MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 18/1983

Probability measures on groups

24.4. bis 30.4.1983

Die siebente Tagung über Wahrscheinlichkeitsmaße auf Gruppen stand wieder unter der Leitung der Herren H. Heyer (Tübingen) und L. Schmetterer (Wien). Es nahmen insgesamt 40 Mathematiker teil; von diesen stammten 29 aus 9 Staaten des Auslands. Neben 3 Übersichtsvorträgen, in denen neueste Forschungsergebnisse über Null-Eins Gesetze der Wahrscheinlichkeitstheorie, Irrfahrten auf homogenen Räumen sowie Konvergenzsätze für Wahrscheinlichkeitsmaße auf Gruppen und Halbgruppen zusammengefaßt wurden, fanden 28 Einzelvorträge statt. Schwerpunkte dieser Ausführungen waren unter anderem die Anwendung der Theorie der Hypergruppen auf wahrscheinlichkeitstheoretische Fragestellungen, stabile und semistabile Maße auf Lie-Gruppen, algebraische Eigenschaften der Menge der Wahrscheinlichkeitsmaße auf einer Gruppe, starke Gesetze der großen Zahlen sowie ausgewählte Probleme der harmonischen Analyse lokalkompakter Gruppen, die in engem Zusammenhang mit der strukturellen Wahrscheinlichkeitstheorie stehen. Physikalisch ausgerichtete Arbeiten über zufällige Schrödingeroperatoren und nicht kommutative Wahrscheinlichkeitstheorie auf freien Algebren zeigen das breit gefächerte Interesse auf, das für die Thematik der Tagung besteht. Lebhaft genutzt wurde die Möglichkeit der Diskussion mit Mathematikern aus dem fernen Ausland, die trotz finanzieller Restriktionen ihren Weg nach Oberwolfach gefunden hatten. Die wissenschaftliche und persönliche Atmosphäre, die nicht zuletzt durch eine begrenzte Teilnehmerzahl gewährleistet wurde, trug dazu bei.

Zum Ende dieses Jahres ist die Herausgabe eines Proceedingsbandes über diese Tagung geplant, der die im folgenden nur kurz zusammengefaßten Vorträge ausführlicher wiedergeben wird.

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Vortragsauszüge

W. BLOOM:

Translation bounded measures and the Orlicz-Paley-Sidon theorem

Let G denote a locally compact abelian group with character group Γ. The space of complex Radon measures on G will be written as M(G). A measure $\mu \in M(G)$ is said to be translation (or shift) bounded if $\{|\mu|(x+K) : x \in G\}$ is bounded for each compact $K \subset G$. Such measures are naturally in the study of convolution. It is known that μ is translation bounded if and only if μ is convolvable with every bounded Radon measure on G. Here we study the space of translation bounded measures, which can be realized as the dual of the amalgam space (C, l^1), and use this description to prove the following: A measure μ is translation bounded iff every $f \in L^2(\Gamma)$ with compact support satisfies $\hat{f} \in L^2(\mu)$. This extends a result previously obtained by V. Ja. Lin for the Euclidean space $\mathbb{R}^n$. We also consider similar results where f is restricted to belong to the family of characteristic functions; for the real line $\mathbb{R}$ our result is

Theorem: Let $\Omega \subset \mathbb{R}$ be compact with nonvoid interior and suppose that $\hat{\xi}_\Lambda \in L^1(\mu)$ for every Borel set $\Lambda \subset \Omega$. Then $\sum_{n=-\infty}^{\infty} (\mu[n,n+1[\,)^2 < \infty$.

The Orlicz-Paley-Sidon theorem obtaines the same conclusion, but with the assumption that $\hat{f} \in L^1(\mu)$ for every continuous function supported in Ω.

P. BOUGEROL

Remarques sur la généralisation du théorème de continuité de Paul Lévy aux groupes localement compacts

Let μ_n be a sequence of probability measures on a locally compact group G such that T_{μ_n} converges weakly to an operator D(T) for every representation T of the reduced dual $\hat{G}_2$ of G. Then μ_n converges to a bounded measure ν of total mass less or equal to 1. The aim is to look for conditions on D(T) which imply that ν is a probability measure.

If G is amenable and D(T) is continuous at $\mathbb{1}$ in a certain sense, ν is a probability measure. If G is not amenable, the convergence of T_{μ_n} to D(T) for all T in $\hat{G}$ and the continuity of D(T) in $\mathbb{1}$ are or are not a sufficient condition for ν to be a probability measure according to the structure of G.

T. BYCZKOWSKI

Decomposition of convolution semigroups induced by processes on locally compact groups

Let G be a second countable locally compact group. We consider a continuous semigroup $(\mu_t)_{t>0}$ of probability measures on $(G^T, \mathcal{B}^T)$ where T is an arbitrary set and $\mathcal{B}^T$ is the product of Borel σ-algebras. The following theorem holds:

Theorem: Assume that H is a normal $\mathcal{B}^T$-measurable subgroup of G^T such that $\mu_t(H) > 0$ for all $t>0$. Then the corresponding generating functional A of $(\mu_t)_{t>0}$ is of the form $A = c(\nu-\delta_e) + A^H$

where $c\geq0$, ν is a probability measure such that $\nu(H)=0$ and A^H is the generating functional of a semigroup of probability measures concentrated on H.

As a corollary we obtain:

Corollary: Let $(\mu_t)_{t>0}$ be a Gaussian semigroup on G^T. Assume that H is a $\mathcal{B}^T$-measurable subgroup of G^T such that $\mu_t(H)>0$ for all $t>0$. Then $\mu_t(H)=1$ for all $t>0$.

The main tool used in the proofs is the Trotter Approximation theorem. Above results are based on a joint paper with A. Hulanicki (to appear in Ann. Prob.).

Y. DERRIENIC

On recurrence and positive harmonic functions for some Markov chains on the real line

The following result is proved:

Let $P(x,dy)$ be a Markov kernel on $\mathbb{R}$. Assume that

1) P is a Feller kernel, that is $Pf(x) = \int f(y)\, P(x,dy)$ is continuous for every continuous bounded f;

2) P is irreducible with respect to open sets, that is $\sum_{n=0}^{\infty} P^n(x,U) > 0$ for every x and every open set U;

3) for every x $P(x,dy)$ is centered at x, that is $\int y\, P(x,dy) = x$;

4) for every x the support of $P(x,dy)$ is compact and the diameter $d(x)$ of this support satisfies the inequality $d(x)\leq|x|+K$ for some constant K.

Then the Markov chain defined by P is recurrent in open sets, that is $P_x(\overline{\lim}\,[X_n\in U]) = 1$ for every x and every open set U.

E. DETTWEILER

Diffusions on Banach spaces

We study the following form of stochastic differential equations on a Banach space E. Suppose that $(Z,\mathfrak{Z},\rho)$ is a σ-finite measure space, ξ is a Gaussian random field on $\mathbb{R}_+\times Z$, and $a: \mathbb{R}_+\times E \to E$, $b: \mathbb{R}_+\times Z\times E \to E$ are two functions which satisfy certain additional conditions (Lipschitz condition etc.). Then the stochastic integral equation

$$X_t = x_0 + \int_0^t a(s,X_s)\,ds + \int_0^t\int_Z b(s,z,X_s)\,\xi(ds,dz)$$

has a (unique) solution if (and only if) the Banach space E has an equivalent 2-uniformly smooth norm. The solution is a Markov process whose infinitesimal generator A has the form

$$Af(t,x) = \frac{d}{dt}f(t,x) + f'_x(t,x)(a(t,x)) + \frac{1}{2}\int_Z f''_x(t,x)(b(t,z,x)^{\otimes 2})\,\rho(dz)$$

for all $f\in C^2(\mathbb{R}_+\times E)$. These results are obtained from inequalities for E-valued martingales which are valid if and only if E has an equivalent 2-uniformly smooth norm (which was proved by Pisier).

T. DRISCH

Stable laws on the Heisenberg group

For characterizing stable probability measures on a locally compact group G, the new concept of Hazod requires the determination of the continuous automorphism groups $\tau:\mathbb{R}_+^* \to \mathrm{Aut}(G)$. If G is a simply connected nilpotent real Lie group, and if τ is contracting, then A is the generating distribution of a stable semigroup iff $xA = \tau_x A$ for all $x>0$.
In the case of the n-th Heisenberg group $\mathbb{H}_n = \mathbb{R}^{2n}\oplus\mathbb{R}$, the contracting τ are of the form

$$\tau_x = \mathrm{inn}(v - x^{m+M}v)\circ\alpha_{x^M}\circ\delta_{x^m} \qquad (x>0),$$

where $v\in\mathbb{R}^{2n}$, $m\in\mathbb{R}_+^*$, $M\in Sp(2n,\mathbb{R})^\wedge$, $\mathrm{spec}(M)\subset\{z\in\mathbb{C}: \mathrm{Re}\, z > -m\}$, and α_s denotes dilatation by s, δ_A rotation of $\mathbb{R}^{2n}$ by the symplectic matrix A, and inn(w) the inner automorphism by w. If, moreover, $m\geq\frac{1}{4}$ and M fulfilles certain spectral conditions, then τ allows stable laws (and conversely). Finally the (more complicated) noncontracting case has been described.

B.-J. FALKOWSKI

An Analogue of the Lévy-Khintchin Formula on $SL(2,\mathbb{C})$

The following theorem is presented

Theorem: Let G be a connected, semisimple Lie group. Then for every pair (δ,b)

consisting of a continuous 1-cocycle δ and a continuous function $b: G \to \mathbb{R}$ satisfying $\operatorname{Im} \langle\delta(g_2),\delta(g_1^{-1})\rangle = b(g_1)+b(g_2)-b(g_1g_2)$ we obtain an infinitely divisible positive definite function f given by $f(y) := \exp(-\frac{1}{2}\langle\delta(y),\delta(y)\rangle - ib(y))$ (*).

Conversely: Given an I.D.P. function f on G there exists a pair (δ,b) as described above, such that (*) is satisfied. Moreover b is uniquely determined and δ up to unitary equivalence.

This theorem in connection with the known 1- and 2-cohomology of $SL(2,\mathbb{C})$ is then exploited to obtain a complete description of I.D.P. functions on $SL(2,\mathbb{C})$. This gives a non-commutative analogue of the classical Lévy-Khintchin formula.

P. FEINSILVER

Bernoulli Processes on $\mathbb{R}^N$

The Brownian martingale $e^{zb(t)-tz^2/2}$ is interesting as it is the generating function of Hermite polynomials $H_n(b(t),t)$. These are analogous to powers x^n as H_n are iterated integrals of the white noise db. The question is to see what processes with stationary indepent increments have the property that the corresponding martingale $e^{zw(t)-tL(z)}$ has an expansion in iterated stochastic integrals. With the change of variable $z \to V(z)$ with inverse $U(v)=z$ we have $e^{w(t)U(v)-tL(U(v))}$ providing:

1) the iterated integrals are in fact functions of $x=w(t)$ and t
2) the generator L satisfies the partial differential system

$L_{jk} = \varepsilon_{jk} + a_{jk}^{\lambda}L_{\lambda} + B_{jk}^{\lambda\mu}L_{\lambda}L_{\mu}$, ε,a,B constants, suscripts denoting partial derivatives.

The equations with general smooth coefficients are interesting in themselves as the coefficients may be interpreted as arising from a differtial geometric structure with ε as a (generalized) metric form, a as connection form (or gauge field) and B as curvature (or Yang-Mills field).

G. FORST

Self-decomposability on $\mathbb{R}$ and $\mathbb{Z}$

The set $L(\mathbb{R})$ of self-decomposable probability measures on $\mathbb{R}$ is studied in terms of characteristic functions using a certain differential operator and its inverse. In particular a natural bijection onto $L(\mathbb{R})$, introduced by S. J. Wolfe, is interpreted via these operators.

In a similar way a bijection of certain sets of probability measures on $\mathbb{Z}$ is discussed, and this leads to a notion of discrete self-decomposability on $\mathbb{Z}$ which extends the notion of discrete self-decomposability on $\mathbb{Z}_+$ as defined by Steutel and van Harn.

L. GALLARDO

The rate of escape of the random walks associated with Gegenbauer's polynomials

Let X_n be a random walk with $\mathbb{N}$ as a state space and with stationnary transition probabilities given by $p(i,i+1)=p_i$, $p(i,i-1)=1-p_i$ for every i. We are interested in the case $p_i=\frac{1}{2}(1+\frac{\lambda}{i}+o(i))$ with $\lambda\in\mathbb{R}$. It is known that X_n is transient iff $\lambda>\frac{1}{2}$. In the particular case $p_i=\frac{1}{2}(1+\frac{\lambda}{i+\lambda})$ and $\lambda>\frac{1}{2}$ we have the following result:
Let $g(n)>0$ be a monotone sequence and $g(t)$ the function such that $g(t)=g(n)$ for $n\leq t<n+1$. Then

$$\liminf_{n\to+\infty} \frac{X_n}{g(n)\sqrt{n}} \overset{a.s.}{=} 0 \text{ or } +\infty \quad \text{according as} \quad \int_0^\infty t^{-1}(g(t))^{2\lambda-1}\,dt \;=+\infty \text{ or } <+\infty .$$

For the proof we show that if we provide $\mathbb{N}$ with an hypergroup structure deriving from Gegenbauer's polynomials, the previous chain is a process with indepent increments with respect to the hypergroup convolution structure.
If we suppose $p_i=\frac{1}{2}(1+\frac{\lambda}{i}+o(\frac{1}{i}))$ we get the following theorem:
If there exists $\varepsilon>0$ such that the integrals $\int_0^\infty t^{-1}(g(t))^{2\lambda+\varepsilon-1}dt$ and $\int_0^\infty t^{-1}(g(t))^{2\lambda-\varepsilon-1}dt$ are at the same time $=+\infty$ or $<+\infty$ then

$$\liminf_{n\to+\infty} \frac{X_n}{g(n)\sqrt{n}} \overset{a.s.}{=} 0 \text{ or } +\infty \quad \text{according as} \quad \int_0^\infty t^{-1}(g(t))^{2\lambda-1}\,dt \;=+\infty \text{ or } <+\infty.$$

W. HAZOD

Zur Definition stabiler Maße auf lokalkompakten Gruppen

Seien G eine lokalkompakte Gruppe sowie $(\tau_t)_{t>0}$ eine Gruppe von Automorphismen mit $\tau_t\tau_s = \tau_{ts}$ für alle t,s>0. Eine Faltungshalbgruppe $(\mu_t)_{t\geq 0}$ mit $\mu_o=\varepsilon_e$ heißt stabil bezüglich (μ_t), falls $\tau_t(\mu_s) = \mu_{ts}$ für alle t,s>0.
Dies ist dazu äquivalent, daß für die erzeugende Distribution A von (μ_t) $\tau_t(A) = tA$ für alle t>0 gilt. Eine Halbgruppe heißt stabil im weiteren Sinne, falls für alle t>0 $\tau_t(A) = tA + X_t$ (mit passendem primitivem X_t).
Dann erhält man (für nicht notwendig kontrahierende (τ_t)):
1) Der Erzeuger einer stabilen Halbgruppe ist auf jede offene τ_t-invariante Umgebung der Eins konzentriert.
2) Im Falle einer Lie-Gruppe ist der Erzeuger auf die Zusammenhangskomponente der Einheit konzentriert, weiter gilt:
3) Es gibt eine τ_t-invariante Borel-Menge B, auf die A konzentriert ist, sodaß

(τ_t) auf B kontrahierend wirkt

4) Für einfach zusammenhängende Lie-Gruppen gibt es zwischen stabilen Erzeugern und den (bezüglich $d\tau_t$) operatorstabilen erzeugenden Distributionen auf der Lie-Algebra von G eine eineindeutige Beziehung.

A. JANSSEN

A Survey about Zero-One Laws for Probability Measures on Groups

We consider probability measures P on a group X , measurable subgroups $G \subset X$ and 0-1 laws in the sense that either P(G) = 0 or 1.

The whole story began in 1951 when Cameron and Graves proved a 0-1 law for Wiener measure on C[a,b] and measurable r-modules $G \subset C[a,b]$. Later around 1970 0-1 laws were proved for further Gaussian processes. This leads to the 0-1 law for Gaussian measures and arbitrary subgroups of separable Banach spaces X. In the next years, these result were extended to stable measures on Banach spaces.

The recent devel opment can be divided in three directions:

1) The Polish school proved 0-1 laws for Gaussian measures on groups,
2) 0-1 laws for semistable distributions,
3) 0-1 laws for continuous convolution semigroups.

Furthermore some applications for the sample space of the underlying process are considered.

E. KANIUTH

*-Regularity of Locally Compact Groups

Let G be a locally compact group and $\hat{G}$ the dual space of G. G is called *-regular if for every closed subset E of $\hat{G}$ and $\pi \in \hat{G} \setminus E$ there exists $f \in L^1(G)$ such that $\pi(f) \neq 0$ and $\rho(f)=0$ for all $\rho \in E$. *-regularity is important for the ideal theory in $L^1(G)$ in so far as it is equivalent to that the canonical mapping Prim $C^*(G) \to$ Prim $L^1(G)$ between primitive ideal spaces is a homeomorphism. In recent years the problem of finding classes of *-regular groups as well as criteria for *-regularity has been attacked. The results which have been obtained up to now and examples are presented.

F. KINZL

Gleichverteilung auf lokalkompakten Halbgruppen

Ausgehend von der klassischen Theorie der Gleichverteilung einer Folge in einer

kompakten Gruppe werden drei Arten von Gleichverteilung eines Netzes (μ_α) von Wahrscheinlichkeitsmaßen auf Halbgruppen studiert: (μ_α) heißt asymptotisch gleichverteilt bzw. uniform asymptotisch gleichverteilt, falls für alle Wahrscheinlichkeitsmaße ν auf einer lokalkompakten Halbgruppe S gilt $|\nu*\mu_\alpha-\mu_\alpha| \to 0$ bzw. $(\nu*\mu_\alpha-\mu_\alpha)(f) \to 0$ für jede links gleichmäßig stetige Funktion f auf S. Schließlich heißt (μ_α) schwach as. gleichverteilt, falls $\sigma*\mu_\alpha$ as. gleichverteilt ist, wobei σ die Gesamtheit der links absolut stetigen Maße durchläuft.
Es werden einige Charakterisierungen für asymptotische und uniform as. Gleichverteilungen angegeben. Desweiteren wird auf interessante Zusammenhänge zwischen der Existenz von asymptotisch gleichverteilten Netzen und der Struktur der Halbgruppe eingegangen. Für as. glvt. Folgen (μ_n) würde gezeigt, daß die Folge der Normen der links gleichmäßig stetigen Anteile von μ_n gegen 1 konvergiert.

J. KISYŃSKI

On the Formula of N. Ikeda and S. Watanabe concerning the Lévy Kernel

Let $(p_{t,x})$ be a weakly continuous Markov transition function on a compact metric space E. Suppose that for each $x \in E$ there is a Radon measure n_x on $E \setminus \{x\}$ such that $\lim_{t\to 0} \frac{1}{t} p_{t,x}(f) = n_x(f)$ uniformly in $x \in E \setminus U$ for each open $U \subsetneq E$ and each $f \in C_c(U)$. Let X_t be an E-valued Markov process governed by $(p_{t,x})$ with right continuous trajectories and left limits.
If $f \in C(E^2)$ has support outside the diagonal of E^2, τ is a stopping time and $\lambda>0$, then we get

$$E_x \sum_{0<s\leq\tau} e^{-\lambda s} f(X_{s-0},X_s) = E_x \int_0^\tau e^{-\lambda s}(nf)\circ X_s \, ds$$

where $(nf)(x) = \int_{E\setminus\{x\}} f(x,y)\, n_x(dy)$.

This is the formula proved first by N. Ikeda and S. Watanabe (by means of the potential theory) which gives the probabilistic interpretation of the Lévy kernel n_x. By standard arguments based on the monotone class theorem the formula may be extended to all non-negative Borel functions on E^2 vanishing on the diagonal of E^2.

J. LACROIX

The Random Schrödinger Difference Operator in a Strip

Let H be the operator defined on the Hilbert space $\mathcal{H}$ of sequences $V = (V_n)_{n\in\mathbb{N}}$ $V_n \in \mathbb{C}^d$ by the formula $(HV)_n = -V_{n-1}-V_{n+1}+A_nV_n$ where $V_{-1}=0$ and A_n is a given random sequence of real symmetric matrices with common law with a density. It is known that for d=1 the spectrum is almost surely pure point so $\mathcal{H}$ has a basis of eigenvectors. When d>1 the same result holds but the proof is much more complicated and only probabilistic methods can work. We show that the problem reduces to the spectral properties of compact operators associated with the action of the symplectic group $Sp(d,\mathbb{R})$ on some Lagrangian flag manifold. The essential tool is the study of such operators, when the cocycle is given by a Poisson kernel and the theory of continuous (or analytic) perturbations of compact operators added to the fact that the lowest positive Lyapounoff exponent of the random product considered is strictly positive.

R. LASSER

On the Lévy-Hinčin Formula for Commutative Hypergroups

There are studied negative definite functions on a compact hypergroup K. It is shown that quadratic forms on K are negative definite. For commutative hypergroups having a dual hypergroup and satisfying a certain property (F) a Lévy-Hinčin formula holds: if (μ_t) is a continuous convolution semigroup, then

$\hat{\mu}_t(\alpha) = \exp(-t\psi(\alpha))$, where ψ is a negative definite function of the form

$$\psi(\alpha) = c + il(\alpha) + q(\alpha) + \int_{K\setminus\{e\}} (1-\mathrm{Re}\ \alpha(x))\, d\eta(x) ,$$

$$c = \psi(1), \quad l = \mathrm{Im}\ \psi , \quad q(\alpha) = \lim_{n\to\infty}\left[\frac{p_\alpha^n(\psi)}{n^2} + \frac{p_\alpha^n x p_{\bar\alpha}^n(\psi)}{2n}\right]$$

For certain hypergroups $(\mathbb{N},\cdot)$ corresponding to orthogonal polynomial sequences, the quadratic forms are determined explicitely.

E. LE PAGE

Empirical Distribution of the Eigenvalues of Random Jacobi Matrices

Let $(X_n)_{n\in\mathbb{Z}}$ be a sequence of independant random variables of law μ with compact support; then consider

a) the random operator $H(\omega)$ defined on $l^2(\mathbb{Z})$ by $(H(\omega)u)_n = -u_{n+1}-u_{n-1}+X_n(\omega)u_n$. This operator is selfadjoined and bounded. Let E_t the resolution of the identity of $H(\omega)$ and $N(t) = E(\langle E_t e_0, e_0\rangle)$,

b) the Jacobi matrices $J_L(\omega) = \begin{pmatrix} X_{-L} & -1 & & (0) \\ -1 & X_{-L+1} & \ddots & \\ & \ddots & \ddots & -1 \\ (0) & & -1 & X_L \end{pmatrix}$

whose eigenvalues are $(\lambda_i^L(\omega))_{0<i\leq L+1}$ and put

$N_L(t) = \frac{1}{2L+1}\sum_{i=0}^{2L} 1_{[\lambda_i^L\leq t]}$. Then we have

1) $\lim_{L\to\infty} \sup_{t\in\mathbb{R}} |N_L(t)-N(t)| = 0$ a.s.

2) if μ has continuous density the sequence of processes $Y_L(t) = \sqrt{2L+1}(N_L(t)-N(t))$ converges to a gaussian process $G(t)$ which is non degenerated, centered and with continuous paths.

G. LETAC

La fonction de Minkowski-Denjoy et les promenades aléatoires dans SL(2,ℝ)

If a_j $(j\geq 0)$ are entiers with $a_o\geq 0$ and $a_j\geq 1$ for all $j\geq 1$, put $x = (a_o,a_1,a_2,\dots) = a_o+\cfrac{1}{a_1+\cfrac{1}{a_2+\dots}}$. The Minkowski-Denjoy function on $]0,\infty[$ can be defined as follows:

$\chi_\alpha(x) = \alpha^{a_o} - \alpha^{a_o}\beta^{a_1} + \alpha^{a_o+a_2}\beta^{a_1} - \alpha^{a_o+a_2}\beta^{a_1+a_3} +-\dots$ with $0<\alpha<1$ and $\beta=1-\alpha$.

If (Y_{2k},Y_{2k+1}) is a sequence of independant $\mathbb{N}^2$-valued random variables of common law γ and if $A_k = \begin{pmatrix} Y_{2k}Y_{2k+1} & Y_{2k} \\ Y_{2k+1} & 1 \end{pmatrix}$ then $Z = \lim_{n\to\infty} A_oA_1\dots A_n x = Y_o+\cfrac{1}{Y_1+\cfrac{1}{Y_2+\dots}}$

(where $\begin{pmatrix} a & b \\ c & d \end{pmatrix}(x) = (ax+b)/(cx+d)$).

The distribution of Z is calculated explicitely representing

$G(a_o,a_1,\dots) = P[Z>(a_o,a_1,\dots)]$ by means of $\phi(x,y) = \sum_{a_o,a_1=o}^{\infty}\gamma(a_o,a_1)x^{a_o}y^{a_1}$:

There exist $A,B:\mathbb{N}^2 \to [0,1[$ such that

$G(a_o,a_1,\dots) = A(a_o,a_1)G(a_2,a_3,\dots) + B(a_o,a_1)$;

this implies the singularity of the distribution of Z.

In the special case $P[Y_o=1,Y_1=0]=\alpha$ and $P[Y_o=0,Y_1=1]=1-\alpha$ we have $P[Z>x] = \chi_\alpha(x)$.

A. MUKHERJEA

Certain results in probability theory and semigroups in the context of probability measures: a survey

In this survey, an interplay between simple semigroup results, results in non-homogeneous Markov chains and convolution iterates of probability measures on semigroups is presented. Among other things, an application of a semigroup result of I. Csiszár (Z.W., 1966) to the tail σ-field structure of non-homogeneous Markov chains with finite states is shown. A further result is the following theorem (Högnäs,Mukherjea):

a) Let f be a bounded real function on a lattice L_d, let (X_i) be an i.i.d. sequence of random variables on L_d with $E(X_i)=0$ and let $S_n=X_1+X_2+\dots+X_n$. Assume that there exist reals α and β such that

$$\alpha \leq \varliminf_{N\to\infty} \frac{1}{N} \inf_{M\in\mathbb{Z}} \sum_{n=M+1}^{M+N} f(n) \leq \varlimsup_{N\to\infty} \frac{1}{N} \sup_{M\in\mathbb{Z}} \sum_{n=M+1}^{M+N} f(n) \leq \beta \text{ , then}$$

$$\alpha \leq \varliminf_{N\to\infty} \frac{1}{N} \inf_{M\in L_d} \sum_{n=1}^{N} f(S_n+M) \leq \varlimsup_{N\to\infty} \frac{1}{N} \sup_{M\in L_d} \sum_{n=1}^{N} f(S_n+M) \leq \beta \text{ almost surely.}$$

b) Let f be a bounded real function on ℝ, let (X_i) be an i.i.d. sequence of real random variables with mean 0 and put $S_n=X_1+\dots+X_n$. Assume that there exist reals α and β such that $\alpha \leq \varliminf_{N\to\infty} \frac{1}{N} \inf_{M\in\mathbb{R}} \int_M^{M+N} f(x)dx \leq \varlimsup_{N\to\infty} \frac{1}{N} \sup_{M\in\mathbb{R}} \int_M^{M+N} f(x)dx \leq \beta$.

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Then for almost all x outside a Lebesgue null set

$$\alpha \leq \varliminf_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(S_n+x) \leq \varlimsup_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} f(S_n+x) \leq \beta \quad \text{almost surely .}$$

If the distribution of X_i is non-singular then we can put 0 for x above.

S. PINCUS

Strong Laws of Large Numbers for Products of Random Matrices

Theorem 1: Let A_i be i.i.d. 2x2 random matrices, $A_i=\begin{pmatrix} t & o \\ o & t^{-1}\end{pmatrix}$ with probability p, $= B$ with probability 1-p. Let $T_n := A_n \dots A_1$ and $\alpha(A,B,p) := \lim_{n\to\infty} \frac{1}{n} \ln \|T_n\|$ a.s.

a) $\lim_{t\to\infty} (\alpha(A,B,p) - p \ln A) = p(1-p) \sum_{n=1}^{\infty} p(1-p)^{n-1} \ln |(B^n)_{1,1}|$

b) Suppose B has real eigenvalues or complex eigenvalues that have argument commensurable with π, and that the right hand side of a) is finite.

Then there exists C such that for all t sufficiently large

$|\alpha(A,B,p) - p \ln t - p(1-p) \sum_{n=1}^{\infty} p(1-p)^{n-1} \ln |(B^n)_{1,1}| \,| < Ct^{-2}$.

Theorem 2: Let T_1, T_2 be two linear transformations of $\mathbb{R}^2 \to \mathbb{R}^2$, det T_1=det T_2= 1, with distinct eigenvalues $\lambda_{11}, \lambda_{12}$ resp. $\lambda_{21}, \lambda_{22}$ and $|\lambda_{11}|>|\lambda_{12}|$, $|\lambda_{21}|>|\lambda_{22}|$. Assume that no eigenvector of T_1 is one of T_2. Let v_{ij} be the eigenpoint of λ_{ij} for T_i on $\mathbb{P}^1$. Then $\mathbb{P}^1 \setminus \{v_{11}, v_{21}\}$ has two connected components U,V. Suppose $v_{12}, v_{22} \in U$ and that $T_1(V \cup \{v_{11}\} \cup \{v_{21}\}) \cap T_2(V \cup \{v_{11}\} \cup \{v_{21}\}) = \emptyset$. Then for each stationary measure π for μ , $\pi(V \cup \{v_{11}\} \cup \{v_{21}\}) = 1$ and π is continuous singular with respect to the Haar measure on $\mathbb{P}^1$.

H. RINDLER

Invariant extensions of Dirac-measure and asymptotical central functions

(joint work with V. Losert)

Let G be a locally compact group; for $f \in L^1(G)$ we put $\|f\| = \int_G |f|$, $\tau_x f = \delta_x * f * \delta_{x^{-1}}$. A net $u_\alpha \in L^1(G)$ is called asymptotical central (as.c.) if $\lim \|\tau_x u_\alpha - u_\alpha\| = 0$ for all $x \in G$. Consider the following assertions

(A) there exists an approximate unit in $L^1(G)$ which is asymptotical central.

(B) there exist (u_α) asymptotical central with $u_\alpha > 0$ and $\|u_\alpha\| = 1$.

Theorem 1: (A) holds iff δ_e admits an extension to a τ-invariant mean on $L^\infty(G)$.

Theorem 2: If G is amenable then (A) holds.

Theorem 3: If (A) holds then G_o, the connected component of G, is amenable.

Theorem 4: If G/G_o is compact then the following assertions are equivalent:

(i) G is amenable,

(ii) (B) holds,

(iii) (C) holds.

I. Z. RUZSA

Infinite Convolutions via Representations

Let $\mu_1, \mu_2, \ldots$ be tight probability measures on a topological group G and consider the convolutions $\nu_n = \mu_1 * \mu_2 * \ldots * \mu_n$. The problem is whether (ν_n) converges with a suitable centering, i.e.

(1) $\nu_n * \delta_{g_n}$ for suitable $g_n \in G$ and measure ν.

Condition (1) cannot happen if the product is dispersing in the sense that

(2) $\sup_{g \in G} \nu_n (X \cdot g) \to 0$ as $n \to \infty$ for every compact $X \subset G$.

For M_1-groups it is known that the alternative (1) or (2) holds (Kloss, Csiszàr, Tortrat). Applying representations it is possible to prove more generally

Theorem: If G is an arbitrary locally compact group then either (1) or (2) holds.

This is closely connected to the problem of shift convergence. Let D be the space of probability measures on G and consider the relation $\sim$ of right translation: $\mu \sim \nu$ if $\mu = \nu * \delta_g$ for some $g \in G$. Consider the factor space $D^* = D/\sim$ with the natural homomorphism $\psi: D \to D^*$. By Csiszàrs method it is easy to show that if (2) fails, then $\psi(\nu_n) \to \psi(\nu)$ for some ν.

Problem: Does $\psi(\nu_n) \to \psi(\nu)$ imply $\nu_n * \delta_{g_n} \to \nu$ for suitable $g_n \in G$?

If G is compact a positive answer can be given.

R. SCHOTT

Random Walks on Homogeneous Spaces (Survey Talk)

For several reasons, the study of random walks on homogeneous spaces is more difficult than on groups. In the first part a dichotomy theorem is given:

If G is a locally compact group with countable base and H is a closed subgroup, if μ is a measure on G which is adapted and spread out and if there exists a relatively invariant measure λ on $M = H \backslash G$ which is excessive for μ, then either all the states of M are transient and the potential of all compact is bounded, either all states are recurrent and the induced random walk on M is recurrent in the sense of Harris with respect to λ.

The result can be proved using Hopf decomposition.

A partial classification of homogeneous spaces having an invariant measure is given: If M is not amenable, then M is transient and has exponential growth.

Conjecture: Let M be amenable, G be simply connected of rigid type. Then M is recurrent if and only if G has polynomial growth of degree at most 2.

If G is a compact extension of a nilpotent group, the conjecture is known to hold.

E. SIEBERT

Semistability and Holomorphy of Convolution Semigroups

A continuous convolution semigroup $(\mu_t)_{t>o}$ on a topological group G is said to be semistable with respect to a continuous homomorphism δ of G if there is some $\beta \in]0,1[$ such that $\delta(\mu_t)=\mu_{\beta t}$ for all $t>0$. This concept (which on the real line was first considered by P. Lévy) has been studied on Euclidean spaces by R. Jajte and A. Luczak. In the following let $(\mu_t)_{t>o}$ be semistable.

Theorem 1: If $\|\mu_s-\mu_t\| < 2$ for some $s \neq t$ then $(\mu_t)_{t>o}$ is quasianalytic (by application of J.W. Neuberger's criterion hence has a common support semigroup).

Theorem 2: If $t \to \mu_t$ is norm continuous on $]0,\infty[$ and if $\|\mu_s-\mu_t\| < 2$ for all $s,t>0$ then $(\mu_t)_{t>o}$ is holomorphic. The converse is also true, even without semistability.

Corollary: Let G be a locally compact group and let all the measures μ_t be absolutely continuous w.r.t. left Haar measure on G. Then $(\mu_t)_{t>0}$ is holomorphic.

G.J. SZEKELY

Hungarian Semigroups: Extensions of Hinčin's Decomposition Theorem

A commutative Hausdorff topological semigroup S is called Hungarian semigroup if

(i) Associates form a closed set in S,

(ii) Denoting by ~ the relation "associate" the set of divisors of any element of S/~ (factor semigroup) form a compact set in S/~,

(iii) Associates are unite multipliers of each other,

(iv) S is first countable.

The main example is the convolution semigroup of (tight) probability measures on an M_1 commutative group. The axioms are motivated by Kendall's delphic semigr.

Theorem: Suppose we are given a set $P \subset S$ of "factors" containing the associates of each of its elements. Then every $s \in S$ has a decomposition

$s = q\, p_1 p_2 \cdots$ where $p_1, p_2, \ldots \in P$ and q is an antifactor (i.e. not effectively divisible by any element of P).

Applying this result to the convolution semigroup $M^1(G)$ Hinčin's decomposition is immediately obtained if G has no non trivial Haar semigroup.

K. URBANIK

Generalized Convolutions

A generalized convolution of probability measures on the positive half-line is a semigroup operation satisfying some additional conditions. For the regular generalized convolutions an analogue of the characteristic function is introduced. The set of all characteristic functions of probability measures can be described in terms of the characteristic measure. Moreover to every generalized convolution there correspond some numerical constants. Relation between these constants completely describe the generalized convolutions.

W. V. WALDENFELS

Lichtemission als quantenstochastischer Prozeß

Man betrachtet ein Atom in einem Strahlungsfeld und modelliert dieses als ein Wärmebad quantenmechanischer Oszillatoren. Das Atom habe zwei Niveaus a und b mit der Übergangsfrequenz ω_o. Der Zeitentwicklungsoperator ist gegeben durch die Differentialgleichung $\frac{d}{dt}U(t) = -i\begin{pmatrix} 0 & F(t) \\ F^*(t) & 0 \end{pmatrix}U(t)$,
dabei ist U(t) eine 2x2-Matrix mit Operatorkoeffizienten, $[F(t),F^*(s)] = \gamma\delta(t-s)$, $\mathbb{E}\, F(t)F^*(s) = (1+\theta)\gamma\delta(t-s)$ und $\theta = \exp(-h\omega_o/kT)/(1-\exp(-h\omega_o/kT))$.

Im klassischen Grenzfall $T \to \infty$ gilt $\theta \to \infty$ und $\gamma\theta \to 1$, sowie
$\frac{d}{dt}U(t) = -i\begin{pmatrix} 0 & F(t) \\ F^*(t) & 0 \end{pmatrix}U(t)$, $[F(t),F^*(s)] = 0$ und $\mathbb{E}\, F(t)F^*(s) = \delta(s-t)$;
F(t) ist ein weißes, komplexes, klassisches Rauschen, U(t) ist Lösung einer stochastischer Differentialgleichung und liefert einen Prozeß auf der unitären Gruppe U(2) mit unabhängigen stationären Zuwächsen.

Im Quantenfall $T < \infty$ ist F(t) ein weißes Quantenrauschen. Das Quantenanalogon zu einem Prozeß auf U(2) wird definiert; die Differentialgleichung kann explizit gelöst werden und liefert einen solchen Prozeß.

M. E. WALTER

Differentiation on the Dual of a Locally Compact Group

It is shown that the classical notion of derivative as a limit of difference quotients when applied to the dual of a group has equivalent formulations in terms of 1) semiderivations, 2) the Lévy-Khinchin formula (generalized to the locally compact group dual setting), 3) generalized function theory, 4) cohomology of continuous, unitary group representations, and 5) von Neumann-Schoenberg "screw"-functions.
An application of this theory is for example the characterisation of Property T groups (of Kazhdan) in a simple "computable" way using the semiderivations which turn out to be realizable by functions of negative type.
Recently papers by other authors have applied this theory to von Neumann algebras (Choda), to arithmetic groups to obtain a result of Margulis and Tits (Watatani), and to K-theory (Kuntz).

W. WOESS

Cogrowth of Groups and Simple Random Walk

Let $G=F_t/N$ be a finitely generated discrete group, where F_t is the free group on $x_1,\dots,x_t$ and N is a normal subgroup of F_t. The cogrowth coefficients γ_n are functions on G: if $\tilde{w}=wN\in G$ $(w\in F_t)$ then $\gamma_n(\tilde{w})$ is the number of words of length n in F_t which ly in wN. Generalizing a result of Grigorchuk (1979) and Cohen (1980), the following is proved:

Theorem: There exists $\gamma\in\,]\sqrt{2t-1},2t-1]$ such that either a) $\lim_{n\to\infty}\gamma_n(w)^{1/n}=\gamma$ for all $\tilde{w}\in G$ or b) $\lim_{n\to\infty}\gamma_{2n+|w|}(\tilde{w})^{1/2n}=\gamma$ and $\gamma_{2n+|w|-1}(\tilde{w})=0$ for all $n\in\mathbb{N}$, $\tilde{w}\in G$.

Grigorchuk and Cohen showed that G is amenable iff $\gamma=2t-1$; the number $\log(\gamma)/\log(2t-1)$ is the cogrowth of G with respect to the presentation $G=F_t/N$.

Theorem: I) If G is infinite then $\lim_{n\to\infty}\gamma_n(\tilde{w})/\gamma^n=0$ uniformly for $\tilde{w}\in G$.

II) If G is finite then either a) $\lim_{n\to\infty}\gamma_n(\tilde{w})/|E_n|=1/|G|$ for all $\tilde{w}\in G$ or b) $\lim_{n\to\infty}\gamma_{2n+|w|}(\tilde{w})/|E_{2n+|w|}|=2/|G|$ for all $\tilde{w}\in G$ according to a),b) above ($|E_n|$ is the number of words of length n in F_t).

The proofs use relations between the cogrowth coefficients and certain "simple" random walks on G and results on random walks by Kesten and Bhattacharya.

HM. ZEUNER

Lévy-Khinchin Formula for Semigroups of Complex Contracting Measures on a Homogeneous Space

Let G be a locally compact group, K a compact subgroup and χ a character on K. Then for the infinitesimal generator T of a continuous convolution semigroup of complex measures μ_t $(t\geq 0)$ on G with $\|\mu_t\|\leq 1$ for all $t\geq 0$ and $\mu_o=\chi\omega_K$ the following decomposition can be proved:

$$T = a\chi\omega_K + P + Q + L_\eta^\Gamma$$

where $a\leq 0$, P is χ-primitive (i.e. T is χ-dissipative and $T^*=-T$), Q is χ-quadratic (i.e. T is χ-dissipative, local, normed and satisfies $T^*=T$), η is the Lévy measure of T and Γ is a Lévy mapping for (G,χ).

L_η^Γ is defined by $L_\eta^\Gamma(f)=i\cdot P_\eta^\Gamma(f)+\int(f-\Gamma f)\,d\eta$ for all χ-invariant $f\in D(G)$ with a suitable χ-primitive distribution P_η^Γ.

Finally conditions are given which characterize the set of all Lévy measures.

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