MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

T a g u n g s b e r i c h t 20/1983

General Inequalities

8.5. bis 14.5.1983

The fourth International conference on General Inequalities was held from May 8 to May 14 at the Mathematisches Forschungsinstitut Oberwolfach. The organizational commitee consisted of L.Losonczi (Lagos and Debrecen) and W.Walter (Karlsruhe). Dr.A.Kovačec served extremely well as a secretary of the conference.

The meeting was attended by 42 participants from 17 countries. In the opening address, W.Walter had to report on the unexpected death of E.F. Beckenbach. He died of a stroke in September 1982, a few days after receiving the award for Distinguished Service to Mathematics from the Mathematical Association of America.

Beckenbach was one of the founding fathers of the General Inequalities conferences. He served with energy and devotion as an organizer and as editor of the proceedings of those conferences. He was also engaged in the preparations for the present conference, which the participants decided should be held in memoriam Edwin F. Beckenbach. In a brief memorial lecture M.Goldberg gave a survey of Beckenbachs mathematical activities and his services to the mathematical community.

Inequalities play a significant role in many branches of mathematics. Correspondingly, the participants represented many different fields among which classical inequalities still provided a steady source of new developments. Lectures also included

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differential and functional inequalities, bounds for eigenvalues, inequalities in functional analysis, convexity and its generalizations, inequalities in number theory and probability theory, as well as mathematical programming and economics.

As in earlier conferences, the problems and remarks sessions produced a vivid exchange of results, methods and hypotheses.

The participants experienced anew the creative, congenial and stimulating atmosphere at the Institute.

The conference was closed by L.Losonczi, who in his resumé expressed the thanks of the participants for the excellent working conditions in the Institute and for the hospitality of its leaders and staff.

Abstracts

R.P.AGARWAL: Difference calcules with Applications to Difference Equations

"Everyone knows" that in discrete case there is no analogue of Rolle's lemma in continuous case, so not all the known results in continuous calculus are expected to have discrete analogues. In this paper we shall discuss some possible ones which we shall need to study qualitive properties of solutions of higher order difference equations. The proofs are based on some simple inequalities.

C.ALSINA: Schur-concave t-norms and triangle functions.

Studying Schur-concavity of t-norms, copulas and triangle functions for probability distribution functions, we have proved the following results:

Theorem 1: Any associative copula is Schur-concave. If T is a strict t-norm, then T is a copula if and only if T is Schur-concave. There are Schur-concave nonstrict Archimedean t-norms which are not copulas.

Theorem 2: If T is a non-strict Archimedean t-norm with concave additive generator t, then T is Schur-convex and, therefore, $T(x,y) \leq \mathrm{Max}(x+y-1,0)$. $W(x,y) = \mathrm{Max}(x+y-1,0)$ is the unique t-norm which is at the same time Schur-convex and Schur-concave.

Theorem 3: The Schur-concavity for triangle functions: $\tau(F,G) \leq \tau(\alpha F+(1-\alpha)G, (1-\alpha)F+\alpha G)$ $(F,G \in \Delta^+, \alpha \in [0,1])$, holds for any triangle function π_C. There exists no copula C such that σ_C or τ_C is a Schur-concave triangle function.

Remark: Compare also the talk given by A.Sklar.

D.BRYDAK: Differential inequalities and generalized convex functions.

We prove, under suitable assumptions, that a function Ψ satisfies the inequality

(1) $\Psi''(x) \geq f(x,\Psi(x), \Psi'(x))$ in an interval I iff it is convex with respect to the two-parameter-family F of all solutions of the equation

$$y'' = f(x,y,y').$$

The convexity with respect to a two-parameter family of functions was defined by E.F.Beckenbach in 1937. The above theorem was proved by M.M.Peixoto in 1949 under the additional assumption of the continuity of the second derivative of Ψ.

As an application of the above theorem we prove that if F is a linear family satisfying suitable conditions, then Ψ is either strictly convex or strictly concave with respect to F iff for every $x_1,x_2 \in I$ there is a unique point $x_0 \in [x_1,x_2]$ such that

$$\Psi'(x_0) = \varphi'(x_0),$$

where $\varphi \in F$ satisfies the equalities

$$\varphi(x_1) = \Psi(x_2), \ \varphi(x_2) = \Psi(x_2).$$

B.CHOCZEWSKI: Stability of some iterative functional equations.

The notion of stability (and interative stability) of iterative functional equations of the form

$$\varphi(f(x)) = g(x)\varphi(x) + h(x)$$

(with the unknown φ) has been introduced in 1970 by D.Brydak. A survey of results will be given and the problem of stability of intervals with respect to the equation will be also mentioned. The talk is based on papers by D.Brydak,E.Turdza, M.Czerni from Kraków and also by R.Wegrzyk and the speaker.

A.CLAUSING: A t-entropy inequality

Let $t>0$ and $p = (p_1,p_2,\dots,p_n)$ a probability vector. The t-entropy function is defined as

$$H_t(p) = -\frac{\sum_{i=1}^{n} p_i^t \log p_i}{\sum_{i=1}^{n} p_i^t}$$

Stolarsky has raised the problem of finding the best, that is, smallest value of t, such that the entropy inequality

$$H_t(p) \leq \log n$$

holds for all p. We prove that this value is given by $t_o(n) = \|\varphi_n\|_\infty$, where

$$\varphi_n(x) = \frac{\log\log(1-x)^{1-n} - \log\log(1+(n-1)x)}{\log(1-x)^{-1} + \log(1+(n-1)x)} \quad x \in (0,1)$$

In contrast with Shannon's inequality ($t=1$), equality can hold for certain $p \neq (\frac{1}{n},\frac{1}{n},\dots,\frac{1}{n})$ if $t = t_o(n)$

W.EICHHORN: Inequalities in the Theory of Economic Inequality

Let $x \in R_+$ be the income before tax, $p: R_+ \to [0,1]$ the income tax rate, i.e. $xp(x)$ the income tax amount, and $x-xp(x) = (1-p(x))x$ the income after tax. For certain reasons p has the following properties:

P.1: "Weak progression of tax rate": p is weakly increasing.

P.2: "Weak increase of income after tax": $(1-p(x))x$ is weakly increasing

P.3: "Weak decrease of income inequality": For all income distributions $(x_1,x_2,\dots,x_n)$ the inequality of $((1-p(x_1))x_1,\dots,(1-p(x_n))x_n)$ is equal or smaller than that of $(x_1,x_2,\dots,x_n)$. The "inequality" mentioned is given by the

<u>Definition</u> (Principle of Majorization): The inequality of $\underline{y} = y_1,y_2,\dots,y_n) \neq \underline{0}$, satisfying $0 \leq y_1 \leq y_2 \leq \dots \leq y_n$ is equal or smaller than that of $\underline{x} = (x_1,\dots,x_n) \neq 0$, $0 \leq x_1 \leq x_2 \leq \dots \leq x_n$, if and only if

$$(1)\quad \frac{\sum_{i=1}^{k} y_i}{\sum_{j=1}^{n} y_j} \geq \frac{\sum_{i=1}^{k} x_i}{\sum_{j=1}^{n} x_j} \quad \text{for every } k = 1,2,\dots,n$$

<u>Theorem:</u> The following statements hold:

(a) P.1 $\not\Rightarrow$ P.3, P.2 $\not\Rightarrow$ P.3

(b) P.1 and P.2 are independent

(c) P.1, P.2 $\Leftrightarrow$ P.3

(d) P.1*, P.2 $\Leftrightarrow$ P.3* (The stars * denote strict versions).

The example $p(x) = \frac{x}{2x+2}$ as well as (b) together with (c) show consistency of P.1, P.2, P.3.

W.N.EVERITT: <u>Hardy-Littlewood Integral-inequalities</u>

This lecture is concerned with three examples of the Hardy-Littlewood type of integral inequalities, viz.

$$\text{(i)}\quad \left(\int_0^\infty f'^2\right)^2 \leq \left[\cos\left\{\frac{\pi(1+\alpha)}{2\alpha+3}\right\}\right]^{-2} \int_0^\infty x^\alpha f^2 \cdot \int_0^\infty x^{-\alpha} f''^2 \quad \text{where } \alpha \in (-1,\infty)$$

$$\text{(ii)}\quad \left(\int_0^\infty \{f'^2 - xf^2\}\right)^2 \leq 4 \int_0^\infty f^2 \int_0^\infty \{f''+xf\}^2$$

$$\text{(iii)}\quad \left(\int_0^\infty \{f'^2+(x^2-1)f^2\}\right)^2 \leq 4 \int_0^\infty f^2 \int_0^\infty \{f'' - (x^2-1)f\}^2$$

All these results are best possible and all cases of equality are known. The lecture reports on joint work with W.D.Evans (Cardiff) and W.K.Hayman (London).

F.FEHÉR: A weak-type inequality and a.e. convergence

A basic theorem in connection with a.e. convergence is the following "Banach Principle":

Let (Ω, Σ, μ) denote a finite measure space, X,Y Banach function spaces of μ-measurable functions on Ω; let $T_k: X \to Y$ be linear, bounded operators (n=1,2,...) and let T denote the maximal operator, defined by

$$(Tf)(x) := \sup_{k \in \mathbb{N}} |(T_k f)(x)| \qquad (x \in \Omega)$$

Moreover, let U be a dense subspace of X. Then the following statements are equivalent:

(1) $\forall f \in X: \quad |(T_k f)(x) - (T_l f)(x)| \to 0 \quad (k,l \to \infty)$

(2) (a) $\forall f \in U: \ |(T_k f)(x) - (T_l f)(x)| \to 0 \quad (k,l \to \infty)$

(b) $\exists C: (0,\infty)$ decreasing, $C(\lambda) \searrow 0 \quad (\lambda \nearrow \infty)$, such that

$$\mu\{x \in \Omega : (Tf)(x) > \lambda\} \leq C\left(\frac{\lambda}{\|f\|}\right)$$

The purpose of the talk is, to give more details on the function C.

I.FENYÖ: Über eine Integralungleichung

Für eine beliebige Lösung z = z(x,y) der Ungleichung vom Gronwall-Typ

$$z(x,y) \leq f(x,y) + \int_0^x a_1(s,y)z(s,y)ds + \int_0^y a_2(x,t)z(x,t)dt +$$

$$+ \int_0^x \int_0^y a_3(s,t)z(s,t)dsdt \qquad (a_i \geq 0)$$

wird eine Abschätzung hergeleitet, welche nicht verbessert werden kann. Das Ergebnis erweist sich als Verallgemeinerung von bekannten Resultaten.

C.H.FITZGERALD: Opial Type Inequalities that involve Higher Order Derivates

Estimates of integrals of the form $\int_a^b |yy'|dx$ are made in terms of integrals of the form

$$\int_a^b [y^{(n)}]^2 dx$$

for functions y satisfying appropriate end point conditions. Certain extremal functions are shown to exist and have necessary monotonicity properties. For each $n = 2,3,4,\ldots$ these extremals can be found explicitly by solving associated systems of linear equations. Sharp constant in the estimates can then be obtained. For example for $n = 2$ and $n = 3$, these extremals and the constants they determine are given.

M.GOLDBERG: New inequalities for ℓ_p-norms.

The ℓ_p norm and the ℓ_p operator-norm of an $m \times n$ complex matrix $A = (\alpha_{ij})$ are given by

$$|A|_p = (\Sigma_{i,j}|\alpha_{ij}|^p)^{1/p}$$

and

$$\|A\|_p = \max\{|Ax|_p : x \in \mathbb{C}^n,\ |x|_p = 1\},$$

respectively. The main purpose of this paper is to investigate the multipicativity of the ℓ_p norms and their relation to the ℓ_p operator-norms.

H.H.KAIRIES: An inequality for Krull solutions

Using a well known result of Krull concerning the uniqueness of convex solutions of certain difference equations we prove: Let $f : \mathbb{R}_+ \to \mathbb{R}$ satisfy the difference equation

(1) $f(x+1) - f(x) = \log x, \quad x \in \mathbb{R}_+$,
and assume that f is convex on $\mathbb{R}_+$, with $f(1) = 0$.
Then necessarily

(2) $f(x) + f(\frac{1}{x}) \geq 0$

for $x \in \mathbb{R}_+$. Moreover, the function $x \to f(x) + f(\frac{1}{x})$ is strictly increasing for $x \geq 1$ and the function

$$y \to f(\tfrac{1}{2}(y+\sqrt{y^2-4})) + f(2(y+\sqrt{y^2-4})^{-1})$$

is strictly increasing for $y \geq 2$.

This result implies the well known inequality $\Gamma(x)\Gamma(\frac{1}{x}) \geq 1$. In the proof we need neither the concept of differentiability nor the concept of an integral.

Heinz KÖNIG: A Dini-type theorem in superconvex analysis

The talk deals with the superconvex theory, a general theory of countable convexity. (Rodé, Arch.Math.34, 452-462 (1980) and 36, 62-72 (1981)). A central result of it is a Dini-type theorem: If on the superconvex space X an increasing sequence of superconvex functions $f_n: X \to \mathbb{R}$ with limit function $f: X \to \mathbb{R}$ is such that at each $x \in X$ one has $f_n(x) = f(x)$ for some $n \in \mathbb{N}$, then $\operatorname{Inf} f_n \to \operatorname{Inf} f$. The condition, that the limit function be attained at each point seems to be severe, but is fulfilled quite often. However, it is shown that it is not essential: It suffices that there exists a sequence of positive numbers $c_n \downarrow 0$ such that $f(x) - f_n(x) = O(c_n)$ for $n \to \infty$ at each $x \in X$. This theorem will be deduced from an inequality of unusual type. Rodé proved his result via a certain intersection theorem which resembles the classical Baire theorem. We retain this procedure and extend the intersection theorem as well, to its ultimate limit as a counter example reveals.

Herrmann KÖNIG: Some inequalities for the eigenvalues of a compact operator.

The approximation-numbers of a compact linear map $T: X \to X$ in a Banachspace X are given by $a_n(T) := \{\|T - T_n\| : \text{Rank } T_n \le n\}$ $n \in \mathbb{N}$. If $(\lambda_n(T))$ denotes the sequence of eigenvalue of T, for any $0 < p < \infty$ there is $c_p \in \mathbb{R}^+$ such that for all T, $n \in \mathbb{N}$

$$\left(\sum_{j=1}^{n} |\lambda_j(T)|^p \right)^{1/p} \le c_p \left(\sum_{j=1}^{n} a_j(T)^p \right)^{1/p}$$

It is shown that $c_p \le 2e \max\left(1, \frac{1}{\sqrt{p}}\right)$. It is an open problem, whether the inequality $\sup_{p \in \mathbb{R}^+} c_p < \infty$ holds.

For single eigenvalues, the following results are known:

$$|\lambda_n(T)| = \lim_{m \to \infty} a_n(T^m)^{1/m}$$

$$|\lambda_n(T)| \le \sqrt{2e}\, (a_m(T))^{1 - \frac{m-1}{n}} \quad (1 \le m \le n) \text{ if } \|T\| \le 1$$

The same formulas hold for the Weyl-numbers instead of the approximation numbers.

A.KOVAČEC: On aspects of the matrix-method for algebraic inequalities

Define $\mathfrak{m} := \{\underline{x} \in R^n : 0 \le x_1 \le x_2 \le x_3 \le \dots \le x_n\}$. Given permutations $\dot{\pi}, \dot{\sigma}, \pi, \sigma \in S_n$ we study the following two problems:

(1) To determine necessary and sufficient conditions for the permutations $\dot{\pi}, \dot{\sigma}, \pi, \sigma$ in order that the inequality

$$\sum_{i=1}^{n} x_i y_{\dot{\pi} i} z_{\dot{\sigma} i} \le \sum_{i=1}^{n} x_i y_{\pi i} z_{\sigma i} \qquad (*)$$

holds for all $\underline{x}, \underline{y}, \underline{z} \in \mathfrak{m}$.

(2) We consider the case that the inequality (*) holds for all $\underline{x}, \underline{y}, \underline{z} \in \mathfrak{m}$ and ask, whether the function $F: \mathfrak{m}\times\mathfrak{m}\times\mathfrak{m} \longrightarrow R$, defined by

$$F(\underline{x}, \underline{y}, \underline{z}) := \sum_{i=1}^{n} x_i y_{\pi i} z_{\sigma i} - \sum_{i=1}^{n} x_i x_{\dot{\pi} i} z_{\dot{\sigma} i},$$

which is nonnegative definite (on $\mathfrak{m}\times\mathfrak{m}\times\mathfrak{m}$) can be rewritten in such a way that it's definiteness becomes obvious: That is, as a sum of products of the form $p(x_{i'} \pm x_i)(y_{j'} \pm y_j)(z_{k'} \pm z_k)$ where $p > 0$ $i' > i$, $j' > j$, $k' > k$. By means of the matrix-method these problems lead to a purely combinatorial question in the theory of finite sets.

If the answer to question (2) is "yes", it follows that very many algebraic expressions (polynomials) $p(x_1, x_2, \dots, x_n)$ that are positive definite whenever $0 \le x_1 \le x_2 \le \dots \le x_n$, can be written as a sum of products of the form

$$p \cdot \prod_{1 \le i < j \le n} (x_j - x_i)^{a(i,j)}, \text{where } a(i,j) \in N \cup \{0\},\ p \geq 0$$

N.KUHN: A note on t-convex functions

A function $f: I \to R$ on an interval $I \subset R$ is for $0 < t < 1$ defined to be

t-convex iff $f((1-t)u+1v) \le (1-t)f(u)+tf(v)\ \forall\ u,v \in I$,
t-affine iff $f((1-t)u+tv) = (1-t)f(u)+tf(v)\ \forall\ u,v \in I$.

One proves in an elementary way

Theorem 1: If $f:I \to \mathbb{R}$ is s-affine for some $0 < s < 1$, then
$$f((1-t)u+tv)+f((1-t)v-tu) = f(u)+f(v) \quad \forall\ 0 < t < 1 \text{ and } u,v \in I.$$

Corollary: If $f:I \to \mathbb{R}$ is s-affine for some $0 < s < 1$, then it is t-affine for all rational $0 < t < 1$.

In view of the Hahn-Banach theorem of Rodé (Arch.Math.31, 474-481 (1974) each s-convex function $f:I \to \mathbb{R}$ is the pointwise maximum of the s-affine functions $\varphi \leq f$. Therefore the above implies the following

Theorem 2: If $f:I \to \mathbb{R}$ is s-convex for some $0 < s < 1$, then it is t-convex for all rational $0 < t < 1$.

Remark: The talk on t-convex functions was given by Heinz König, in absence of N.Kuhn.

M.KWAPISZ: Functional inequalities and existence results for fixed point equations in function spaces.

In the paper we will show a wide class of operators in $C(I,\mathbb{R}^n)$ for which a fixed point result can be established by the use of the Schauder theorem if we are able to solve some functional inequality related to this operator. Having a solution of this inequality one can define a compact and convex subset of $C(I,\mathbb{R}^n)$ which remains invariant with respect to the operator mentioned.

A number of examples of operators and inequalities related which have solutions will be shown. They include the integral, integro-functional and functional equations.

The paper will show the importance of functional inequalities for more detailed discussion of existence problems for functional equations.

V.LAKSHMIKANTHAM: Differential inequalities at resonance

It is well known that the comparison results for the initial and boundary value problems have been very useful in the theory of differential equations. It is natural to expect that comparison results for problems at resonance will be useful in proving, for example, existence results for periodic boundary value problems. In this paper, we develop systematically general comparison results of various types for boundary value problems at resonance and show some applications of these results.

L.LOSONCZI: Inequalities of Young type

A function $\varphi:[0,\infty)$ is called a Young function if

(i) φ is increasing and right continuous on $[0,\infty)$

(ii) $\lim_{x\to\infty} \varphi(x) = \infty$.

The right inverse $\varphi^{(-1)}$ of a Young function φ is defined by

$$\varphi^{(-1)}(y) = \begin{cases} 0 & \text{if } y\in[0,\varphi(0)) \\ \sup\{x\geq 0 \mid \varphi(x)\leq y\} & \text{if } y\in[\varphi(0),\infty). \end{cases}$$

We prove the following

Theorem: Let f,g be arbitrary real valued functions on $(0,\infty)$. The Young type inequality

(1) $xy \leq f(x)+g(y) \quad (x,y > 0)$

is satisfied if and only if there exist nonnegative functions p,q on $(0,\infty)$, a real constant α and a Young function φ such that

$$f(x) = \int_0^x \varphi(t)dt + p(x)+\alpha \quad (x>0),$$

$$g(y) = \int_0^y \varphi^{(-1)}(s)ds + q(y) - \alpha \quad (y>0) .$$

We also investigate (1) if $x\in(a,\infty)$, $y\in(b,\infty)$ and mention a generalization of (1).

E.R.LOVE: Links between some generalizations of Hardy's integral inequality

J.Kadlec and A.Kufner*(1967) used Hardy's inequality in their studies of functions with zero traces, and developed extensions like following to deal with certain singular cases. If $p \geq 1$, and other conditions hold,

$$\left(\int_0^1 \left|\frac{F(t)}{t}\right|^p \chi(t)dt\right)^{\frac{1}{p}} \leq C\left(\int_0^1 |f(t)|^p \chi(t)dt\right)^{\frac{1}{p}} ,$$

where $F(t) = \int_0^t f(u)du$ and $\chi(t) = t^{p+\beta}\{\log(R/t)\}^{\gamma}$.

When $\beta = -1$ the form of the inequality is slightly modified.

E.T.Copson[+](1976) independently gave several inequalities which might be described, rather inexactly, as the outcome of a general monotonic substitution $t = \Phi(x)$ in the above results after putting $\gamma = 0$.

I propose to present two inequalities which include most of the above as quite special cases. The two main directions of generalization are that F becomes a fairly general integral transform of f, and that $\{\log(R/t)\}^{\gamma}$ is replaced by an almost arbitrary monotonic function. The method of proof is different from that of the above authors, but it yields the same constants C in the special cases.

*MR 35 # 5924 +MR 56 # 559

R.N.MOHAPATRA: Inequalities related to sequence space ces[p,q]

In this paper we consider inequalities which lead to inclusion relation between two sequence spaces. In an earlier paper we have defined that the space of sequences ces [p,q] is the collection of all sequences whose (N,q) transform belong to ℓ_p when $q \equiv \{q_n\}$ is a non-negative sequence. Copson had given generalization of Hardy's inequality which is somewhat similar to the kind of inequalities considered in this paper.

In this paper we shall be concerned with inclusion among sequence spaces with the help of our inequalities. Some relations with Copson's results will be brought out.

R.J.NESSEL: Some negative results in connection with Marchaud-type inequalities

Continuing our previous investigations on quantitative uniform boundedness and condensation principles, the present paper, which presents joint work with W.Dickmeis and E.van Wickeren, is concerned with some negative results in connection with Marchaud-type inequalities. The existence of the relevant counterexamples follows by means of a general theorem, given in terms of operators in Banach spaces. The method of proof essentially consists of a quantitative version of the familiar gliding hump method.

Zs.PÁLES: Inequalities for comparison of means

The mean values M and N are said to be comparable on the interval I if

(1) $M(x_1,x_2,\ldots,x_n) \le N(x_1,x_2,\ldots,x_n)$ for $n \in \mathbb{N}, x_1,x_2,\ldots,x_n \in I$ (1)

If M and N are quasiarithmetic means then a well-known result of Jensen states that (1) holds if and only if

$$M(x,y) \le N(x,y) \tag{2}$$

for any $x,y \in I$

If M and N are quasiarithmetic means, weighted by a weight function, then (2) does not imply (1), but it is known, that the following inequality is already necessary and sufficient:

$$M(\underbrace{x,x,\ldots,x}_{k},\underbrace{y,y,\ldots,y}_{l}) \le N(\underbrace{x,x,\ldots,x}_{k},\underbrace{y,y,\ldots,y}_{l}) \tag{3}$$

for any $k,l \in \mathbb{N}$, $x,y \in I$

In the talk we investigate that in other classes of means is a necessary and sufficient condition in order that (1) is valid.

J.RÄTZ: On unilaterally bounded orthogonally additive mappings

For a real inner product space X of dimension at least 2, we call a mapping $f:X \longrightarrow \mathbb{R}$ orthogonally additive if it satisfies the conditional Cauchy functional equation

(*) $f(x_1+x_2) = f(x_1) + f(x_2)$ for all $x_1,x_2 \in X$ with $x_1 \perp x_2$.

For solutions f of (*), the following questions are answered:

1) When is f bounded below?
2) When does f assume a minimum?
3) When does every f which is bounded below assume a minimum?

The details will appear in "Aequationes Math",1983 or 1984, under the title "On orthogonally additive mappings".

D.K.ROSS: Inequalities for ratios of integrals

A repeated integration by parts procedure is used to develop a number of stronger as well as new inequalities for the ratios of integrals of the form

$$\int_0^x K_{\alpha+m}(t)dt, \ \alpha \in R, \ m = 0,1,2,3,...$$

where $K_{\alpha+m}(t)$ is a positive kernel and $f(t)$ is an m-times differentiable function with $f^{(m)}(t) \not\equiv 0$ on the non-null interval $I := \{t : 0 \le t \le x\}$

The method is illustrated by taking as the kernel function $K_\beta(t) := t^\beta e^{-pt}$, with $\beta, p \ge 0$. A generalization of a converse to the Cauchy-Schwarz-Buniakowski inequality is found.

D.C.RUSSEL: Remark on an inequality of N.Ozeki

In a paper (in Japanese) in J.Coll.Arts.Sc.Chiba Univ.(1968), N.Ozeki stated without proof an inequality of the form

$$\sum_{k=1}^{n} |a_k|^p \ge c_{np} \min_{1 \le i < j \le n} |a_i - a_j|^p$$

where $a_1, a_2, ..., a_n$ are real numbers, and $p > 0$. A proof was later supplied by D.S. Mitrinović and G.Kalajdžić, Univ.Beograd Publ. Fak.(1980); their proof and the best constant, stated by Ozeki, turns out to be valid only for $p \ge 1$. However (as remarked by S.B.Prešić), we can replace the left side of (1) by

$$\min_{x \in R} \sum_{k=1}^{n} |a_k - x|^p,$$

which allows us to derive from (1) an apparently improved result. A proof can be given, with the correct (and sharp) constant c_{np}, which extends to the full range $0 < p < \infty$, namely:

Theorem: Let $p > 0$, $n \in \{2,3,...\}$, $e_p := \min\{2^{1-p}, 1\}$,

$$c_{np} := \begin{cases} 2(1^p + 2^p + ... + [\frac{1}{2}(n-1)]^p), & n \text{ odd} \\ e_p(1^p + 3^p + ... + (n-1)^p), & n \text{ even} \end{cases}$$

If $a_1, a_2, ..., a_n$ are real numbers and $d := \min_{1 \le i < j \le n} |a_i - a_j|$ then

$$\min_{x \in R} \sum_{k=1}^{n} |a_k - x|^p \ge c_{np} d^p$$

B.SAFFARI: On the best constant in a remarkable inequality of Delange

The following theorem is due to Delange [Bulletin des Sciences Mathematiques, 1982]:

Let $S := \prod_{j=1}^{k} \frac{1}{(1-x_j)}$ denote the sum of the k-tuple series $\sum_{\substack{n_1\geq 0 \\ \dots \\ n_k \geq 0}} x_1^{n_1} \dots x_k^{n_k}$

where $0 \leq x_j < 1$ $(j=1,\dots,k)$. Let H be any hyperplane of $\mathbb{R}^k$ not containing the origin. Then the partial sum corresponding to those k-tuples $(n_1,\dots,n_k)$ which belong to H satisfies the inequality

$$(1) \qquad \sum_{(n_1,\dots,n_k)\in H} x_1^{n_1} \dots x_k^{n_k} < e^{-1} . S$$

This result provides an elegant and short proof of a difficult theorem of Erdös, Ruzsa and Sárközi [Acta Arithmetica, 1973]: If a real (or complex)-valued function defined on the set of positive integers is "completely additive" (i.e. $f(mn) = f(m)+f(n)$ unconditionally), then the set $\{n:f(n) = a\}$ has asymptotic density $\leq e^{-1}$ whenever $a \neq 0$. (A deeper result with "$< e^{-1}$" instead of "$\leq e^{-1}$" was subsequently proved by Ruzsa).

Delange's method of proof, which is based upon an analogue of (1) concerning the exponential function and then a Laplace transform, does not provide the best possible constant (in terms of k) in the right hand side of (1) for fixed k, although e^{-1} is indeed the best possible absolute constant. Our observation is that the best possible constant is

$$\left(\frac{k}{k+1}\right)^{k+1},$$

with equality only if $\lambda_1=\dots=\lambda_k= m$ and $x_1=\dots=x_k= 1/(k+1)$. Our proof is based on a double induction argument.

S.SCHAIBLE: An application of Farkas' Lemma to nonconvex optimization duality

A dual problem is introduced for the following quasiconvex optimization problem: minimize the maximum of finitely many convex-concave ratios subject to convex inequality constraints. The dual is a quasiconcave optimization problem where the minimum of concave-convex ratios is to be maximized subject to convex constraints. The dual is obtained with help of Farkas' Lemma and its generalization to convex inequalities. Duality relations between the primal and the dual are established which resemble those in convex optimization. This work was done in collaboration with R.Jagannathan, University of Iowa.

W.SCHEMPP: Über eine Ungleichung der Radarortung

Es bezeichne $\tilde{A}(\mathbb{R})$ die reelle nilpotente Heisenberg-Gruppe mit eindimensionalem Zentrum $\tilde{Z}$. Die als "peak property" der Radar-Mehrdeutigkeitsfläche bezeichnete Ungleichung

$$H(f;,x,y) \leq H(f;,0,0) \qquad (x,y) \in \mathbb{R} \times \mathbb{R}$$

wird bewiesen, indem die Radar-Autokorrelationsfunktion $H(f;,.,.)$ zur Signaleinhüllenden $f \in \mathcal{S}(\mathbb{R})$ als positiv-definite Funktion auf $\tilde{A}(\mathbb{R})/\tilde{Z}$ aufgefaßt wird. Der radiale Fall, d.h. der Fall, daß die Radar-Ambiguityfunktion $H(f,.,.)$ $SO(2,\mathbb{R})$-invariant ist, wird eingehend betrachtet.

A.SKLAR: Extension of functions satisfying certain systems of inequalities

The inequalities in the title are those, used to define n-monotonic functions. A function F from an appropriate subset of $\mathbb{R}^n$ into $\mathbb{R}$ is n-monotonic, if

$$(*) \quad \Delta_{1,y_1,z_1} \ldots \Delta_{n,y_n,z_n} F \geq 0$$

for all 2n-tuples $(y_1,y_2,\ldots,y_n,z_1,z_2,\ldots,z_n)$ such that $(y_1,y_2,\ldots,y_n)$ and $(z_1,z_2,\ldots,z_n)$ are in Dom F and $y_m \leq z_m$ for $m = 1,\ldots,n$. The difference operators Δ_{m,y_m,z_m} in (*) are defined by

$$(\Delta_{m,y_m,z_m} F)(x_1,x_2,\ldots,x_n) = F(x_1,\ldots,x_{m-1},z_m,x_{m+1},\ldots,x_n)$$
$$-F(x_1,\ldots,x_{m-1},y_m,x_{m+1},\ldots,x_n)$$

for all $(x_1,\ldots,x_n)$ in Dom F. An n-dimensional distribution function (briefly an n-df) is a function $F:\bar{\mathbb{R}}^n \to [0,1]$ such that $F(x_1,\ldots,x_n)=0$ if any $x_m=-\infty$, $F(\infty,\infty,\ldots,\infty) = 1$ and F is n-monotonic.

The margins of an n-d.f. F are the functions $F_m:\bar{\mathbb{R}} \to [0,1]$ defined by $F_m(x_m) = F(\infty,\infty,\ldots,\infty,x_m,\infty,\ldots,\infty)$. An n-copula is a function $C:[0,1]^n \to [0,1]$ such that $C(x_1,\ldots,x_n) = 0$ if any $x_m = 0$, $C(1,1,\ldots,1,x_m,1,\ldots 1) = x_m$ for $m = 1,\ldots,n$ and C is n-monotonic.

An n-copula is necessarily continuous. A fundamental result in probability theory is that, given any n-d.f.F with margins $F_1,\dots,F_n$, there is an n-copula C such that

$$(**)\quad F(x_1,\dots,x_n) = C(F_1(x_1),\dots,F_n(x_n))$$

for all $(x_1,x_2,\dots,x_n)$ in $\bar{R}^n$. The authors proof of this result proceeds by first showing, that

$$F(x_1,\dots,x_n) = C^*(F_1(x_1),\dots,F_n(x_n)),$$

where C^* is an n-monotonic function defined on Ran $F_1 \times\dots\times \mathrm{Ran} F_n$, and then showing that C^* can be extended to an n-copula C. An outline of the proof of this extension theorem will be given.

Remark: Compare also the talk given by C.ALSINA.

B.SMITH: An inequality for dyadic rearrangements

$\overset{d}{\leq}$ denotes a dyadic reordering of $\{1,2,\dots,N\}$. This is gotten by a dyadic grid on $[0,N]$. We are allowed to change the order of the elements of grid member I_1 with respect to those of I_2, if I_1,I_2 have the same larger member of the grid. Let $Q_1,Q_2,\dots,Q_q$ be a partition of

$$A:=\{\frac{2\pi}{N},\frac{2\pi.2}{N},\dots,\frac{2\pi.N}{N}\}: A=\bigcup_{l=1}^{q} Q_l,\ Q_l\cap Q_m=\emptyset \text{ if } l\neq m.$$

Then:

$$\sup_{z_1\overset{d}{\leq}z_2\overset{d}{\leq}\dots\overset{d}{\leq}z_q}\left(\sum_{l}\left(\sum_{\substack{a\in Q_l\\ b\in Q_m\\ (m>l)}}\frac{\sin (a-b)z_l}{a-b}\right)\right)\leq \mathrm{Konst}.N^2$$

This is used to prove theorems on pointwise convergence of Fourier series under rearrangements.

R.SPERB: Inequalities in elliptic problems derived from maximum-principles

Let u be a solution of

$(^*)$ $\Delta u + f(u) = 0$ in Ω, where Ω is a domain on a Riemannian manifold $\mathcal{M}$.
A number of interesting inequalities can be derived in problem $(^*)$

in the following way. One shows that the quantity $P := g(u)|\nabla u|^2+h(u)$ satisfies a maximum principle if g and h are suitably selected. Possible applications are: lower bounds for the first positive eigenvalue λ, for $f(u) = \lambda u$, or the critical value λ^* for $f(u) = \lambda\hat{f}(u)$.

R.L.STENS: Error estimates for sampling approximation

E.T.Whittaker's cardinal series theorem, also known as C.E.Shannon's sampling theorem, states that every entire function f of exponential type $\leq \sigma$ has the representation

$$(1) \quad f(t) = \sum_{k=-\infty}^{\infty} f(\tfrac{k\pi}{\sigma}) \frac{\sin(\sigma t-k\pi)}{(\sigma t-k\pi)} \qquad (t \in \mathbb{R})$$

If f is not an entire function of exponential type, then (1) may hold at least approximately, i.e., in the limit for $\sigma \to \infty$. In this case one is interested in the error

$$(2) \quad E_\sigma(f;t) := \left| f(t) - \sum_{k=-\infty}^{\infty} f(\tfrac{k\pi}{\sigma}) \frac{\sin(\sigma t-k\pi)}{(\sigma t-k\pi)} \right| \quad .$$

The aim of the talk is to give estimates of (2) provided f satisfies certain smoothness conditions.

By similar methods of proof one can deduce bounds for the error when the derivatives $f^{(r)}$ or the Hilbert transform $f^{\sim}$ are approximated by cardinal series.

G.TALENTI: Estimates of eigenvalues of Sturm-Lionville problems

We consider the following problem:

$$-u''+q(x)u = \lambda u \text{ for } -1<x<+1, \; u(-1) = u(+1) = 0;$$

where q is nonnegative and integrable. Let $\lambda(q)$ be the smallest eigenvalue of such a problem. We compute

$$\max \{\lambda(q) : q(x) \geq 0, \; \int_{-1}^{+1} q(x)dx = A\} \quad ,$$

where A is any positive constant.

E.TURDZA: Stability of an iterative linear equation

Some sufficient conditions for stability and iterative stability of linear equations

$$\varphi(x) = G(x)\,\varphi(f(x)) + F(x)$$

and

$$\varphi(f(x)) = g(x)\,\varphi(x) + h(x)$$

will be given. The range of considered functions are subsets of a Banach space.

P.M.VASIĆ: <u>The Jensen-Steffensen inequality as a function of the index set</u>

In 1963 W.N.Everitt has started investigations regarding refinements of general inequalities given in the form of index set functions. Such results for the Jensen inequality were obtained by P.M.Vasić and Z.Mijalković in 1976. An analogous result for the Jensen-Steffensen inequality was not known hitherto. In the lecture, with certain modifications, the corresponding results for the Jensen-Steffensen inequality are obtained.

P.VOLKMANN: <u>Konvergenz der sukzessiven Approximation für Systeme gewöhnlicher Differentialgleichungen</u>

Es sei $T>0$ und $w(t,s) : [0,T]\times[0,\infty) \to [0,\infty]$ eine bezüglich s schwach wachsende Funktion mit folgender Eigenschaft: Ist $N \in \{1,2,3,\ldots\}$, $a \in \mathbb{R}^N$ und $f:[0,T]\times\mathbb{R}^N \to \mathbb{R}^N$ stetig, beschränkt mit $\|f(t,x) - f(t,y)\| \leq w(t,\|x-y\|)$ (wo $\|.\|$:=Maximumsnorm im $\mathbb{R}^N$), so habe das Anfangswertproblem $u(0) = a$, $u' = f(t,u)$ genau eine Lösung $u:[0,T]\to\mathbb{R}^N$. Unter diesen Voraussetzungen wird (mit Hilfe von Ungleichungsmethoden) ein neuer Beweis des Folgenden gegeben: Sind N,a,f wie oben beschrieben, ist $u_0:[0,T]\to\mathbb{R}^N$ stetig und definiert man rekursiv

$$u_n(t) := a + \int_0^t f(\tau,u_{n-1}(\tau))\,d\tau \quad (n=1,2,3,\ldots),$$ so gilt

$$u_n(t) \to u(t) \text{ gleichmäßig auf } [0,T]$$

C.L.WANG: <u>Inequalities and mathematical programming II</u>

The lecture is a continuation of the paper "Inequalities and Mathematical Programming" which was presented at the third International Conference on General Inequalities in 1981. More examples, (e.g. in economics) along a similar line will be given. In particular special attention will be directed to the concept of a transition constraint of a mathematical programming problem.

Such a constraint is one designed to be consistent with the original one(s), so as to facilitate solving the problem. Finally, the van der Waerden inequality concerning permanents will be discussed.

R.J.WALLACE: Optimal strategies for locating zeroes of derivatives

The aim here is to find an optimal strategy for locating the zero of the k'th derivative of a well behaved function on a prescribed finite interval.

The method of subolividing the interval is the problem at hand, and it depends on a particular integer sequence $\{L_k(n)\}$, n=0,1,2,... For any given nonnegative integer k, it is known that $L_k(n) \leq U_k(n)$ for all n = 0,1,2,... ; with equality when k = 0,1,2,3,4, and 6 and strict inequality when k = 5. The integer sequence $\{U_k(n)\}$, n = 0,1,2,... is defined by the initial conditions $U_k(0) = U_k(1) = \ldots = U_n(k)$ and by the rule

$$U_k(N+k+1) = \min_{i=0,1,\ldots,[\frac{k}{2}]} \{U_k(N+i) + U_k(N+k-i)\},\ N = 0,1,2,\ldots .$$

In 1957 R.Bellman described these types of problems as "extraordinarily difficult" but a closed form solution for $U_k(n)$ is herewith presented for the cases $k \equiv 2 (\mathrm{mod}\ 4)$, $k \geq 2$. Although the techniques can be extended to when $k \equiv 0 (\mathrm{mod}\ 4)$ $k \geq 0$, the cases k odd, $k \geq 1$ are presently proving somewhat more difficult.

K.ZELLER: Positivity in summability

Wir betrachten dreieckige Matrixverfahren A, bei denen die Abschnitte oder gewisse bewichtete Abschnitte positive Operatoren darstellen (bezüglich $As \geq 0$). Bei solchen Verfahren erhält man aus leicht beschreibbaren Faktoren positive Linearformen bzw. Operatoren. Besonders interessant ist der autopositive Fall (obige Gewichte durch die $a_{nk} \geq 0$ gegeben); hier findet man weitere strukturelle Aussagen über positive Linearformen (z.B.Multiplikation von Faktoren). Beispiele bieten u.a. die Cesàro-Verfahren C_p (für $0 \leq p \leq 1$ bzw. $p \geq 1$); ein auf Faktoren beruhender Vergleichssatz $(B \supseteq C_p)$ illustriert die Anwendungen. Die Ergebnisse stehen in Zusammenhang mit dem Second Theorem of Consistency (für Riesz-Mittel bzw. stetige Cesàro-Verfahren). Positiv-Dekomposition führt zu einer Ausweitung der Resultate.

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