKAHASHI:

Combinatorial Identities Related to the Principal Series of Sp (1,1)

The principal series representation $U^{n,1}$ of the group Sp(1,1) can be realized in the following way: for $n \in \frac{1}{2}\mathbb{N}$, let (ρ^n, U^n) be an irreducible unitary representation of the group $\mathbb{U} = \{u \in \mathbb{H}: |u| = 1\} \approx SU(2)$ of dimension 2n+1. For $f \in L^2_{V^n}(\mathbb{U})$, space of square-integrable functions on $\mathbb{U}$, with values in V^n:

$$\left[U^{n,s}\begin{pmatrix} a & b \\ c & d \end{pmatrix} f\right](u) = |-\bar{b}u+\bar{d}|^{-2s-3}\rho^n((-\bar{b}u+\bar{d})/|-\bar{b}u+\bar{d}|)^{-1} f((\bar{a}u-\bar{c})(-\bar{b}u+\bar{d})^{-1})$$

The talk gives an explicit computation of some matrix coefficients and yields i) some combinatorial identities involving the Clebsch-Gordon coefficients of SU(2), and ii) irreducibility proof for n = O ($s \neq -3/2, -5/2, \ldots$), $n=1/2$ ($s \neq 0, -1, -2, \ldots$) and $n=1$ ($s \neq 1/2, -1/2, -5/2 \ldots$). The same method can be applied to the computation of the intertwining operator given by

$$[J(s)f](v) = c_n(s)^{-1} \int_{\mathbb{U}} |1-\bar{v}u|^{2s-3} \rho^n((1-\bar{v}u)/|1-\bar{v}u|) f(u)\,du \quad (n=0,1,2,\ldots)$$

normalized by the condition : $J(s)v = v$ for $v \in V^n$. For the irreducible K-type (l,l'), $|l-n| \leq l' \leq n + l$, corresponding scalar value is equal to

$$\gamma^{l,n,l'}(s)$$

$$= \frac{2s+2n+1}{2l'+1} \frac{(\frac{1}{2}+s)_n}{(\frac{1}{2}-s)_n} \sum_{\substack{-n\leq\Lambda\leq n \\ -l\leq\lambda\leq l}} (-1)^{n+\Lambda} \frac{(\frac{1}{2}-s)_{|2\lambda+\Lambda|}}{(\frac{1}{2}+s)_{|2\lambda+\Lambda|}} \frac{(2\lambda+\Lambda)^2-(\frac{1}{2}+s)\frac{1}{2}}{(2\lambda+\Lambda)^2-(\frac{1}{2}+s)^2} |C^{l,n,l'}_{\lambda,\Lambda,\lambda+\Lambda}|^2$$

Conjecture: $\gamma^{l,n,l'}(s) = \frac{(\frac{1}{2}+|p|+s)_{n-|p|}}{(\frac{1}{2}+|p|-s)_{n-|p|}} \frac{(\frac{1}{2}+n-s)_{2l-n+p}}{(\frac{1}{2}+n+s)_{2l-n+p}}$, if $l' = l+p$

(True if $n = 0$, 1 or 2).

A. TERRAS:

Special Functions for the Symmetric Space of Positive Matrices

The talk presents a summary of results in harmonic analysis on the symmetric space $\mathfrak{P}_n$ of the general linear group $GL(n,\mathbb{R})$. The special functions involved are matrix argument analogues of y^s, spherical harmonics, J- and K-Bessel functions, Eisenstein series. The derivation of the fundamental solution of the heat equation for $\mathfrak{P}_n$ is considered. The central limit theorem for $O(n)$ - invariant random variables in $\mathfrak{P}_n$ is shown to depend on an asymptotic formula relating spherical functions and matrix argument J - Bessel functions for $\mathfrak{P}_n$. Such a result is of interest in such diverse fields as electrical engineering and demography.

K. TRIMECHE:

Fonctions Moyenne-Périodiques Associées à un Opérateur Différentiel Singulier sur $(0,\infty)$

On considère un opérateur différentiel Δ sur $(0,\infty)$ qui généralise les opérateurs de Bessel, de Jacobi et la partie radiale de l'opérateur de Laplace-Beltrami sur un espace riemannien symétrique de type non compact de rang 1.

On définit les fonctions moyenne-périodiques généralisées pour un produit de convolution associé à l'opérateur Δ et on démontre un théorème de représentation de ces fonctions.

La méthode utilisée pour démontrer ce théorème consiste à lier les fonctions moyenne-périodiques généralisées aux fonctions moyenne-périodiques sur $\mathbb{R}$, paire, à l'aide de transformations intégrales associées à l'opérateur Δ, puis appliquer le théorème de représentation de L. Schwartz pour les fonctions moyenne-périodiques sur $\mathbb{R}$, paires.

Berichterstatter: F.-J. Delvos (Siegen)

Adressen der Tagungsteilnehmer

Askey, Richard	Department of Mathematics Van Vleck Hall University of Wisconsin Madison, WI 53706 U.S.A.
Auslander, Louis	Department of Mathematics Graduate Center City University of New York 33W 42 St New York, N.Y. 10036 U.S.A.
Bannai, Eiichi	Department of Mathematics Ohio State University Columbus, Ohio 43210 U.S.A.
Caboz, Régis	Laboratoire de Physique Appliquée Université de Pau Avenue Salle Nave F-64000 Pau France
Calogero, Francesco	Dipartimento di Fisica Università di Roma I Piazzale Aldo Moro 00189 Roma Italia
Delvos, Franz.-J.	Lehrstuhl für Mathematik I University of Siegen D-5900 Siegen West-Germany
Dunkl, Charles F.	Department of Mathematics Math.-Astron. Bldg. University of Virginia Charlottesville, VA 22903 U.S.A.
Durand, Loyal	Physics Department University of Wisconsin 1150 University Avenue Madison, WI 53706 U.S.A.

Foata, Dominique	Département de Mathématique Université de Strasbourg 7, rue René-Descartes F-67084 Strasbourg France
Gasper, George	Department of Mathematics, Lunt Bldg. Northwestern University Evanston, IL 60201 U.S.A.
Greiner, Peter C.	Mathematics Department University of Toronto Toronto, Ont. Canada M5S 1A1
Grünbaum, F. Alberto	Department of Mathematics University of California Berkeley, Calif. 94720 U.S.A.
Hazewinkel, Michiel	Mathematical Centre P.O. Box 4079 1009 AB Amsterdam The Netherlands
Hermann, Robert	53 Jordan Road Brookline, Mass 02146 U.S.A.
Hoogenboom, Bob	Mathematical Centre P.O. Box 4079 1009 AB Amsterdam The Netherlands
Kadell, Kevin W.J.	Department of Mathematics Arizona State University Tempe, AZ 85287 U.S.A.
Koornwinder, Tom H.	Mathematisch Centrum P.O. Box 4079 1009 AB Amsterdam The Netherlands
Kramer, Peter	Institut für Theoretische Physik Universität Tübingen Auf der Morgenstelle 14 7400 Tübingen 1 West-Germany

Lasser, Rupert	Mathematisches Institut Technische Universität München Arcisstraße 21 8000 München 2 West-Germany
Letac, Gérard	Université Paul Sabatier Mathématiques F-31062 Toulouse France
Louck, James D.	Los Alamos National Laboratory MSB 284 Los Alamos, NM 87545 U.S.A.
Louis, Alfred K.	Institut für Nümerische Mathematik Universität Münster 4400 Münster West-Germany
Macdonald, I.G.	Department of Pure Mathematics Queen Mary College Mile End Road London E1 4NS England
Metha, Madan Lal	Service Physique Théorique C.E.N. de Saclay F-91191 Gif-Sur-Yvette Cedex France
Miller, Willard, Jr.	School of Mathematics University of Minnesota Minneapolis, MN 55455 U.S.A.
Milne, Stephen C.	Department of Mathematics Texas A & M University College Station, TX 77843 U.S.A.
Mizony, Michel	Département de Mathématiques 2ème cycle Université Lyon 1 43 Bd. du 11 Novembre 1918 F-69621 Villeurbanne Cedex France
Onofri, Enrico	Istituto di Fisica Università degli Studi di Parma Via M. D'Azeglio, 85 I-43100 Parma Italia

Rahman, Mizan	Department of Mathematics and Statistics Carleton University Ottawa, Ontario Canada K1S 5B6
Reimann, Martin	Mathematisches Institut Universität Bern Sidlerstr. 12 CH-3012 Bern Switzerland
Ronveaux, André	Département de Physique Facultés Universitaires Notre-Dame de la Paix B-5000 Namur Belgium
Schempp, Walter	Lehrstuhl für Mathematik I University of Siegen D-5900 Siegen West-Germany
Schiffmann, Gérard	Département de Mathématique Université Louis Pasteur 7, rue René-Descartes F-67084 Strasbourg Cedex France
Seidel, J.J.	Department of Mathematics Technological University Den Dolech 2 5600MB Eindhoven The Netherlands
Sprinkhuizen-Kuyper, Ida G.	Boterbloemstraat 34 1562 RX Krommenie Nederland
Stanton, Dennis	School of Mathematics University of Minnesota Minneapolis MN 55455 U.S.A.
Takahashi, Reiji	Department of Mathematics Sophia University Kioi-cho, Chiyada-ku Tokyo 102 Japan

Terras, Audrey

C-012, Mathematics Department
University of California at San Diego
La Jolla, California 92093
U.S.A.

Trimèche, Kalifa

Faculté des Sciences de Tunis
Département de Mathématiques
Campus-Belvédère
1060 Tunis
Tunisie