MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 17/1983

Mathematische Logik

17.4. bis 23.4.1983

Die Tagung fand unter der Leitung von Herrn W. Felscher (Tübingen) und Herrn H. Schwichtenberg (München) statt. An ihr nahmen 44 Wissenschaftler aus 8 Ländern teil, darunter 23 aus Deutschland. Wegen des großen Interesses, auf das diese Tagung gestoßen war, konnten einige Interessenten nicht eingeladen werden.

Das Thema der Tagung sprach bewußt keine Teilgebiete der mathematischen Logik an. Die 33 gehaltenen Vorträge stellten deshalb neue Entwicklungen aus allen wichtigen Teilgebieten der mathematischen Logik und ihren Beziehungen zur Informatik dar. Dabei ließen sich folgende Schwerpunkte feststellen: Die Modelltheorie war besonders mit modelltheoretischer Algebra und der Analyse von schwachen Systemen der Arithmetik vertreten. In der Beweistheorie sind drei Vorträge in Verbindung mit Girards neuentwickelter Π_2^1-Logik zu nennen, wobei in diesem Zusammenhang auf einen eindrucksvollen Erfolg der Logik-Tagungen in Oberwolfach hingewiesen werden kann: Vor allem an dieser Stelle konnte Girard die Entwicklung seiner neuen Theorie darstellen, mit der sich inzwischen Logiker in mehreren Ländern beschäftigen. Die Rekursionstheorie war mit international führenden Wissenschaftlern aus den USA, Großbritannien und Italien vertreten. Weitere Schwerpunkte bildeten die Mengenlehre und Verbindungen der mathematischen Logik zur Informatik.

Vortragsauszüge

Z. ADAMOWICZ: Primes and rationals

Let M be a model of PA^-. We can define rationals over M as follows: If $\langle x,y\rangle$, $\langle z,w\rangle$ are pairs in $M \times M - \{0\}$ then we define $\langle x,y\rangle$, $\langle z,w\rangle$ to be equivalent if $xw = yz$. We take M^* to be the collection of equivalence classes $[\![\langle x,y\rangle]\!]$ for $x \in M$, $y \in M - \{0\}$.

We define
$$[\![\langle x,y\rangle]\!] + [\![\langle z,w\rangle]\!] = [\![\langle xw+yz, yw\rangle]\!]$$
$$[\![\langle x,y\rangle]\!] \cdot [\![\langle z,w\rangle]\!] = [\![\langle xz, yw\rangle]\!].$$

For any countable model M of PA^- we find a model M' of PA^- such that $M^* \cong (M')^*$ and in M' the set of primes is bounded.

K. AMBOS - SPIES: Weak-truth-table reducibility as a tool for studying the r.e. Turing degrees

Es wird dargestellt, wie sich mit Hilfe der "weak-truth-table" Reduzierbarkeit neue Erkenntnisse über die algebraische Struktur der rekursiv aufzählbaren Turing-Grade gewinnen lassen. Diese Methode wurde erstmals von Ladner und Sasso 1975 angewendet. Die Grenzen, aber auch die überraschend weit reichenden Möglichkeiten dieser Methode werden anhand neuerer Ergebnisse aufgezeigt.

E. BÖRGER: Logical Decision Problems: Computational Complexity and Completeness

We present a method by which it becomes possible to formulate in a natural, simple and explicit way a uniform reason underlying many famous undecidability or lower complexity bound results in first order or propositional logic. This mehtod builds upon ideas from A. Turing and R. Büchi and has been developed independently by S.O. Aanderaa and myself in 1971 for solving particular cases of the Entscheidungsproblem of first order predicate logic. Essentially it gives an economical and natural way to embed computations into logical deductions by a step-by-step translation of the effect of single instructions into logical implications describing how the computational situation encoded by the premisse is transformed by execution of the instruction into the configuration described by the conclusion.

We have discovered that by appropriate slight variations of this technique to produce logical descriptions of computations one gets uniform and technically almost trivial proofs for the following fundamental theorems:

Theorem (Church 1935, Turing 1936). Hilbert's Entscheidungsproblem is unsolvable, indeed the class of first order deducible formulae is Σ_1-complete.

Theorem (Cook 1970). The satisfiability decision problem for propositional logic is NP-complete.

Theorem (Trachtenbrot 1950, Büchi 1962). The emptiness problem for first order spectra (i.e. the finite satisfiability problem) is unsolvable, indeed Π_1-complete.

Theorem (Bennett 1962, Rödding + Schwichtenberg 1972, Jones + Selman 1974). The class of first order spectra coincides with respect to unary (resp. binary) representation with the NP- (resp. NEXPTIME-) sets of positive integers.

Theorem (Fagin 1974). The encodings of finitely first order axiomatizable projective classes of finite type (so called generalized spectra) coincide with the NP-sets of nonempty words.

Theorem (Rödding + Schwichtenberg 1972, Christen 1974). The spectra of type formulae or order $n+1$ are precisely the NTIME (θ_{n+1})-acceptable sets of positive integers, where $\theta_o(x) = x$, $\theta_{n+1}(x) = 2^{\theta_n(x)}$.

Theorem (Bennett 1962). All rudimentary predicates have a first order representation in finite domains, therefore the infinity problem for first order spectra is Π_2-complete.

Theorem (Aanderaa 1971). The program description is axiom of an essentially undecidable and incomplete first order theory (resp. a satisfiable formula without recursive models) if the program enumerates recursively unseparable r.e. sets.

The same logical description of computation processes underlies half of the proofs of the following theorems:

Theorem (Aanderaa + Börger 1981). For arbitrary Boolean functions Horn complexity and network complexity are polynomially equivalent.

Theorem (Börger + Kleine Büning 1980). The decision problem with respect to satisfiability for the class of all prenex universal Horn formulae of extended Skolem arithmetic restricted to formulae with only monadic predicates, only binary disjunctions and without more than one occurence of a variable per term is equivalent to the reachability problem for Petri nets and therefore recursive. Leaving out one of these 3 restrictions yields classes with many-one complete decision problems.

By still more refined encoding techniques combined with that description method many lower complexity bounds and indeed completeness results for decidable cases of first order decision problems have been obtained by various authors including Denenberg, Fürer, Lewis. For exact references and technical outlines of the above discussed subject see my paper "Decision problems in predicate logic", to appear in the Proceedings of the Florence Logic Colloquium 1982 (North-Holland, Edited by G. Lolli).

P. CLOTE: Partition Relations in Arithmetic

Recall the folklore result that the scheme of the definable version of Ramsey's Theorem is provable in Peano arithmetic. Here we refine this by giving combinatorial equivalences of induction and collection schemes.

The language of Peano arithmetic is $\{+ , \cdot , 0 , 1 , <\}$ and as usual Δ_0 is the class of bounded formulae, also denoted Σ_0 and Π_0 and Σ_{n+1} is the class of formulae of the form

$$\exists x_1 \ldots x_m \Theta \ , \ \Theta \in \Pi_n$$

and Π_{n+1} is the class of formulae of the form

$$\forall x_1 \ldots x_m \Theta \ , \ \Theta \in \Sigma_n \ .$$

Let the Σ_n induction scheme be

$$\forall\vec{u}[(\varphi(0,\vec{u}) \wedge \forall x(\varphi(x,\vec{u}) \rightarrow \varphi(x+1,\vec{u}))) \rightarrow \forall x\varphi(x\vec{u})]$$

for $\varphi \in \Sigma_n$

and the Σ_n collection or bounding scheme be

$$\forall\vec{u}[\forall x < a\exists y\Phi(xy\vec{u}) \rightarrow \exists b\forall x < a\exists y < b\Phi(x,y,\vec{u})]$$

for $\Phi \in \Sigma_n$.

Throughout M will denote a countable model of $P^- + I\Sigma_o$, where P^- denote the usual set of axioms for $+ , \cdot , 0$, "successor" $+1$, and $<$, not including induction axioms.

DEF $M \underset{\Delta_m}{\rightarrow} (M)^n_{<M}$ means for any Δ_m definable cofinal subset X of M and Δ_m partition $F : [X]^n \rightarrow a$ where $a \in M$ there is a set $Y \subseteq X$ which is cofinal in M and homogeneous for F , i.e. if $\{y_1,\ldots,y_n\}_<$ and $\{z_1,\ldots,z_n\}_<$ are two n-element subsets of Y , written in increasing order, then

$$F(y_1\ldots y_n) = F(z_1\ldots z_n) .$$

Similarly

$M \underset{\Delta_m}{\rightarrow} (k)'_{<M}$ means for any Δ_m definable partition F of a Δ_m definable cofinal set X of M into M-finite many classes, one of the classes has at least k elements.

Recall the

Proposition (Kirby - Paris) For $n \geq 1$

$$M \models B\Sigma_{n+1} \quad \text{iff} \quad M \underset{\Delta_n}{\rightarrow} (M)'_{<M}$$

We can easily show the

Proposition For $n \geq 1$ the following are equivalent:

(1) $M \underset{\Delta_n}{\rightarrow} (z)'_{<M}$

(2) $M \models \forall a\exists b\forall x \leq a(\exists y\Phi \leftrightarrow \exists y < b\Phi)$ where Φ is Π_{n-1}

(3) $M \models \forall a\exists b_1\ldots b_n\forall x \leq a(Q_1x\ldots Q_nx_n\Theta \leftrightarrow Q_1x_1 < b_1\ldots Q_nx_n < b_n\Theta)$ where Θ is Δ_o

(4) $(M,\mathrm{Def}_oM) \models$ weak Σ_n comprehension $(\omega\Sigma_nCA_o)$

(5) $M \models I\Sigma_n$.

Cor The Σ_n definable points are not cofinal in any non-standard model of $P^- + I\Sigma_n$, for $n \geq 1$.

Cor $I\Sigma_n$ is equivalent to $I\mathbb{B}_n^o$, where $\mathbb{B}_n^o$ is the closure of the set of boolean combinations of Σ_n formulae under bounded quantification, and $I\mathbb{B}_n^o$ the induction scheme for that class of formulae.

Remark It is easy to see that

$$M \underset{\Delta_o}{\longrightarrow} (z)^{\prime}_{<M} \quad \text{iff} \quad M \models I\Sigma_1 .$$

In view of the fact due to Wrathall that the LINEAR TIME HIERARCHY equals the class of Δ_o definable subsets of $\mathbb{N}$, G. Wilmers asked whether

$$M \underset{E_1}{\longrightarrow} (z)^{\prime}_{<M} \quad \text{is weaker than} \quad M \underset{E_{n+1}}{\longrightarrow} (z)^{\prime}_{<M}$$

where E_n is the subclass of Δ_n consisting of those formulae having at most n blocks of bounded quantifiers and beginning with bounded existential quantifiers.

A. Wilkie showed that for n sufficiently large

$$M \underset{E_n}{\longrightarrow} (z)^{\prime}_{<M} \quad \text{iff} \quad M \models I\Sigma_1 .$$

The value of n depends on the number of alternating blocks of bounded quantifiers necessary to define the graph of the exponential function (BENNETT).

Finally we can show that for $n,m \geq 1$

THM $M \underset{\Delta_m}{\longrightarrow} (M)^n_{<M}$ iff $M \models B\Sigma_{n+m}$.

The direction from left to right uses the

Limit Lemma If $M \models B\Sigma_{k+1}$ for $k \geq 1$ and f is a Σ_{k+1} definable total function then there exists a Σ_k definable total function g of two variables, such that

$$f(x) = \lim_s g(x,s) .$$

Cor If $M \models B\Sigma_{k+1}$ for $k \geq 0$ then $M \models I\Delta_{k+1}$.

This corollary is used to formalize the Jockusch - Soare low basis theorem: if $T \subseteq \omega^{<\omega}$ is a recursive tree which is infinite and finite branching, then there is an infinite branch f whose jump f' is recursive in $0''$.

W. CRAIG: Is there a good equational logic for partial functions?

In this talk I raise, but do not answer, related questions about several fragments of the logic of partial functions. To any term t and any algebra $\mathfrak{A}$ of appropriate similarity type there corresponds an ω-ary operation $t^{\mathfrak{A}}$ on the universe A of $\mathfrak{A}$. When $\mathfrak{A}$ is a partial algebra then $t^{\mathfrak{A}}$ is in general a partial operation. Traditional equational logic concerns ordinary algebras $\mathfrak{A}$ and total operations $t^{\mathfrak{A}}$. As Birkhoff showed, it is complete for identities $s^{\mathfrak{A}} = t^{\mathfrak{A}}$ in the following sense: If, for each ordinary $\mathfrak{A}$, $s^{\mathfrak{A}} = t^{\mathfrak{A}}$ is a consequence of certain other identities, then the corresponding assertion can be derived. For partial $\mathfrak{A}$, analogous completeness questions arise about various relationships. Among there are: Again $s^{\mathfrak{A}} = t^{\mathfrak{A}}$; $s^{\mathfrak{A}} \subseteq t^{\mathfrak{A}}$; $\mathrm{dom}\, s^{\mathfrak{A}} \subseteq \mathrm{dom}\, t^{\mathfrak{A}}$; combinations of these.

Some laws carry over from ordinary to partial $\mathfrak{A}$ and also from $=$ to $\subseteq$. Substitution, however, must be restricted. I do not know whether these adjustments result in systems that are complete.

D. van DALEN: Internal Glueing of Beth Models

The glueing of Beth models was introduced for the purpose of modeltheoretic proofs of DP and EP (disjunction and existence property) of analysis-theories (with choice sequences). The old version used classical logic all the way and, as a result, was cumbersome. The present version is carried out within Krol's model (Zeitschr. 1978), so that the principles that one wants to preserve under glueing are established by applying the same principle externally. Since in Krol's model the arithmetical fragment is classical we could apply one of the refined completeness theorem (e.g. Friedmans). As a result we obtained DP and EP for $\mathrm{El} + \mathrm{AC} \cdot \mathrm{NF} + \mathrm{WC} + \mathrm{KS} + \mathrm{BI}_m$. A slight extension of the method yielded the definability property ($\vdash \exists \xi \varphi(\xi) \Rightarrow \vdash \varphi(f)$, f some definable function) for the above theory with Kripke's Schema.

J. DILLER: Σ_1^0-akzeptable Strukturen

Eine Struktur $\mathfrak{A}$ ist Σ_1^0-akzeptabel, wenn sie ein Kodierschema $\mathcal{C} = (N, <, \langle\, \rangle)$ besitzt, so daß $<$ $\exists$-definierbar ist und das Sequenz-Prädikat Im$\langle\, \rangle$, die Längen- und die Dekodierfunktion $\Sigma_1^0(\mathfrak{A})$ sind. Dabei entstehen die $\Sigma_1^0(\mathfrak{A})$-Formeln aus quantorenfreien Formeln von $L(\mathfrak{A})$ mit $\wedge, \vee, \exists$ und $\forall x < t :\equiv Nt \wedge \forall x(x < t \rightarrow \ldots)$. Dies ist eine Abschwächung von Aczel's $\exists$-Akzeptabilität. $\mathfrak{N}$ (Davis 1958) und das Kontinuum $\mathfrak{R}$ sind Σ_1^0-akzeptabel, $\mathfrak{N}$ ist nach Matijasevic sogar $\exists$-akzeptabel. Sei nun $\mathfrak{A}$ Σ_1^0-akzeptabel, und die Nachfolgerfunktion sei $\exists$-induktiv auf $\mathfrak{A}$. Nach Aczel's Handbook - Kapitel 1979 gibt es eine Aufzählungsrelation $U^{\mathfrak{A}} \in \Sigma_1^0(\mathfrak{A})$ für die $\Sigma_1^0(\mathfrak{A})$-Relationen. Wir bezeichnen die Expansion $(\mathfrak{A}, U^{\mathfrak{A}})$ als jump von $\mathfrak{A}$ und setzen nach Post $\Sigma_{n+1}^0(\mathfrak{A}) := \Sigma_1^0(\mathfrak{A}^{(n)})$, wobei $\mathfrak{A}^{(n)}$ der n-fache jump von $\mathfrak{A}$ ist. Ist $N \in \Delta_1^0(\mathfrak{A})$, so erhalten wir die übliche Präfix-Darstellung von $\Sigma_n^0(\mathfrak{A})$-Formeln und ein elementares Hierarchie-Theorem, das für $\mathfrak{A} = \mathfrak{N}$ die arithmetische und für $\mathfrak{A} = \mathfrak{R}$ die analytische Hierarchie beschreibt.- Diese Untersuchungen wurden gemeinsam mit M. Hülsermann begonnen.

E. ENGELER: Equations in Combinatory Algebras

Let L be the set of terms generated from the atom 0 by the binary function symbol g. If $M \subseteq L$ define $y \leqslant M$ recursively by $0 \leqslant M$, $g(x,y) \leqslant M$ iff $x \in M$ and $g \leqslant M$. For $M, N \subseteq L$ let $M \cdot N = \{x : \exists y \leqslant N \cdot g(x,y) \in M\}$. Then $\underline{L} = \langle 2^L, \cdot \rangle$ is a combinatory algebra.

Let E be a finite set of equations with parameters A_i and unknowns X_i. Theorem. If all A_i are recursively enumerable and E has a solution in $\underline{L}$, then there is a solution in $\underline{L}$, when all X_i are recursively enumerable.

U. FELGNER: Aristotelian Syllogistics

Man kann oft lesen, daß von den 24 von Aristoteles angegebenen Syllogismen nur 15 gültig seien (cf. Hilbert-Ackermann et al.). Weit verbreitet ist auch die Ansicht, daß zur lückenlosen Durchführung der bei Aristoteles (Analytica priora) angegebenen

Beweise die Verwendung aussagenlogischer Gesetze notwendig sei (Lukasiewicz et al.).
(1) Es wurde eine Interpretation der Syllogistik in einem 3-sortigen Prädikatenkalkül (mit Gleichheitszeichen) angegeben, unter der alle 24 Aristotelischen Syllogismen gültig sind. Das Problem des "Existential Import" ist auf diese Weise in den Modell-Begriff verlagert worden.
(2) Die Beweisverfahren wurden in einem Sequenzen-Kalkül (à la Gentzen) dargestellt. Hier können die Beweise (ohne Hinzufügung von Aussagenlogik) wie bei Petrus Hispanus durchgeführt werden. Metathesis praemissarum (M), Conversio Syllogismi (C), Conversio simplex (S) und Conversio per accidens (P) sind als spezielle Sequenzen formuliert.
Aus (1) und (2) ergibt sich, daß die Aristotelische Syllogistik auch im Rahmen moderner Logik-Systeme ohne Abstriche (cf.(1)) und ohne Zusätze (cf.(2)) durchgeführt werden kann. Die Lösung (1) wurde unabhängig auch von T. Smiley vorgeschlagen. Das von Corcoran vorgeschlagene System der "Natürlichen Deduktion" (Gentzen) gestattet weitgehend das Aristotelische Beweisverfahren isomorph wiederzugeben. Das hier vorgeschlagene System eines Sequenzenkalküls ist demgegenüber mit dem Vorgehen von P. Hispanus isomorph. Dies ist das klassische System mit den mnemotechnischen Ausdrücken Barbara, Celarent, Bamalip, etc.

J. FLUM: Charakterisierung von Logiken

Die folgende Charakterisierung der Quantoren "es gibt mindestens $\aleph_\alpha$ viele" im Bereich der monotonen Quantoren wird bewiesen.
Satz. Sei L eine reguläre Logik, $L_{\omega\omega} < L$ und $L = L_{\omega\omega}(Q)$ für einen monotonen Quantoren Q. Hat L die Löwenheimzahl $\aleph_\alpha$, so haben L und $L_{\omega\omega}(Q_\alpha)$ die gleiche Ausdrucksstärke. - Dabei wird von einer regulären Logik gefordert, daß sie die Relativierungseigenschaft besitzt.

J.-Y. GIRARD: The ordinal analysis of Π^1_2 - CA

I develop the analysis of Π^1_2 - CA on the model of what I did for Π^1_1 - CA. The crucial step is the analysis of the property

$$I^1(P,D) \neq \emptyset$$

where P is a predilator and D is a dilator.

L. GORDEEV: Generalized predicative Proof Theory

Presented:

Purely intuitionistic proofs of the theorems

Thm. $S \vdash A$ implies $T \vdash A$ for any arithmetic statement A

for various intuitionistic formal systems, particularly:

1. $T = HA$ and $S = EM_0 \upharpoonright + J$ (proposed and studied by S. Feferman) and / or S = strong formal version of P. Martin-Löf's Type Theory without universes.
2. $T = HA + TI(< \overline{\theta}\varepsilon_0 0)$ and $S = EM_0 + J$ and / or S = P. Martin-Löf's Type Theory with one universe (where $\overline{\theta}\varepsilon_0 0$ = ordinal of the theory $RA_{<\varepsilon_0}$).

All these results are new.

The proofs are based on suitable generalized version of the cut-elimination procedure, which has been precisely explained during this winter sem.-course at München University.

W. GUZICKI: Generic Families

A family of Cohen-generic reals (over a model M) is called a generic family if every finite sequence of members of it is a Cohen-generic sequence and if every forcing condition is extended by a real in the family.

The following problems are considered:

1. What are cardinals of maximal generic families?
2. How many pairwise incompatible (i.e. such that they do not belong to any model of the height of that of M) generic families exist?
3. Does a generic familiy generate a smallest (or minimal) model of ZFC ?

Problem 2 is answered in the best possible way by showing 2^{2^ω} pairwise incompatible generic families of power 2^ω. Problem 3 is answered negatively (pointly with K. Ciesielski) by showing that no smallest such model exists, though the case of minimal models is still open. However there is no such minimal model among Cohen-extensions of M.

C.G. JOCKUSCH, JR.: Recursion theory and the problems of Ramsey and van der Waerden

The work on van der Waerden's theorem is joint work with Iraj Kalantari. It is shown that if a recursively enumerable set A fails to contain arbitrary long arithmetic progressions, then for each k there is an r.e. set B_k disjoint from A which contains infinitely many pairwise disjoint progressions of length k . On the other hand, it can happen that A has no arithmetic progression of length 3 but no r.e. set disjoint from A contains arbitrarily long arithmetic progressions.

In regard to Ramsey's theorem, the early work of Specker and mine on definable homogeneous sets for recursive partitions of finite exponent was reviewed. Then extensions to recursively enumerable partitions as well as definable partitions of infinite exponent were considered, including work of Solovay, Simpson, Clote, Kechris, and Woodin.

A. KRAWCZYK: Non-duality of measure and category

Let A(c) "says" union of fewer then continuum many meager sets is meager.

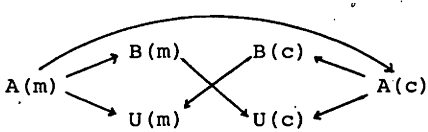
B(c) "says" [0,1] is not union of fewer then continuum many meager sets.

U(c) "says" every subset of [0,1] of cardinality less then continuum is meager.

Let A(m) , B(m) , U(m) be obtain from A(c) , B(c) , U(c) by interchanging the term "meager" by "null".

We investigate possible implications between this sentences.

We have the following diagram



Any combination of this propositions and its negations not contradictory with this diagram is consistently true excluded

$U(c) + \neg U(m) + \neg B(m)$.

Implication $A(m) \rightarrow A(c)$ was proved recently by T. Bortosipinski.

G. LONGO: One and half approaches to Recursion Theory in higher types

A very simple approach to Recursion Theory in the integer types is presented. The approach uses intermediate types. Its relation to Ershov's computable objects is sketched. By this the approach is compared to the Kleene-Kreisel countable functionals and to the HEO.
If there is enough time, an other way of looking at higher type functionals may be presented. This is inspired by the semantics of typed lambda-calculus and also corresponds to Ershov's approach via $\mathcal{f}$-spaces.

H. LUCKHARDT: A remark on boolean functions

Upper bounds for the complexity of SAT-decisions on characteristic subclasses are really noteworthy. Known results are discussed and somewhat extended. The general case seems to be wide open. Problem: Is there a polynomial transformation to normal forms preserving satisfiability and depending ondy on the number of variables?

K.L. MANDERS: Two projects in foundations

(1.) To give formal theories of mathematical reasoning which would render clear (within a restricted range of application) the utility of changes of setting, such as the algebraic constructions (adèle rings, $\mathcal{L}$-functions,...) used to study the natural numbers. The ability of existing logical theory to illuminate such matter was critically evaluated, and some approaches suggested.
(2.) A method was proposed to simultaneously explain existence postulates in mathematics and applicability of mathematics by analysing existence postulates in specific cases in terms of (i) informal intuitive data and (ii) methodological principles of theorizing, which together would force commitment to specifie mathematical structures. Examples discussed included the classical space-time postulates in mathematical physics.

K.L. MANDERS: Model Completions in Geometry

(i) $\mathrm{Th}\{\mathbb{P}^n(k): k \models T\}$ is model complete iff T is;
(ii) $\mathrm{Th}\{\mathbb{A}^n(k): k \models T\}$ is model complete iff T is,

where $\mathbb{P}^n(k)$ is projective n-space over k formalised as point set with collinearity relation, and $\mathbb{A}^n(k)$ is affine n-space over k, but formalised as point set with quaternary parallelism relation $ab \parallel cd$. If we formalise affine spaces with collinearity relation and restrict to fields of characteristic zero, as assumed henceforth, the relation $ab \parallel cd$ is never existentially definable, and model completeness fails. (Embed $\mathbb{A}^n(k)$ in $\mathbb{P}^n(k)$; any finite subset of $\mathbb{P}^n(k)$ can again be embedded in $\mathbb{A}^n(k)$.)

We characterize all embeddings among affine collinearity spaces, and show (writing $\mathbb{P}^n(T)$ for $\mathrm{Th}\{\mathbb{P}^n(k): k \models T\}$ and similarly $\mathbb{A}^n(T)$):

(iii) $\mathbb{A}^n(T)$ has the amalgamation property iff T has;
(iv) $\mathbb{P}^n(T^*)$ is the model completion of $\mathbb{A}^n(T)$ iff T* is the model completion of T.

D. MUNDICI: New Methods and results in Abstract Model Theory

We describe new techniques and results in Abstract Model Theory. We study in particular those topological spaces which naturally generalize the familiar Stone space over the set of complete theories in first-order logic.

P. PÄPPINGHAUS: Primitive recursive ptykes

For every finite type σ a category PT^σ has been defined by Girard. PT^0 is the category of ordinals. An object of $PT^{\sigma\to\tau}$ is a functor from PT^σ to PT^τ preserving direct limits and pull-backs. A morphism in the category $PT^{\sigma\to\tau}$ is a natural transformation between two such functors. With every closed term of Gödel's T one can associate a ptyx of corresponding type in a natural way, and one obtains a model of Gödel's T in this way. A natural assignment of ordinals to closed terms of type $0\to 0$ is now to assign $t(\omega)$ to t. We prove Girard's conjecture: $\{t(\omega)\,|\,t \text{ closed term of type } 0\to 0 \text{ in } T\} = \eta_0$, where

η_0 denotes the so-called Bachmann-Howard-ordinal. For "$\subseteq$" we use a valuation function for an extension of Gödel's T shown to be provably total in ID_1. For "$\supseteq$" we define the Bachmann hierarchy $(\varphi_\alpha)_{\alpha<\varepsilon_{\Omega+1}}$ in an extension of T.

A. PELC: Saturation of invariant and idempotent ideals on abelian groups

A proper non-trivial ideal I on a group G is invariant iff $Ag \in I$ for any $A \in I$, $g \in G$ and idempotent iff $\{g \in G : Ag^{-1} \notin I\} \in I$ for $A \in I$.

Theorem.

Let G be an abelian group of cardinality κ.

1. There is no σ-complete κ-saturated invariant ideal on G.
2. If κ carries a κ-complete κ^+-saturated ideal then there is a κ-complete κ^+-saturated invariant ideal on G.
3. There is no σ-complete prime idempotent ideal on G.
4. If $\lambda < \kappa$, λ is an uncountable regular cardinal and κ carries a κ-complete λ-saturated ideal then there is a κ-complete λ-saturated idempotent ideal on G.

A. PRESTEL: Definable closed subsets of $\mathbb{R}^n$

We gave a proof of the following

Theorem: Every definable (= semi algebraic) closed subset of $\mathbb{R}^n$ can be defined by some formula $\bigvee_i \bigwedge_j g_{ij}(\vec{X}) \geq 0$ where g_{ij} are finitely many polynomials from $\mathbb{R}[X_1,\dots,X_n]$. This important theorem from real algebraic geometry was independently proved by Recio (1977), Corte-Corte (1979), and Delzell (1980) using geometric arguments and the Tarski-Seidenberg Principle. Later in 1982, van den Dries gave a purely model theoretic proof. The presented proof essentially follows that of van den Dries. We also discussed an analog of this theorem over the p-adic numbers $\mathbb{Q}_p$. There, the resulting formula is of the type $\bigvee_i (f_i = 0 \wedge \bigwedge_j g_{ij} | h_{ij})$ with $f_i, g_{ij}, h_{ij} \in \mathbb{Q}_p[\vec{X}]$ and $a|b$ being defined by $v_p(a) \leq v_p(b)$.

W. RAUTENBERG: Interpolation and tableaus for intermediate logics

This is a short survey on recent results concerning interpolations in extensions of intuitionistic propositional logics (mainly due to L. Maximova). It is surprising that there are but 7 intermediate logics (including intuitionistic LI and classic LK) with interpolation. We add some polution results on extensions of minimal logic LJ obtained by tableaus. Besides for LJ we show interpolation for $LG = LJ(((0 \to p) \to 0) \to 0)$ (the smallest of extensions $L \supseteq LJ$ having the Glivenko property $p \in LK \Rightarrow \neg\neg p \in L$), and some other extensions of LJ.

M.M. RICHTER: The Theory of Superinfinitesimals

Starting from axiomatic nonstandard analysis using Nelson's axioms the theory of monads is revisited. Most of the results are due to B. Benninghofen. The monads are characterized as those external sets which satisfy some formula in which the standard predicate occurs at most negatively. Then the notion of a monad is extended to nonstandard filters (relatively to a standard family of filters). Using a general transfer theorem of Benninghofen the generalized monads are described syntactically. Applications are in the theory of monads as well as in analysis (e.g. the nontrivial part of the Bernoulli-L'Hospital rule, the Alaoghi-Bourbaki-theorem and the multiplication of distributions).

B. SCARPELLINI: Untere Schranken für Längen von Formeln

(I) Sei $\exp(x) = \exp_1(x) = 2^x$, $\exp_{k+1}(x) = \exp(\exp_k(x))$. Sei $\log(x)$ der Logarithmus zur Basis 2 mit der Konvention $\log(1) = 1$. Sei $\log_1(x) = \log(x)$, $\log_{k+1}(x) = \log(\log_k(x))$.
Definition: f gehört zur Funktionenklasse F_k genau dann, falls Zahlen $p > 1$, $a,b > 0$ existieren mit $f(x) = \exp_k(a(\log_k(x))^p+b)$. Wir setzen $F = \cup F_k$.
Bemerkung: Die Funktionen $f \in F$ wachsen stärker als alle Polynome, aber schwächer als alle Exponentialfunktionen $2^{\varepsilon n^\gamma}$ $\varepsilon > 0$, $\gamma > 0$.

(II) Wir betrachten Formeln aus der Sprache der Presburger Arithmetik. Eine Formel $(Ey_1)\ldots(Ey_t)L(x_1,\ldots,x_s, y_1,\ldots,y_t)$, L quantorenfrei, universell.

Satz: Es gibt eine natürliche Zahl $s > 0$ mit folgender Eigenschaft. Es gibt keine Funktion $f \in F$ die folgende Bedingung (C) erfüllt:

(C) Für jede Formel $(\forall x)L$ mit höchstens s freien Variablen und L existentiell existiert eine logisch äquivalente existentielle Formel G mit $\text{Länge}(G) \leq f(\text{Länge}((\forall x)L))$.

U. SCHMERL: Diophantine equations in weak systems of arithmetic

We study the following problem: Given an impossible diophantine equation $s(x_1..x_n) \neq t(x_1..x_n)$, is it possible to find out, how complex a derivation must be in order to be able to prove $\forall x_1..x_n[s(x_1..x_n) \neq t(x_1..x_n)]$. An answer is given for certain classes of diophantine equations such as linear equations, $x^2 = 2(y+1)^2$, $(x+1)^3 + (y+1)^3 = (z+1)^3$. The corresponding derivations (in the language of $0, S, +, \cdot$) use open induction and induction on restricted Σ_1^0-formulas.

P.H. SCHMITT: Model Theory of ordered Abelian groups

With every ordered Abelian group G we associated for each $n > 2$ a linear order $Sp_n(G)$ with additional unary predicates. The main part of the talk was spent on giving the precise definition of $Sp_n(G)$ and providing motivational background. The main theorem asserted that two ordered Abelian groups G,H are elementarily equivalent if and only if for all n $Sp_n(G)$ and $Sp_n(H)$ are elementarily equivalent. It was shown that this reduction is fruitful by quoting applications: criteria for model - and substructure completeness of elementary classes of ordered Abelian groups, decidability of the elementary and $L(Q_\alpha)$ - theory of the class of all Abelian ordered groups and the theorem that no complete theory of ordered Abelian groups does have the independence property.

R.I. SOARE: Recursively Enumerable Sets and Degrees

A survey lecture was given on recursively enumerable (r.e.) sets and degrees, including questions of algebraic structure, automorphisms, undecidability of elementary theory, and priority methods. More details can be found in the following papers by the author:

- Recursively enumerable sets and degrees, Bull. A.M.S., vol. 84, 1978, pp. 1149 - 1182.
- Tree methods in recursion theory and the 0'''- priority method, A.M.S. Summer Research Institute, July 1982, at Cornell University (to appear).
- Computational Complexity of recursively enumerable sets, Information and Control, vol. 52, 1982, pp. 8 - 18.
- Recursively enumerable sets and degrees, book to appear in Omega Series in Logic, Springer-Verlag.

A. SOCHOR: <u>Models of the alternative set theory</u>

(1) $\mathfrak{A} \models \mathrm{AST} \rightarrow (V, \in)_{\mathfrak{A}}$ is recursively saturated.

(2) $\mathfrak{M} \models ZF_F$ is resplendent iff $(\exists t \in M)(\forall \alpha$ nonstandard in $\mathfrak{M})$ $(\exists \mathfrak{A} \models \mathrm{AST})((V, \in)_{\mathfrak{A}} = \mathfrak{M}\ \&\ \mathfrak{A} \models (FN \subseteq \alpha\ \&\ t \cap FN = \{\varphi ; V \models \varphi\}))$.

(3) $\mathfrak{M} \models ZF_F$ countable & I nonstandard. $\mathfrak{M}$ is expandable to a model $\mathfrak{A}$ of AST with $FN_{\mathfrak{A}} = I$ iff $(\mathfrak{M}, I)$ is recursively saturated & $(I, +, ., P_{\mathfrak{M}}(I))$ is expandable to the third order arithmetic & $(\mathfrak{M}, I) \models (\forall a \in I)\Phi(a) \rightarrow \mathrm{Cons}(\overline{ZF_F} + \Phi(a))$.

(4) For $\mathfrak{A} \models \mathrm{AST}$ there is $\mathfrak{B} \succ \mathfrak{A}$ under the following conditions:

	$N_{\mathfrak{A}} = N_{\mathfrak{B}}$	$N_{\mathfrak{A}}$ cofinal in $N_{\mathfrak{B}}$	$N_{\mathfrak{A}}$ bounded in $N_{\mathfrak{B}}$
$FN_{\mathfrak{A}} = FN_{\mathfrak{B}}$	$\mathfrak{A}$ countable & $\mathfrak{A} \models$ strong schema of choice	/	countable
$FN_{\mathfrak{A}} \subset_{\ell} FN_{\mathfrak{B}}$	/	$\mathfrak{A}$ countable	$\mathfrak{A}$ countable or $\mathfrak{A} \models$ schema of choice or $FN_{\mathfrak{A}}$ standard
$FN_{\mathfrak{A}}$ cofinal in $FN_{\mathfrak{B}}$ & & $FN_{\mathfrak{A}} \neq FN_{\mathfrak{B}}$	/	$\mathfrak{A}$ countable & $FN_{\mathfrak{A}}$ nonstandard	
none of above	/		$FN_{\mathfrak{A}}$ nonstandard

A.S. TROELSTRA: Some examples of truly intuitionistic model theory

Several examples of metamathematical results on intuitionistic systems obtained by classical semantical methods are discussed. It is shown how the model-theoretic treatment can easily be made constructive in these cases. The examples are 1^o) addition of classes to first-order theories is conservative; 2^o) addition of descriptors to logic with existence predicate is conservative; 3^o) Scott's theorem on the relation between ordinary predicate logic and logic with existence predicate; 4^o) the characterization of the f - free fragment of the theory of a single Skolem - function f .

The basic tools are: the Lindenbaum construction, constructive completion of Heyting algebras,

S.S. WAINER: Π^1_2 approach to hierarchies

The "slow-growing" hierarchy $G_x(0) = 0$, $G_x(\alpha + 1) = G_x(\alpha) + 1$, $G_x(\lambda) = G_x(\lambda_x)$, extended to a maximal set Ω of abstract ordinal notations, provides a natural assignment of ordinals to Kleene-computations in the maximal type-structure over N . The motivation comes from Girard's Π^1_2-logic.

More precisely, each computation over N occurs as the collapse, under G , of an identical computation over Ω . Thus recursion-theoretic hierarchies defined over Kleene's O occur as the collapse of corresponding hierarchies defined over a complete Π^1_2-set of ordinal notations over Ω . In particular, the Grzegorczyk hierarchy occurs as the collapse of the Bachmann hierarchy, and we can then read off results of Girard, assigning large ordinals to small hierarchies.

V. WEISPFENNING: Integer-like rings

A Z-group is a model of additive (= Presburger) arithmetic $Th(Z^+)$, where $Z^+ = (\mathbb{Z}, 0, 1, +, -)$. The good model theoretic properties of Z-groups suggest a study of Z-rings as weak models of arithmetic: A Z-ring is a ring (= comm. ring with 1) R whose additive part R^+ is a Z-group. In terms of the $\mathbb{Z}$-adic topology, Z-rings can be characterized as rings of characteristic 0 containing Z as pure subgroup and dense subring. Let $D = \cap nR$ denote the divisible ideal of R ; R is reduced if $D = \{0\}$; $\hat{R}$ is the $\mathbb{Z}$-adic completion of R . Then the following properties carry over from Z-groups:

Prop. Let R be a Z-ring.

(i) If $D \supseteq I$ is an ideal of R , then R/I is a Z-ring.

(ii) If S is a divisible ring of char 0, then $R \times S$ is a Z-ring.

(iii) If R is reduced then there is a unique embedding of R into $\hat{\mathbb{Z}}$.

(iv) The only reduced, Z-ad. complete Z-ring is $\hat{\mathbb{Z}}$.

Cor. Let G be a Z-group.

(i) G has at most one expansion to a Z-ring iff G is reduced.

(ii) G has at least one expansion to a Z-ring, provided G/D is cyclic, or G is Z-adically complete, or G is resplendent.

Our main result is:

Theorem. There are infinitely many pairwise inequivalent primitiv recursively decidable nonreduced Z-rings and infinitely many pairwise inequivalent decidable reduced Z-rings, among them $\hat{\mathbb{Z}}$.

The method of proof is adapted from the author's proof for the decidability of the adele-ring of an algebraic number field, using the fact that $\hat{\mathbb{Z}}$ is isomorphic to ΠO_p , O_p p-adic integers. The second part also applies decidability results of v.d. Dries and Ersov on multiply valued fields.

We give an example of a diophantine equation unsolvable in $\mathbb{Z}$ but solvable in $\hat{\mathbb{Z}}$.

Problem: Is there a decidable Z-integral domain?

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