MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 19/1983

Gruppentheorie unter besonderer Berücksichtigung proendlicher Gruppen

1.5. bis 7.5.1983

Die Tagung fand unter der Leitung der Herren K. W. Gruenberg (London) und O. H. Kegel (Freiburg) statt. Naturgemäß standen Fragen über unendliche, residuell endliche Gruppen und ihre Beziehungen zur Körpertheorie im Vordergrund des Interesses. Es wurden aber auch Fragen über endliche und lokal endliche Gruppen und darstellungstheoretische Ergebnisse diskutiert. - Wie stets bei diesen Tagungen waren die informellen Diskussionen besonders anregend.
Der Vortragsauszug von O. V. Mel'nikov (Minsk) erscheint im Folgenden, auch wenn der Verfasser nicht an der Tagung teilnehmen konnte.

Vortragsauszüge

B. Baumslag

Magnus's method applied to a free product of locally indicable groups

The method of Magnus was used to prove a Freiheitssatz for the free product of locally indicable groups with one relator, a result originally due to Howie & Brodskiî.

R. Bieri

More about the geometry of characters induced by valuations

Let $K = k(G)$ be a field generated by a subfield k and a finitely generated multiplicative subgroup $G \leq K^{\times}$. Let $\Delta \subseteq G^{*} = \mathrm{Hom}(G, \mathbb{R}_{add}) \cong \mathbb{R}^n$ be the set of all characters of G induced by valuations $v: K \to \mathbb{R}_{\infty}$ with $v(k^{\times}) = 0$. Last year I reported on joint work with John Groves: we proved that Δ is a homogenous, totally concave rational polyhedron. Here is another result:
Let $A \leq G$ be the subgroups of all algebraic elements and $B \leq G$ the minimal subgroup with K purely transcendental over $k(B)$. Then $A \leq B$ and

$B^* \subseteq \Lambda \subseteq A^*$. In fact A^* <u>is the subspace spanned by</u> Λ, <u>and</u> B^* <u>is the maximal subspace of</u> G^* <u>which is a cartesian factor of</u> Λ. The case when $A \neq 1$ or $B \neq G$ is a degenerate situation. In the generic case $A = 1$ and $B = G$ one has the following rigidity Theorem for Λ: Let $\mathcal{C}(\Delta)$ be the set of all subspaces $X \leqslant G^*$ with $\dim(X \cap \Delta) = \dim \Delta$ (= "carrier of Δ"). G. M. Bergman has shown that $\mathcal{C}(\Lambda)$ is finite.
<u>Theorem:</u> If $A = 1$ and $B = G$ then G^* is spanned by the 1-dimensional intersections of subspaces in $\mathcal{C}(\Lambda)$.
As an application one obtains the Brewster-Roseblade result that the group of all Galois automorphisms of K stabilizing G is finite.

Brunella Bruno

<u>Groups with Abelian by Finite Proper Subgroups</u>

We study groups G in which all proper subgroups are Abelian-by-Finite. If a rather weak finiteness condition is imposed (locally graded), then the periodic groups with the above property have a fairly satisfactory classification.
In particular, such groups are extensions of an Abelian group by a group of type C_{r^∞} and from this fact finer aspects of the classification can be given.

D. J. Collins

<u>Automorphisms of free products</u>

For a free group F of finite rank, there is a theorem of Whitehead that provides algorithms to decide
(1) of any element $g \in F$ if g is of minimal length in its automorphism class
(2) of any two elements $g,h \in F$ whether they are equivalent under an automorphism.
We show that a close analogue of Whitehead's theorem is valid for a free product $G = \ast_{i \in I} G_i$ (I finite) and derive an algorithm that solves the analogue of (1). However, at present we can only conjecture that the analogue of problem (2) can be solved - although it is at least clear that a solution exists when all G_i are infinite cyclic or finite.

S. Donkin

Forms on some simple modules for simple groups

Let G be a group, k a field and V a self dual, absolutely irreducible kG module of finite k-dimension. There is a G invariant bilinear form on V determined up to multiplication by a scalar. We use the rational representation theory of algebraic groups to determine in some specific cases whether the form is alternating or may be defined by a G-invariant quadratic form on V. This is useful in the proof of a theorem of M. W. Liebeck:
Let G_o be a classical simple group $(A_\ell, {}^2A_\ell, B_\ell, C_\ell, D_\ell, {}^2D_\ell)$ with natural projective module V of dimension n over GF(q) and G a group such that $G_o \triangleleft G \leq Aut(G_o)$. Let H be a maximal subgroup of G. Then either H is a known group with well described action or $|H| < q^{2n+3}$.

Lou van den Dries

Model theoretic aspects of profinite groups

We associate to a profinite group G an $\mathbb{N}$-sorted relational structure S(G); roughly speaking it is the inverse system of finite quotients of G; for each $n \in \mathbb{N}$ we have variables $V_{n1}, V_{n2}, \ldots$, ranging over the cosets of the open normal subgroups of index $\leq n$. In this way model theory (of $\mathbb{N}$-sorted structures) becomes applicable to profinite groups.

Theorems.

(1) Given $e \in \mathbb{N}$ the theory of projective profinite groups of rank $\leq e$ is decidable.

(2) The theory of (projective) profinite groups is undecidable.

(3) Two profinite groups with EP (= "the embedding property") are elementarily equivalent iff they have the same finite quotients.

(4) Given $\varkappa \geq \aleph_o$, the profinite completion of the free discrete group on $\varkappa$ generators is isomorphic to the free profinite group on $2^\varkappa$ generators.

(5) Two pro-p groups with EP are isomorphic iff they have the same cardinal number of open subgroups and the same finite quotients.

The first 3 theorems were proved in collaboration with Cherlin and Macintyre. The last 2 are due to Chatzidakis.

W.-D. Geyer

Jede endliche Gruppe ist Automorphismengruppe einer endlichen Erweiterung $K/\mathbb{Q}$

Dieser Satz wurde 1978 erstmals von E. Fried & J. Kollár bewiesen unter

Benutzung von Siegels Satz über die endliche Anzahl ganzzahliger Punkte auf nicht rationalen Kurven. 1980 wurde der Beweis von M. Fried korrigiert und vereinfacht, er benutzte Hilberts Irreduzibilitätssatz. Hier wird ein Beweis gegeben, der in eine gewöhnliche Algebra-Vorlesung paßt.

Fritz Grunewald

Remarks on the Hasse-principle for finitely generated nilpotent groups

Let G,H be finitely generated nilpotent groups. $G_{\mathbb{Q}}, G_{\mathbb{Q}_p}$ stand for the rational or p-adic completions of G,H. Here the following question is discussed:

Does $G_{\mathbb{Q}_p} \cong H_{\mathbb{Q}_p}$ $(\forall p)$ imply that $G_{\mathbb{Q}} \cong H_{\mathbb{Q}}$?

Fritz Grunewald

On S_3

S_3 stands for the symmetric group on 3 symbols. Let K be a numberfield and L a Galois extension of K with Galois group S_3. In case $K = \mathbb{Q}(i)$ I have discussed a suggestion for a decomposition law in such extensions.

D. Haran

Real projective groups

A profinite group G is called real projective, if every finite real embedding problem for G is solvable; the "finite real embedding problem" for G is a diagram $\begin{array}{ccc} & & G \\ & & \downarrow \varphi \\ B & \xrightarrow{\alpha} & A \end{array}$ where α is an epimorphism of finite groups such that for every $\varepsilon \in G$ of order ≤ 2 there is a $\beta \in B$ of order ≤ 2 with $\alpha(\beta) = \varphi(\varepsilon)$; a "solution" is a homomorphism $J: G \to B$ such that $\alpha \circ J = \varphi$.

A field K is PRC of every absolutely irreducible variety V over K such that every ordering of K extends to K(V), has a K-rational point.

Theorem: $\{G(K) \mid K \text{ is PRC}\} = \{G \mid G \text{ is real projective}\}$.

The theorem is obtained via so-called Artin-Schreier structures, defined in the talk.

B. Hartley

Profinite and residually finite groups

The basic facts about profinite completions of residually finite groups were reviewed. The point of view was that we may try to use the profinite completion as a tool to prove results about residually finite groups. As an example of transferring results from finite group theory to the profinite case, the theory of projectors was mentioned, giving Sylow subgroups in general, and Hall subgroups, Carter subgroups, etc. for prosoluble groups. The work of Grunewald, Pickel and Segal on polycyclic groups with the same finite quotients, and the theory of local conjugacy in periodic FC-groups were mentioned as examples of situations when profinite completions help to prove results about abstract ("discrete") groups.

Hermann Heineken

<u>On the subnormal embedding of complete groups</u>

Let A be a finite complete group which is subnormal in a group G. This is a report on joint work done together with J. C. Lennox about this situation. Our main result is the following:
Assume that A is a directly indecomposable complete subnormal subgroup of the finite group G. Denote the nilpotent residual of A by A^+ and the Fitting subgroup of A^G by F. If A is not the holomorph of a cyclic 3-group then
A^+ is normal in A^G and $(A^+)^G$ is the direct product of all conjugates of A^+,
AF is normal in A^G and A^G/F is the direct product of all conjugates of AF/F,
and there is a supplement U of A^G in G such that $U \cap A^G = F$.

Wolfgang N. Herfort

<u>Gruppen mit proendlicher Arithmetik</u>

This is joint work with Siegfried Grosser (Univ. Wien).
The well known example by Adian-Novikov invalidates both the Burnside conjecture and a conjecture by O. Ju. Schmidt: "There is an infinite abelian subgroup in every infinite group". Thus the Schmidt conjecture defines a restrictive condition. One is far from knowing all groups that satisfy it.
For locally compact [IN]-groups the question reduces to one for compact groups. The topological versions of finiteness conditions have given rise to the rather extensive theory of compactness conditions [1,2]. In this sense, one may formalize the conditions refered to above as follows:

$[AF]^-$ = class of locally compact groups G whose closed abelian subgroups are compact.

$[CF]^-$ = class of locally compact groups G with compactcentralizers $C_G(x)$ of the elements $x \neq e$.

Theorem 1 For a Lie-group G the following holds: $G \in [AF]^- \leftrightarrow G_0$ is compact and $G/G_0 \in [AF]^-$.

Theorem 2 If $G \in [CF]^- \cap [SIN]$ then G is either totally disconnected or compact.

Theorem 3 If $G \in [CF]^- \cap [Moore]$ then G is either compact or is a finite extension of a p-group in [Moore] (p a prime).

REFERENCES

[1] Grosser, S.-Moskowitz, M.: Compactness conditions in topological groups; J. reine and ang. Math., 246 (1971). 1-40.

[2] Palmer, T.W.; Classes of Nonabelian Noncompact Locally Compact groups; Rocky Mtn. of Math. 4 (1974)., 683 - 741.

Johannes Huebschmann

Normality of algebras over commutative rings and the Teichmüller class in the cohomology of finite and profinite groups

Let S be a commutative ring, Q a group, and $\varkappa: Q \to Aut(S)$ an action of Q on S. A suitable generalisation of the classical notion due to Teichmüller and Eilenberg-Mac Lane yields that of a Q-normal central S-algebra. As in the classical case, with a Q-normal algebra a class in $H^3(Q,U(S))$ can be associated. The purpose of the talk is to discuss the significance of this class. It will be shown how it provides sort of a classification of Q-normal algebras, which generalises the known results of Teichmüller and Eilenberg-Mac Lane in the classical case. Moreover the Teichmüller class may be used to produce exact sequences of the Chase-Rosenberg-Auslander-Brumer type.

M. Jarden

Projective Groups

Definition: A profinite group G is said to be projective if for every diagram

$$\begin{array}{ccc} & & G \\ & & \downarrow \pi \\ B & \xrightarrow{\alpha} & A \end{array}$$

where A,B are profinite (equivalently: finite) and α, π are epimorphisms there exists a homomorphism $\gamma: G \to B$ such that $\alpha \cdot \gamma = \pi$.

Theorem: A profinite group G is projective $\Leftrightarrow$ G is isomorphic to a closed subgroup of a free profinite group.

Corollary: Every closed subgroup of a projective group is projective.

Theorem: A profinite group G is projective $\Leftrightarrow$ Every p-Sylow subgroup of G is p-free.

Theorem-Definition: Every profinite group G has a cover $\tilde{\varphi}: \tilde{G} \to G$ (i.e. $\tilde{\varphi}$ is an epimorphism), unique up to an isomorphism satisfying the following equivalent conditions:

a) The map $\tilde{\varphi}$ is a projective Frattini cover of G.

b) The map $\tilde{\varphi}$ is the largest Frattini cover of G.

c) The map $\tilde{\varphi}$ is the smallest projective cover of G.

Here a cover $\varphi: H \to G$ is said to be Frattini if $\varphi(H_o) = G$ for no proper closed subgroup H_o of H.

Example: If G is a finite p-group of rank e, then $\tilde{G} \cong \tilde{F}_e(p)$.

Problem: Describe $\tilde{G}$ for other finite groups, e.g. for simple groups.

Definition: A field K is said to be PAC if for every variety ever K:

$$V(\tilde{K}) \neq \emptyset \Rightarrow V(K) \neq \emptyset$$

(Here $\tilde{K}$ is the algebraic closure of K).

Theorem: If K is a PAC field, then G(K) is projective.

If G is a projective group, then $\exists$ a PAC field K such that $G(K) \cong G$.

Applications to the model theory of fields:

a) The theory of Frobenius fields is decidable.

b) The theory of PAC fields of bounded rank is decidable.

c) The theory of all PAC fields is undecidable.

W. Jehne

A class of pronilpotent groups arising from number theory

Let k be an algebraic number field,

π a set of rational primes,

S a set of prime divisors of k,

$L = k_S^{nil}(\pi)$ the maximal pronilpotent S-ramified π-extension of k.

For the Galois group $G = G(L/k)$ let

$$G \rhd G_1 = G^1 \rhd \ldots \rhd G_n \ldots$$

be the lower central series.

Theorem: If S is ample for π then all central extensions

$$A_n \hookrightarrow \overline{G}_{n+1} \twoheadrightarrow \overline{G}_n := G/G_n$$

are stem covers, i.e. Schur multipliers $\pi\overline{G}_n \xrightarrow{\sim} A_n$.

This follows from general genus theory.

J. C. Lennox

A fixed point theorem for modules over f.g. nilpotent groups

The following result is proved.

Theorem. Suppose that A is a Noetherian module for a f.g. nilpotent group Γ and that $C_A(x)$ is nontrivial for all $x \in \Gamma$. Then there exists a subgroup Δ of finite index on Γ such that $C_A(\Delta)$ is nontrivial.

C. R. Leedham-Green

The classification of pro-p-groups by co-class

More conjectures than theorems. Sample conjecture: every pro-p-group of finite co-class is soluble. If the conjectures are true, all pro-p-groups of finite co-class are 'essentially' the p-adic completion of space groups.

This would have strong implications in the theory of finite p-groups.

P. A. Linnell

Relation modules of abelian groups

Let G be a group and let

$$1 \to R \to F \to G \to 1$$

be a free presentation for G. Then $\bar{R} = R/R'$ is a $\mathbb{Z}G$-module, called the relation module associated with the above presentation. We say that $\bar{R}$ is a minimal relation module for G if $d(F) = d(G)$ in the above presentation, where $d(G)$ denotes the minimum number of elements required to generate G. Peter Webb has obtained a formula for the number of minimal relation modules for any finite abelian group [P. Webb, The minimal relation modules of a finite abelian group, J. Pure Appl. Algebra 21 (1981), 205 - 232]. I will be concerned with deriving a formula for the number of minimal relation modules of any finitely generated abelian group.

Alex Lubotzky

Pro-finite groups applied to discrete groups

Let Γ be a discrete group, $\hat{\Gamma}$ (resp: $\hat{\Gamma}_p$) its pro-finite (resp: pro-p) completion. We shall describe several ways of obtaining results on the discrete group Γ using $\hat{\Gamma}$ and for $\hat{\Gamma}_p$. Usually, the topology of $\hat{\Gamma}$ or $\hat{\Gamma}_p$ is used to obtain algebraic properties of Γ. These methods will be illustrated by presenting new proofs to known results as well as new

results concerning: the residual finiteness of Γ, automorphism groups of Γ (Aut Γ and Out Γ), the representation theory of Γ, presentations of Γ by generators and relations, the congruence subgroup problem for Γ (in case Γ is an arithmetic group), etc.
The above point of view suggests looking at some questions concerning the "behavior" of the finite index (f.i.) subgroups of Γ, e.g. the group of "virtual" automorphisms of Γ, the limit of rank (Δ) where Δ runs over the f.i. subgroups of Γ and $(\Gamma:\Delta) \to \infty$, etc. Some results and open problems will be presented.

B. H. Matzat

Zwei Aspekte konstruktiver Galoistheorie

I. Über Darstellungen von $\Lambda = \mathrm{Gal}(\bar{\mathbb{Q}}/\mathbb{Q})$: $K/\mathbb{Q}$ aufgeschlossener algebraischer Funktionenkörper, $\bar{K} = K\bar{\mathbb{Q}}$, $\mathcal{S} \subseteq \mathbb{P}(\bar{K}/\bar{\mathbb{Q}})$ mit $|\mathcal{S}| < \infty$, $\bar{M}_{\mathcal{S}}/\bar{K}$ maximaler außerhalb $\mathcal{S}$ unverzweigter Erweiterungskörper, $\mathfrak{F}_m$ freie proendliche Gruppe vom Rang m.

Satz 1: $\prod_{p \in \mathcal{S}} p$ über K definiert $\rightsquigarrow \bar{M}_{\mathcal{S}}/K$ galoissch mit
$\mathrm{Gal}(\bar{M}_{\mathcal{S}}/K) \cong \mathrm{Gal}(\bar{M}_{\mathcal{S}}/\bar{K}) \rtimes \mathrm{Gal}(\bar{K}/K)$.

Satz 2: Es gibt Homomorphismen $d_m: \Lambda \to \mathrm{Out}(\mathfrak{F}_m)$ mit d_1 surjektiv und d_m injektiv für $m \geq 2$.

Satz 3: Λ' operiert vermöge d_m auf $\mathfrak{F}_m/\mathfrak{F}_m'$ trivial.

II. Rationalitätskriterien für Galoiserweiterungen von $\bar{\mathbb{Q}}(t)$: $\bar{N}/\bar{\mathbb{Q}}(t)$ galoissch mit Gruppe G, $|G| = n$, und Verzweigungsdaten $(\sigma_1,\dots,\sigma_s) \in G^s$, $C_i := [\sigma_i]$, $\mathfrak{C} := (C_1,\dots,C_s)$, $\mathfrak{S} := \bigcup_{a \in (\mathbb{Z}/n\mathbb{Z})^*} \mathfrak{C}^a$ Verzweigungsstellen von $\bar{N}/\bar{\mathbb{Q}}(t)$.

Bedingung (Z): $Z(G)$ besitze ein Komplement in $N_G(G_T)$, G_T Trägheitsgruppe von $\bar{N}/\bar{\mathbb{Q}}(t)$.

Satz 4: a) $\ell^a(\mathfrak{S}) = \ell \rightsquigarrow \bar{N}/\bar{\mathbb{Q}}(t)$ über $K(t)$ mit $(K/\mathbb{Q}) \leq \ell$ definiert.
b) Unter Bedingung (Z): $\ell^i(\mathfrak{S}) = \ell \rightsquigarrow \bar{N}/\bar{\mathbb{Q}}(t)$ über $K(t)$ als Galoiserweiterung definiert.

Satz 5: Unter Bedingung (Z): $\ell^i(\mathfrak{C}) = 1 \rightsquigarrow \bar{N}/\bar{\mathbb{Q}}(t)$ über $\mathbb{Q}^{ab}(t)$ als Galoiserweiterung definiert.

Als Anwendungsbeispiele werden die Resultate von Belyi, Thompson und dem Referenten über Realisierung von Gruppen als Galoisgruppen über $\mathbb{Q}$ bzw. $\mathbb{Q}^{ab}$ zusammengestellt.

O. V. Mel'nikov

On products of powers in free pro-p-groups

Theorem 1. Let $a_1,\ldots,a_m$ be such elements of a free pro-p-group F_p that $a_1^{n_1}a_2^{n_2}\ldots a_m^{n_m} = 1$ for some $n_i \in p\mathbb{Z}_p$, $i = 1,\ldots,m$. Then the rank of the closed subgroup of F_p generated by $a_1,\ldots,a_m$ is $\leq m/2$.

As an obvious consequence we get a slight generalization of the proposition 6.6 from [1]. Namely, it remains valid for the relation $u_1^{n_1}u_2^{n_2}\ldots u_m^{n_m} = 1$, $n_i \in \mathbb{Z}$, if the greatest common divisor $(n_1,\ldots,n_m) > 1$.

Theorem 1 is used in the proof of the following

Theorem 2. Any one-relator pro-p-group G admits a presentation $G = \langle X \| r = 1\rangle$ where r is not a p-th power in the free pro-p-group $F(X)$.

The first examples of such presentation for some one-relator pro-p-groups with torsion were constructed in [2].

[1] R. C. Lyndon, P. E. Schupp. Combinatorial Group Theory, Springer, Berlin, 1977;

[2] D. Gildenhuys, Invent. math., 5 (1968), 357 - 366.

Richard E. Phillips

Locally finite simple groups

Throughout, G is a countable, infinite, locally finite simple group. Such a group has an approximating series of finite subgroups $\{F_n\}$; $1 \leq F_1 \leq \ldots \leq F_n \leq \ldots$; $\cup F_i = G$ such that for each n $\exists$ a maximal normal subgroup $M_n < F_n$ with $F_n \cap M_{n+1} = 1$ $\forall n$. Thus F_n is embedded in the finite simple group F_{n+1}/M_{n+1}. Thanks to the classification of finite simple groups, the simple quotients F_n/M_n are determined and in fact, we may assume that all of the quotients F_n/M_n are either alternating or belong to one of 16 families of groups of Lie type. The group of Lie type are determined by a rank parameter t and a field parameter. If the rank parameter t is uniformly bounded with n, then G is linear and hence G itself is a group of Lie type. Thus, if G is not linear, we may assume that $\forall n$ F_n/M_n is alternating, or F_n/M_n is a group of Lie type with t increasing with n. From this, we see that there are seven possible families to which the quotients F_n/M_n can belong and these are all reasonably well understood. The real roadblock at this point is the complexity of the groups M_n.

For example, in trying to prove the following (Conjecture. Suppose every proper section of G is non-simple. Then either $G \cong PSL(2,F)$ or $G \cong Sz(F)$ where F is a minimal infinite locally finite field.) one has little difficulty in the case where the M_n are all = 1.

So, one important question appears to be:

How complicated can the M_n's be?

In this regard we have a 2nd:

Conjecture. Every G as above has an approximating sequence $\{F_n\}$ where each F_n is "central-by-semisimple".

Finally, we provide an infinite collection of groups G on which there are no approximating sequences $\{F_n\}$ with each $M_n = 1$.

P. Plaumann

Polythetic groups

A topological group is called polythetic, if it has a finite subnormal series with monothetic factors, i.e. factors having dense cyclic subgroups. Totally disconnected polythetic groups have a greatest compact normal subgroup provided they are locally compact. So often these groups can be studied via compact groups, polycyclic groups and extenstions. With the help of the Ascoli-theorem one proves

Theorem A: The automorphism group of a locally compact, totally disconnected polythetic group G is again locally compact. If G is compact, then Aut G is even compact.

Major results on polycyclic groups which can be carried over to special classes of these include:

Theorem B: If G is (polythetic and noetherian)-by-finite then every solvable subgroup of Aut G is polythetic and noetherian.

Theorem C (Koderisch): Every (poly-p-adic)-by-finite group can be embedded in a linear group over the ring of p-adic integers.

Stephen J. Pride

Spelling theorems and generalized 2-complexes

Let $A = \mathop{*}\limits_{i=1}^{m} A_i$, let R be a cyclically reduced element of A of length at least 2, and let $n \geq 1$. Let G denote $A/\{R^n\}^A$. A Gurevich word is a word of the form $aT^{n-1}T_1b$ where T is a cyclic permutation of $R^{\pm 1}$; $T = T_1T_2$ with T_2 non-empty; a (resp. b) is a non-trivial element of the factor containing the last (resp. first) term of T_2; aT,b has occurences of each group A_i involved in R.

Theorem (James Howie, S. J. Pride) Suppose each A_i is locally indicable (and non-trivial). If W is a non-empty cyclically reduced element of A which defines the identity of G, then some cyclic permutation of W has one of the following forms: $R^{\pm n}$; $U_1V_1U_2V_2$ (U_1, U_2 Gurevich words); U_1aU_2V (U_1a, aU_2 Gurevich words); aU_1bU_2 (aU_1b, bU_2a Gurevich words).
This generalizes the well-known "Spelling Theorem" of B. B. Newman, G. A. Gurevich and others, which concerns "ordinary" one-relator groups. As a consequence of our theorem we solve the word problem (WP) and related decision problems for G, provided n > 1 and each A_i has solvable WP. The theorem stated above is actually a very special case of our work, which is concerned with the fundamental groups of objects we call "generalized 2-complexes". The techniques we use are almost entirely geometric.

V. N. Remeslennikov

Elementary theory of finitely generated pro-p-groups

In this lecture the problem of elementary equivalence of two pro-p-groups and the question of decidability of the theory of a pro-p-group were discussed.

Luis Ribes

On Frattini covers

Assume a class $\mathcal{C}$ of finite groups satisfies:
1) $\mathcal{C}$ is closed under homomorphic images, and
2) $\mathcal{C}$ is saturated, i.e. $G/G^* \in \mathcal{C}$, G finite $\Rightarrow G \in \mathcal{C}$ (G^* = Frattini subgroup of G).

Definition. If G is a profinite group, we say the epimorphism $H \xrightarrow{\gamma} G$ of profinite groups is a Frattini cover if $\ker \gamma \leq H^*$.
It is well-known that every profinite group has a (unique) projective Frattini cover.
Theorem. Let $\mathcal{C}$ be as above. Let $H \xrightarrow{\gamma} G$ be a Frattini cover of profinite groups, and assume G is a pro-$\mathcal{C}$ group. Then H is a pro-$\mathcal{C}$ group.
Corollary. (Haran-Lubotzky, Cherlin-Macintyre-van den Dries)
If $H \xrightarrow{\gamma} G$ is a projective Frattini cover, then for each prime p,
$p \mid |G| \Leftrightarrow p \mid |H|$.
Lemma. Let p be a prime and t a natural number (fixed). Consider the class $\mathcal{C}$ of finite groups which are p-nilpotent and such that the minimal number $d(G_p)$ of generators of a p-Sylow subgroup of $G \in \mathcal{C}$ is at most t. Then $\mathcal{C}$

satisfies conditions 1) and 2) above.

Theorem. Let p be a prime and let G be a p-nilpotent profinite group with finite p-Sylow subgroups. Let $H \overset{\gamma}{\twoheadrightarrow} G$ be a projective Frattini cover. Let $d = d(G_p)$ and $A = \ker \gamma$. Then the p-Sylow subgroup A_p of A is free pro-p of rank $1 + |G_p|(d - 1)$.

Corollary. (Generalizes a bit a result of Ershov)
Let G be a prosupersolvable group with finite Sylow subgroups. Let $H \overset{\gamma}{\twoheadrightarrow} G$ be a projective Frattini cover of G. Then for any prime p, the p-Sylow subgroup A_p of $A = \ker \gamma$ is free pro-p of rank $1 + |G_p|(d(G_p)-1)$.

L. Scott

On the isomorphism problem for integral group rings

Some recent activity on the isomorphism problem is discussed, especially an approach of the author and K. Roggenkamp for group rings of p-groups over the p-adics. An essential ingredient is nonabelian 1-cohomology with coefficients in various profinite unit groups.

D. Segal

Decision procedures for S-arithmetic groups

Let S be a finite set of primes and Γ an S-arithmetic group. We describe some algorithms which decide various algebraic or arithmetic questions about Γ, e.g. the conjugacy problem; or more generally, the question of when two points in some rational module are in the same orbit under Γ.

The method involves the study of Bruhat-Tits buildings for p-adic algebraic groups, as well as our usual apparatus (i.e. reduction theory over $\mathbb{R}$) for arithmetic groups.

This is joint work with Fritz Grunewald.

B. A. F. Wehrfritz

Some residually finite p-groups

Let X be a finitely generated group of matrices over a division ring generated (as division ring) by its centre and a polycyclic-by-finite subgroup of its multiplicative group. It is implicit in recent work of Alexander Lichtman that such a group X is residually finite. In fact the following is true:
There exists a prime p and a subgroup Y of X of finite index such that Y

is residually a finite p-group. If the characteristic is zero we can choose p to be almost any prime. Otherwise p is the characteristic. The proof requires much more work than the residual finiteness of X. For matrices over an arbitrary division ring X need not even be residually finite. However, the above theorem can be extended to cover a wider class of division rings than those specified above.

J. S. Wilson

On the structure of profinite torsion groups

The proof and some consequences of the following result will be discussed: if G is a profinite torsion group, then G has a finite series of closed characteristic subgroups in which each factor either is a pro-p-group for some prime p or is isomorphic (as a topological group) to a Cartesian power of a finite simple group.

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