MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 21/1983

Kommutative Algebra und algebraische Geometrie

16. 5. bis 22. 5. 1983

Die Tagung stand unter der Leitung von E.Kunz (Regensburg), H.-J. Nastold (Münster) und L.Szpiro (Paris).

Ziel der Tagung war es, neuere Ergebnisse aus der kommutativen Algebra und der algebraischen Geometrie darzustellen. Insbesondere sollten Fragen diskutiert werden, die sich aus beiden Gebieten gemeinsam ergeben.

Folgende Einzelgebiete wurden vor allem behandelt:
Raumkurven, Vektorbündel projektiver Varietäten, Deformation von Singularitäten, Liaison lokaler Ringe, Blowing-up von Ringen.

Die Tagung fand auch im Ausland großes Interesse und so waren über die Hälfte der Teilnehmer ausländische Gäste; davon kamen u.a. 9 aus Frankreich, 11 aus Nordamerika und je 2 aus England, Italien und Japan.

Vortragsauszüge

B. ANGENIOL

Global Atiyah classes and Riemann Roch Theorem
(joint work with M. Lejeune)

We prove a Grothendieck-Riemann-Roch theorem for proper (non necessarily projective) morphisms of algebraic schemes or for analytic varieties. Namely, let X,Y be algebraic smooth schemes or analytic varieties over $\mathbb{C}$, $\mathcal{M}$ a perfect complex of $\mathcal{O}_X$-modules, $f : X \to Y$ a morphism, proper on the support of $\mathcal{M}$.

<u>Theorem.</u> $f_*(\mathrm{ch}\,\mathcal{M}\,.\,\mathrm{Todd}(T_X)) = \mathrm{ch}(Rf_*\mathcal{M})\,.\,\mathrm{Todd}\,T_Y$.

The main tool to prove this theorem is the use of the Atiyah classes of $\mathcal{M}$, namely $\gamma^p_{\mathcal{M}} \in \mathrm{Ext}(\mathcal{M}, \mathcal{M} \otimes \Omega^p_X)$ defined as follows $\gamma_{\mathcal{M}}^1$ corresponds to the extension via principal parts of $\mathcal{M}$ and $\gamma^p_{\mathcal{M}}$ is the p^{th} power of $\gamma^1_{\mathcal{M}}$. Then we show that the trace of $\sum_p \frac{\gamma^p\mathcal{M}}{p!}$ is the Chern character of $\mathcal{M}$ in Hodge cohomology. Computing the Atiyah classes rising simplicial resolutions, we give explicit computations for Chern classes in Čech cohomology. To prove the theorem, we treat separately the case of a closed immersion and the case of a projection. In the first case, we go by explicit computation. In the second case, we use the diagonal immersion of X in $X \times_Y X$, and using duality for the first projection from $X \times_Y X$ to X, Künneth formula and explicit computation, we deduce the theorem in this case from the theorem for the diagonal immersion.

J. BINGENER

<u>The local moduli problem for 1-convex spaces</u>

Let X be a 1-convex complex space with exceptional subset $E \subseteq X$. Then, if the germ(X,E) of X around E has a (formal) semi-universal deformation, the support of $\mathcal{T}^1(X,\mathcal{O}_X)$ is contained in E.

Conversely, one conjectures, that (X,E) has a convergent semi-universal deformation, if this necessary condition is satisfied. In a very special case ($\dim(X) = 2$, X smooth) this was shown by Laufer. We prove the above conjecture in the following weaker form:

Theorem. Let $\mathrm{Supp}(\mathcal{T}^1(X,\mathcal{O}_X)) \subseteq E$ and suppose X is locally a complete intersection. Then (X,E) has a convergent formally semi-universal deformation.

M. BRODMANN

Local cohomology and connectedness

For a noetherian scheme X define the following invariant, which measures in which dimension X is connected:

$c(X) = \min\{\dim(Y) \mid Y \subsetneq X,\ Y \text{ closed},\ X - Y \text{ not connected}\}$.

c(X) is called the connectedness-dimension of X. If A is a noetherian ring, we set $c(\mathrm{Spec}(A)) = c(A)$. Now, essentially using an argument of Rung (1979), which bases on the Mayer-Vietoris sequence for local cohomology and the Hartshorne-Lichtenbaum theorem for local cohomology we may prove

Theorem 1. Let $(A,\mathfrak{m})$ be noetherian and local, $I \subsetneq A$ an ideal. Then $c(A/I) > c(\hat{A}) - \alpha(I) - 1$, where $\alpha(I)$ denotes the arithmetic rank of I.

(Recall that $\mathrm{ark}(I) = \alpha(I) := \min\{r \mid \exists\, x_1,\dots,x_r \in I \text{ with } \sqrt{(x_1,\dots,x_r)} = \sqrt{I}\}$). As a corollary we have:

Corollary 2. Let X be a normal, excellent scheme, let $Z \subseteq X$ be a closed set and let $J \subseteq \mathcal{O}_X$ be the ideal of sections vanishing at Z. Then, if Z is connected, it holds $c(Z) \geq \dim(X)-1-\max\{\alpha(J_x)/ x \in Z\}$. Another consequence of (1) is:

Corollary 3. Let $X, Y \subseteq \mathbb{A}^n_k$ (k = alg. closed field) two affine algebraic varieties which meet in a closed point x. Then

$c(\mathcal{O}_{X\cap Y,x}) \geq c((\mathcal{O}_{X\times Y,x\times x})^{\wedge}) - n - 1.$

To get this result, one in fact only has to pass to an embedding into the diagonal and then to apply (1). Now we get

Theorem 4. (Fulton-Hansen, 1979). Let $V, W \subseteq \mathbb{P}^n_k$ (k algebraically closed) be two irreducible projective varieties. Then it holds

$c(V \cap W) \geq \dim(V) + \dim(W) - n - 1.$

This is immediate from (3) in passing to affine cones.

In the sequel, let $(A, \mathfrak{m})$ be local, $S = A \oplus S_1 \oplus S_2 \oplus \cdots$ a noetherian graded A-algebra and consider the morphism $\pi : X = \mathrm{Spec}(S) \to \mathrm{Spec}(A)$. Using Mayer-Vietoris, Hartshorne-Lichtenbaum and an argument on gradings we prove the following result of Grothendieck

Proposition 5. $\hat{X} = X \times_{\mathrm{Spec}(A)} \mathrm{Spec}(\hat{A})$ connected $\Longleftrightarrow \pi^{-1}\{\mathfrak{m}\}$ connected.

Combining (2) and (5) we get the following sharpened version of the Zariski connectedness theorem

Corollary 6. Assume that $\hat{X}$ is connected, let $J \subsetneqq A$ be an ideal and put $Z = \pi^{-1}(V(J))$. Then Z is connected and satisfies:

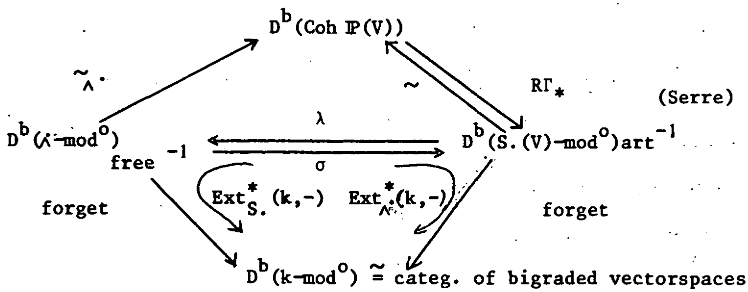
$c(Z) \geq c(\hat{X}) - 1 - \max\{\alpha(J\mathcal{O}_{X,x}) / x \in Z\}.$

R.-O. BUCHWEITZ

A characterization of modules over the Weyl-algebra

Let V be a finite-dimensional vectorspace over a field k, $\mathbb{P}(V)$ the projective space of lines in V, $\Lambda^{\cdot}(V)$ the exterior-algebra on V, $S.(V) = \mathrm{Sym}_*(V^*)$ the polynomialring on V^*.

In 1978, Bernstein-Gelfand-Gelfand showed that the following diagram is commutative and consists of equivalences of categories (in the upper triangle):



If $N = \bigoplus_i N_i$ is in $\Lambda^{\cdot}$-mod^o (the category of graded $\Lambda^{\cdot}$-modules and degree zero morphisms), $\tilde{N}^{\wedge}$ is the complex $\tilde{N}_i^{\wedge} = N_i \otimes \mathcal{O}_i$ and the differential is determined by the action of $V = \Lambda^1$ on N_i, $N_i \otimes V \to N_{i+1}$.

We prove an analogous statement in the following context:

Let $W(V) = T_k(V \oplus V^*) / ([v^*,v] = v^*(v),\ [v,v'],\ [v^*,v'^*])_{v,v' \in V,\ v^*,v^* \in V^*}$ be the Weyl-algebra over a field k of characteristic zero,

$$\mathcal{H} = T_k(V + V^* + kz) \Big/ ([v^*,v] = V^*(v)z^2,\ [v,v'],\ [v^*,v^*],\ [v,z]\ ,[v^*,z])$$

the Heisenberg algebra, i.e. the enveloping algebra of the (super-)Lie algebra $\mathfrak{h}$ on $(V + V^* \oplus ku)_+ \oplus (kz)$ with the only non-trivial commutators $[v^*,v] - v^*(v)u$, $[z,z] = 2u$. Let $\Delta = \mathcal{H}^*(\mathfrak{h}, k) = \mathrm{Ext}^*_{\mathcal{H}}(k,k)$. Then we have the following commutative diagram.

$$\begin{array}{ccc}
D^b(\Lambda\text{-mod}^o) & \underset{\lambda}{\overset{\sigma}{\rightleftarrows}} & D^b(S.\text{-mod}^o) \\
\text{forget} \uparrow\downarrow\ r = \mathrm{RHom}_\Lambda(\Delta,-) & & \text{forget} \downarrow\uparrow\ \text{right adjoint } r = \mathrm{RHom}_{\mathcal{H}}(S,-) \\
D^b(\Delta\text{-mod}^o) & \underset{\sigma}{\overset{\varkappa}{\rightleftarrows}} & D^b(\mathcal{H}\text{-mod}^o) \\
\downarrow & & z \downarrow \\
D^b(\Delta\text{-mod}^o)_{r(\Lambda\text{-mod}^o)} & \rightleftarrows & D^b(W\text{-mod}) \simeq D^b(\mathcal{H}\text{-mod}^o)_{\text{forget}(D^b(S\text{-mod}^o))}.
\end{array}$$

$\varkappa$ and σ are naturally defined by the fact that $\mathcal{H}$. and Δ are Koszul-algebras in the sense of Löfwall et al.

For the proof we use results by Bernstein, Björk and others on modules over W as well as Quillen's results on the cohomology of graded Lie-algebras.

It still remains to characterize the holonomic (= of the Bern-stein-class), holonomic and tame, monochromic modules over W as an (abelian) subcategory of $D^b(\Delta\text{-mod}^o)_{r(\Lambda^\cdot\text{-mod}^o)}$.

E.D. DAVIS

<u>Hilbert-function-complete-intersections and the Cayley-Bacharach theorem</u>

(joint work with A.V. Geramita and P. Maroscia)

Our effort to come to a fresh understanding of P.Dubreil's work of fifty years ago led us to an elementary proposition concerning the two variable polynomial algebra modulo a regular sequence of forms. Among its several applications are: new and simple proofs of Dubreil's estimates of the minimal number of forms required to generate perfect homogeneous polynomial ideals of height 2; further developments, more strongly emphasizing the role of the Hilbert function, of Dubreil's treatment of space curves of the first kind; a complete analysis of those 0-dimensional subschemes of $\mathbb{P}^2$ having the Hilbert function of a complete intersection. The talk will concentrate on the last of these applications, in particular presenting a definitive solution to the problem of determining which among those finite subsets of $\mathbb{P}^2$ having the "Cayley-Bacharach" property are in fact complete intersections.

M. DESCHAMPS

Propriétés de descente des variétés à fibré cotangent ample.

On démontre la généralisation suivante du théorème de Manin pour les courbes:

Théorème. Soit k un corps algébriquement clos de caractéristique 0, L un corps de fonctions sur k. Soit X une variété propre et lisse sur L, telle que le fibré cotangent $\Omega^1_{X|L}$ soit ample. Alors si X(L) (ensemble des points rationnels de X) est Zariski-dense dans X, il existe une variété X_o sur k, et un L-isomorphisme

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$X_o \times_k L \tilde{\to} X$. De plus $X(L) - X_o(k)$ est fini.

Rappelons qu' un fibré E sur une variété X est ample si et seulement si le fibré de rang 1 $\mathcal{O}_p(1)$ (quotient universel de E), est ample sur $\mathbb{P}(E)$. Cette définition est due à Hartshorne (Ample Vector Bundles, Publ. Math. I.H.E.S. 1966).

Exemples de variétés à fibré cotangent ample:

1) Toute courbe lisse de genre $g \geq 2$.

2) (exemple du à Bogomolov) Soit X une surface lisse, intersection complète dans $\mathbb{P}^n$, telle que $c_1^2 - c_2 > o$. On considère sur le produite $X^d (d \geq \sigma)$ un diviseur très ample H, et soit $Y = H^{2d-2}$ intersection de (2d-2) hyperplans assez généraux que est une surface projective et lisse. Alors Ω_y^1 est ample.

3) Soit $f : X \to C$ une fibration au-dessus d'une courbe lisse et propre C, telle que les fibres soient des courbes propres, lisses et irréductibles, de genre $g \geq 3$, et telle que la flèche de Kodaira-Spencer: $\Theta_c \to R^1 f_* \omega_{X|C}$ soit partout non nulle sur C (Il existe de telles fibrations, à cause des propriétés du module des courbes). Alors $\Omega^1_{X|k}$ est ample.

Esquisse de la démonstration du théorème: on le démontre dans le cas où L est le corps de fonctions d'une courbe C sur k, le cas général s'en déduit par récurrence sur le degré de transcendance de L sur k.

On considère la suite exacte de modules de différentielles, associée au morphisme structural $\pi : X \to L$

$$0 \to \pi^* \Omega_{L|k} \simeq \mathcal{O}_X \to \Omega_{X|k} \to \Omega_{X|L} \to 0$$

et on montre, en utilisant le fait que $\Omega_{X|L}$ est ample et que $X(L)$ est dense, que cette suite est scindée, ce qui équivaut à montre que le morphisme de projection $\mathbb{P}(\Omega_{X|k}) - \mathbb{P}(\Omega_{X|L}) \to X$ admet une section.

Ce scindage de la suite donne un champ de vecteurs sur X, tangent à tous les points rationnels de X(L), excepté un nombre fini.

On se fixe ensuite un plongement n-canonique de X (Il en existe car le faisceau $\omega_X = \bigwedge^{\max} \Omega^1_X$ est ample) et on relève le champ de vecteurs sur X en un champ de vecteurs sur l'espace projectif. En choisissant bien le système de coordonnées dans l'espace projectif, l'image de X est alors défini par un idéal "défini sur k", ce qui fournit la variété X_o cherchée.

P. EAKIN

Relations among n + 1 power series in n variables

(joint work with G. Harris)

Let k be an algebraically closed field of characteristic zero and Ψ a k-homomorphism of $k[[X_1,\dots,X_p]]$ into $k[[Y_1,\dots,Y_n]]$. Let $\Psi(X_i) = \varphi_i$ and $J = \left(\frac{\partial \Psi(X_i)}{\partial Y_j}\right)$ the jacobian matrix. These are equivalent:

1. If rank J = s then the p-tuple $(\varphi_1,\dots,\varphi_p)$ can be modified to $(Y_1,\dots,Y_s,0,\dots,0)$ by a finite sequence of changes of the following types:

   (a) $(\varphi_1,\dots,\varphi_p) \to (\sigma(\varphi_1),\dots,\sigma(\varphi_p))$, σ an automorphism of

$k[|Y_1,\ldots,Y_n|]$.

(b) for some i, $\varphi_i \to \varphi_i + g(\varphi_1,\ldots,\hat{\varphi}_i,\ldots,\varphi_p)$ where $g \in k[|X_1,\ldots,X_{p-1}|]$.

(c) for some i, $\varphi_i \to {}^{\varphi_i}/\varphi_j$ provided the ratio is a non unit in $k[|Y_1,\ldots,Y_n|]$.

(d) for some i, $q \in \mathbb{N}$, $\varphi_i \to \varphi_i^{1/q}$ provided the root exists in $k[|Y_1,\ldots,Y_n|]$.

2. By a finite sequence of blow-ups of the maximal ideal one can arrive at $k[|Y_1,\ldots,Y_n|] \subseteq k[|Z_1,\ldots,Z_n|]$ where $\operatorname{rank}_{k((z))}\left(\frac{\partial Y_i}{\partial Z_i}\right) = n$ and $\{\varphi_1,\ldots,\varphi_p\} \subset k[|Z_1,\ldots,Z_s|]$.

If, moreover, the φ_i are convergent then the operations of (1) and (2) can be carried out within the convergent series and the following are equivalent:

3. $s < p$.

4. There exists a divergent series $F(X_1,\ldots,X_p)$ such that $F(\varphi_1,\ldots,\varphi_p)$ is convergent in $Y_1,\ldots,Y_n$.

5. There exist a family of polynomials $\{f_n\}_{v=1}^{\infty} \subset k[|X_1,\ldots X_p|]$ and a polydisc $U \subseteq k^n$ such that

(i) f_v is of degree at most v.

(ii) $\max\{\|\xi\| \mid \xi$ is coefficient of $f_v\} = 1$

(iii) $\lim\limits_{v\to\infty} \max\limits_{u\in U} \|f_v(\varphi_1(u),\ldots,\varphi_p(u))\|^{1/v} = 0$

Since the steps in (2) are reversible we see that the p's zeroes produced from $(\varphi_1,\ldots,\varphi_p)$ represent that many formal relations among the φ_i's which will generally be expressible as formal power series in rational functions of the φ_i. Then (5) tells us that analytic functions $\{\varphi_1,\ldots\varphi_p\}$ have a non vanishing jacobian matrix if and only if the zero function cannot be too well approximated by (normalized) polynomials in $\{\varphi_1,\ldots,\varphi_p\}$.

G. FALTINGS

Semistable vector bundles on Mumford-Curves

Let V be a discrete valuation ring, X_η a Mumford-curve over the generic point η = Spec(R) of V. To any representation $p : \Gamma \to GL(r,R)$ of the fundamental group Γ of X_η there is associated a vecturbundle $\mathcal{E}_p$ on X_η. We prove, that this gives a bijection between a certain subclass of representations of Γ, and the set of semistable vectorbundles on X_η.
The proof uses a total order on Γ, well ordered sets and local cohomology.

(Außerhalb des eigentlichen Programmes berichtete G. Faltings noch in informeller Weise über "The Tate conjecture on homomorphisms of abelian varieties over a number field". In Weiterentwicklung der dabei benutzten Methoden konnte er kurze Zeit später die Mordell-Vermutung beweisen.)

H. FLENNER

Restrictions of semistable bundles to hypersurfaces

Suppose X is a normal projective variety over the field k and $\mathcal{O}_X(1)$ is a very ample sheaf on X. Then we have discussed the following question in this talk. If $\mathcal{E}$ is a semistable torsion free sheaf on X, is then the restriction of $\mathcal{E}$ to a general hypersurface of degree d also semistable? It is well known and seen by obvious examples that this is not true in general. On the other hand Maruyama has shown such a restriction theorem if $rk(\mathcal{E}) < \dim X$, and Mehta and Ramanatan have proven that this is true if $d = d(\mathcal{E})$ is very large; here d is always dependent on $\mathcal{E}$. In this talk we have shown: If char $k = o$ and if

$$\frac{\binom{n+d}{d}-1-d}{d} > \frac{\deg X}{4} r^2 \text{ , } r = rk(\mathcal{E}), \ n = \dim X$$

then $\mathcal{E}_{|H}$ is semistable for a general hypersurface of degree d. In order to prove this result we stated a generalized version of the Grauert-Mühlich-theorem: If $\mathcal{E}$ is semistable on X, and if $\mathcal{E}_o \subsetneqq \mathcal{E}_1 \subsetneqq \dots \subsetneqq \mathcal{E}_{|H}$ is the Harder-Narasimhan filtration of $\mathcal{E}_{|H}$ then

$$o < \mu(\mathcal{E}_i/\mathcal{E}_{i-1}) - \mu(\mathcal{E}_{i+1}/\mathcal{E}_i) \leq \frac{d^2 \deg X}{\binom{n+d}{d}-1-d} .$$

R. FOSSUM

Factorial Rings in characteristic $p > o$

Let k be a field and $F : k[[T]] \to k[[X,Y]]$ a 1-dimensional formal group law over k. Let V be a k[[T]]-module of finite length. Let $S.(V) = \bigoplus_{r \geq o} \mathrm{Sym}_r(V)$. Then k[[T]] acts on S.(V) via F by extending from the 1-forms according to the rule

$$T(fg) = (Tf)g + f(Tg) + \sum_{i,j\geq 1} c_{ij}\,(T^i f)T^j g$$

(where $F(X,Y) = X + Y + \Sigma\, c_{ij}X^iY^i$).

<u>Theorem.</u> Let $S.(V)^F := \{f \in S(V) : Tf = o\}$. This is a normal noetherian <u>factorial</u> domain. If char $k = o$, then Kdim $S.(V)^F = rk_k V - 1$. If char $k = p > o$, then Kdim $S.(V)^F = rk_k V$.

If $F = X + Y$ and $V_q := k[[T]]/(T^q)$, where $q = p^\ell$, then the Hilbert Poincaré series of the graded ring $S.(V)^F$ is

$$(1-t^p)^{-p^\ell - 1} + p^{-\ell}((1-t)^{p^\ell} - (1-t^p)^{-p^\ell - 1}).$$

The H.P. series for V_{q-1} is

$$(1-t^p)^{-p^{\ell-1}} + (1-t)_p^{-\ell}\,((1-t)^{p^\ell} - (1-t^p)^{-p^{\ell-1}}).$$

Thus $S.(V)^{X+Y}$ is not Cohen-Macaulay when $g - 1 \geq 4$.

H.-B. FOXBY

<u>The support of a non-finitely generated module</u>

For a (non-f.g.) module M over a Noetherian (commutative) ring A the following notation $\mathrm{supp}_A M := \{p \in \mathrm{Spec}A \mid \exists \ell : \mathrm{Tor}_\ell^A(A/p, M)_p \neq 0\}$, the <u>small support</u> of M, has, in many ways, nicer properties than the usual support $\mathrm{Supp}_A M$ (= $\{p \in \mathrm{Spec}A \mid M_p \neq 0\}$). The following hold:

1). $\mathrm{Ass}_A M \subseteq \mathrm{supp}_A M$.

2). $\mathrm{supp}_A M \neq \emptyset \Leftrightarrow M \neq 0$.

3). $\mathrm{supp}_A M \subseteq \mathrm{Supp}_A M$ with equality if M is f.g.

4). $\mathrm{supp}_A(M \overset{L}{\otimes}_A N) = \mathrm{supp}_A M \cap \mathrm{supp}_A N$, when also N is an A-module.

5). $\mathfrak{m} \in \mathrm{supp}_A \mathrm{RHom}_A(M,N) \Leftrightarrow \mathfrak{m} \in \mathrm{supp}_A M \cap \mathrm{supp}_A N$, when $\mathfrak{m}$ is a maximal ideal.

6). $\mathrm{depth}_{A_p} M_p + \dim A/p = \dim A$ for all $p \in \mathrm{supp}_A M$ if and only if M is a balanced big CM module (and that is, each s.o.p. for A is a regular sequence for M). This is provided A is local.

7). If $A \to B$ is a morphism of Noetherian rings and N is a f.g. B-module, then $\{\mathfrak{q} \cap A \mid \mathfrak{q} \in \mathrm{Supp}_B N\} = \mathrm{supp}_A N$.

Applications of 6).: Let M be a balanced big CM module and A be local. Then the following hold:

a) If A is catenary and $p \in \mathrm{supp}_A M$, then M_p is a balanced big CM as A_p-module.

b) If N is an A-module of finite flat dimension, then $\mathrm{Tor}_i(M,N) = 0$ for $i > o$.

[(7) was suggested by Matsumura. (a) has been proved by Sharp, when A, in addition, is a domain].

A. GERAMITA

<u>The Position of Points in $\mathbb{P}^n(k)$</u>

Let $k = \bar{k}$, $R = k[x_o,x_1,\dots,x_n]$, $S = k[x_1,\dots,x_n]$ and $P_1,\dots,P_o$ be distinct points in $\mathbb{P}^n(k)$. If $P_i \leftrightarrow p_1 \subseteq R$, $I = p_1 \cap \dots \cap p_s$ then $A = {}^R/_I$ is the homogeneous coordinate ring of $P_1,\dots,P_s$ in $\mathbb{P}^n$. Since $A = \oplus A_i$, let $H(A,t) = \dim_k A_t$ be the Hilbert function. We say $P_1,\dots,P_s$ are in <u>generic position</u> in $\mathbb{P}^n$ if $H(A,t) = \min\{\binom{t+n}{n}, s\}$, $\forall n$. The s-tuples of points in generic position describe an open ($\neq \emptyset$) set

$V_{n,s} \subseteq (\mathbb{P}^n)^s$. We consider the functions $\nu(I)$ = min. number generators of I, and $r(A)$ = Cohen-Macaulay (C-M) type of A, as functions on $V_{n,s}$ and seek the "generic" value of these functions on $V_{n,s}$. There is an expected value and the main object of the talk was to show how verifying that the expected value occur is related to some combinatorial problems about monomial ideals.

If $S_n(d)$ denotes the graph whose vertices are the $v_n(d) = \binom{d+n-1}{n-1}$ monomials of deg = d in S and whose edge set is defined by: the monomials x^α and x^β are adjacent if $\exists i,j \ni : x^\alpha \left(\frac{x_i}{x_j}\right) = x^\beta$.

A subset $T \subset S_n(d)$ is a clique if any two distinct elements of T are adjacent. <u>Fact:</u> A clique in $S_n(d)$ contains $\leq n$ elements.

The cliques with exactly n elts. are: upward cliques (obtained by multiplying a monomial of deg = d - 1 by $x_1, x_2, \ldots, x_n$ (respectively)); downward cliques (obtained by dividing a monomial of deg = d + 1 of the form $x_1 \cdot \ldots \cdot x_n f$ by $x_1, \ldots, x_n$ respectively).

<u>Definition.</u> $f_n(d)$ = min $|\mathfrak{F}|$, $\mathfrak{F}$ a family of upward cliques in $S_n(d) \ni$: every vertex of $S_n(d)$ is on a member of $\mathfrak{F}$;

$\alpha_n(d)$ = max. number of mutually non-adjacent vertices of $S_n(d)$.

$\tau_n(d)$ = min $|\mathfrak{F}|$, $\mathfrak{F}$ a family vertices $S_n(d) \ni$ every upward clique contains a vertex of $\mathfrak{F}$.

<u>Theorem.</u> i) Write $s = \binom{d+n}{n} + \lambda$, $o < \lambda < \binom{d-1+n}{n}$. The expected value of $r(A)$ occur on $V_{n,s}$ when $o < \lambda < \alpha_n(d-n)$ and when $\tau_n(d) \leq \lambda < \binom{d-1+n}{n-1}$.

ii) Write $s = \binom{d+n}{n} - \lambda$, $o < \lambda < \binom{d-1+n}{n-1}$. The expected value for

$\nu(I)$ occur when $o < \lambda \le \alpha_n(d)$ and when $p_n(d+1) \le \lambda < \binom{d-1+n}{n-1}$.

We have obtained the following values for α, p, τ:

$\tau_3(d) = \left\lfloor \frac{\upsilon_3(d)}{3} \right\rfloor$, $d \ge 2$; $\alpha_3(d) = \left\lfloor \frac{\upsilon_3(d)}{3} \right\rfloor$, $d \neq 3$, $d \ge 5$; and

$\frac{\upsilon_3(d)-\alpha_3(d-1)}{2} \le p_3(d) \le \left\lfloor \frac{\upsilon_3(d+2)}{3} \right\rfloor - 3$, $d \ge 3$. Also,

$$\alpha_4(d) = \begin{cases} \frac{\upsilon_4(d)}{4}, & d \text{ odd} \\ \frac{\upsilon_4(d)}{4} + \frac{3d+6}{8}; & d \text{ even.} \end{cases}$$

Also, $\forall n \ge 3$, $\tau_n(3) = \left\lfloor \frac{\upsilon_n(3)}{n} \right\rfloor$, $f_n(3) = \left\lfloor \frac{n^2+2n}{4} \right\rfloor$ and

$\alpha_n(3) = n + \left\lfloor \frac{n}{3} \left\lfloor \frac{n-1}{2} \right\rfloor \right\rfloor - \varepsilon$, $\varepsilon = 1$ if $n \equiv 5 \pmod 6$, $\varepsilon = o$ otherwise. The calculation of $\alpha_n(3)$ is the counting of Steiner triple systems in an n-set and $p_n(3)$ is Turan's theorem stating that a bipartite graph on n vertices (whose vertex sets are as equal in size as possible) has more edges than any other graph on n verties having no 3-cycle.

(This is joint work with L. Roberts and D. Gregory).

S. GRECO

Normal singularities and rational surfaces

(Joint work with A. Vistoli)

Let x be a singular normal point of the complex analytic surface X, and assume that there is a resolution $\tilde{X} \to X$ of x whose exceptional curve is smooth of genus g. Assume further that

$c^2 < 4 - 4g$, and that one of the following conditions is satisfied:

(a) C is a plane curve; (b) $g \leq 7$; (c) $c^2 < -g^2 + 3g + 6$;

(d) C is hyperelliptic.

Then there is a compact rational analytic surface Y , with a singular point y, such that the germs (X,x) and (Y,y) are isomorphic. If moreover $g \leq 1$, then Y can be an algebraic rational surface.

G.-M. GREUEL

Another characterization of simple curve singularities

(Joint work with H. Knörrer.)

The simplest smooth projective curve is certainly $\mathbb{P}^1(\mathbb{C})$, at least for two reasons: 1st $\mathbb{P}^1$ has no moduli, 2nd the line bundles on $\mathbb{P}^1$ have no moduli. Now, asking for the simplest singularities on curves one is led, from the first point of view, to the simple singularities of Arnold. Recall, that an isolated singularity is called simple if, in its semiuniversal deformation space there are only finitely many (analytic) isomorphism classes of singularities. Classifying these simple singularities Arnold found in 1972 the famous list (of hypersurface singularities) named

$$A_k (k \geq 1),\ D_k (k \geq 4),\ E_6\ ,\ E_7\ ,\ E_8\ ,$$

a list which has occurred since then in many other different contexts.

Formulating the second point of view for singularities we define a curve singularity to be <u>module-simple</u> if there are only finitely many isomorphism classes of finitely generated rank 1 torsion free modules over its local ring. The following theorem, proved by computing and case by case checking is (at least for the moment) another miracle in the history of simple singularities.

<u>Theorem.</u> A <u>plane</u> curve singularity is simple if and only if it is module-simple.

Moreover we gave a complete list of all modules which occur.

R. HARTSHORNE

<u>Curves on quartic surfaces in $\mathbb{P}^3$ (after Mori).</u>

<u>Theorem.</u> $(k=\mathbb{C})$. Let $d \geq 1$, $g \geq o$ be integers. There exists an irreducible nonsingular curve C of degree d and genus g in $\mathbb{P}^3$, lying on a n.s. quartic surface X (depending on C), if and only if either

(a) $g = \frac{1}{8} d^2 + 1$ or

(b) $g < \frac{1}{8} d^2$ and $(d,g) \neq (5,3)$.

Case (a) corresponds to a complete intersection curve.
The necessity in case (b) is proved using the Hodge index theorem.

To prove existence, one first constructs an abstract K3-surface and then embeds it with degree 4 in $\mathbb{P}^3$. First one takes a product of elliptic curves, and constructs divisor classes H_o, C_o on the associated Kummer surface such that $H_o^2 = 4$, $H_o C_o = d$, $C_o^2 = 2g - 2$. Then one takes a sufficiently general deformation of the Kummer surface so as to get on K3-surface with similar divisor classes H , C , and with the further property that pic X $\cong$ $\mathbb{Z}H \oplus \mathbb{Z}C$. Then one shows that H is very ample and |C| contains an irreducible nonsingular curve, using the results of Saint-Donat on K3-surfaces. The embedding of X by |H| then gives a quartic surface in $\mathbb{P}^3$ with the desired curve on it.

J. HERZOG

On the divisor class group of blowing-up rings

(joint work with Vasconcelos)

Theorem. Let I be an ideal in a normal domain whose associated graded ring $gr_I(R)$ is a domain. Let g = grade I $\geq$ 2, then

(a) The Rees-algebra $S = R[It] = \underset{n\geq o}{\oplus} I^n t^n$ is normal

(b) There is an exact sequence

$$0 \to \mathbb{Z}[S_+] \to Cl(S) \to Cl(R) \to 0$$

where S_+ denotes the irrelevant ideal of S.

(c) The sequence (b) splits if I is generically a complete intersection, i.e. $Cl(S) = Cl(R) \oplus \mathbb{Z}[S_+]$.

(d) If, moreover, R is Gorenstein and S is Cohen-Macaulay, then $[K_S] = -(g-2)\ [S_+]$, where K_S is the canonical module of S. In particular (S Gorenstein $\Longleftrightarrow$ g = 2) and ($Cl(S) \simeq \mathbb{Z}[K_S] \Longleftrightarrow$ g = 3).

(e) Under the assumptions of (d) one has $K_S \cong (x,xt)^{g-2}$, where

$x \in I$.

(f) If, in contrary to the above assumptions $g = 1$, then $Cl(R) \cong Cl(S)$.

M. HOCHSTER

Modules of finite length and finite projective dimension with negative intersection multiplicities

Let $S = K[X_1,X_2,X_3,X_4]_{\mathfrak{n}}$, where K is any field and $\mathfrak{n} = \sum_{i=1}^{n} X_i S$ and let $R = S/(X_1X_4-X_2X_3)$.

A family of modules of length 15 and finite projective dimension 3 is constructed over R such that for the module M in the family we have

1) $\mathfrak{m}^3 M + (x_2,x_4)\mathfrak{m} M = 0$, where $\mathfrak{m} = (x_1,x_2,x_3,x_4)R$

2) each M has Betti numbers 6, 17, 16, 5,

i.e. the minimal resolution of M is of the form

$0 \to R^5 \to R^{16} \to R^{17} \to R^6 \to M \to 0$

3) $\chi(M,R/P) = -1$, where $P = (x_1,x_2)R$

and χ denotes the Serre multiplicity

$$\chi(M,N) = \sum_{i=0}^{pdM} (-1)^i \, l(\mathrm{Tor}_i(M,N)),$$

defined when $pdM < \infty$ and $l(M \otimes N) < \infty$.

It is shown that for modules killed by $\mathfrak{m}^3 + (x_2,x_4)\mathfrak{m}$ such that $pdM < \infty$ and with $\chi(M,R/P) \neq 0$, one must have $l(M) \geq 15$; in fact $l(M) \geq 15|\chi(M,R/P)|$.

This disproves the generalized Serre Conjecture on multiplicities and various other conjectures. E. g. one can also construct M of finite projective dimension 2 over a 2-dimensional ring with $\chi_1(M,N) = l(Tor_1(M,N)) - l(Tor_2(M,N)) < 0$, and that the Grothendieck group $\mathcal{K}(R)$ of modules of finite length and finite projective dimension is not generated by the classes $[R/(u_1,u_2,u_3)]$, where u_1,u_2,u_3 is a maximal R-sequence.

A. HOLME

<u>A computer approach to smooth codimension 2 subvarieties of $\mathbb{P}^N$, $N \geq 6$</u>

It is an interesting open question if all such varieties are complete intersections. For $N \geq 7$ this would follow from a more general conjecture by Hartshorne, in "Varieties of low codimension in projective space", Bull. Amer. Math. Soc. Vol. 80, 1974, pp. 1017 - 1032.

We consider here the problem of showing that all such varieties have the Chern numbers of complete intersections, i.e., are "of numerical c.i. type" or equivalently that the associated rank 2 bundle E have Chern numbers $c_1 = a + b$, $c_2 = ab$, $a,b \in Z$. This is partially verified for $d = c_2 \leq 3000$ by a computational method. The domain of possible counterexamples left by the computation is rather small, and diminishes rapidly as N increases.

For N = 6 the first few such values of $c_2(E) = \deg(X)$ and $c_1(E)$ are: (54,19), (60,20), (66,23), (68,24), (72,23), (74,15), (74,21). The first case with negative discriminant is (74,15).

An interesting question is whether an E with Chern numbers as above is always splitt. This is a weakened version of a conjecture by Grauert and Schneider.

G. HORROCKS

A bundle on $\mathbb{P}^6$

The simple extensions $\mathrm{Ext}^1(T^{(r)}_{(i)}, T^{(s)}_{(j)})$ of twisted exterior powers of the tangent bundle T are classified by elements ω of $\Lambda^{s-r+1}V$, where V is (n+1)-dimensional vector space and T is the tangent bundle to $\mathbb{P}^n$. Such an extension has a "free" direct summand of rank $\omega \wedge \Lambda^r V$. So the complementary summand is zero or of rank at least $n-1$. The latter is the Pfaffian or null-correlation bundle. The only other exterior powers - up to duality - which have dense orbits are $\Lambda^3 V$ (n=5,6,7). Our bundle M is on $\mathbb{P}^6$; it is of rank 9 and since ω is stabilized by G_2 it carries a G_2-action. Its chern polynomial is

$$c(M) = 1 + 3h^2 - 28h^6 = (1-h^2)^3(1+6h^2+15h^4) .$$

The polynomial on the right satisfies Schwarzenberger's congruences. So it is a natural question to look for α, β

$$3\mathcal{O}(-1) \xrightarrow{\alpha} M + \mathcal{O} \xrightarrow{\beta} 3\mathcal{O}(1)$$

with $\mathrm{Ker}\beta/\mathrm{Im}\alpha$ of rank 4. I have not solved this question.

The existence of α, β depends on the structure of the mapping

$$\Gamma(M(-1)) \times \mathrm{Hom}(M, \mathcal{O}(1)) \to \mathcal{O}(2)$$

given by composition of maps. It is canonically isomorphic as a G_2-mapping to a non-associative (in fact non-power associative) commutative algebra structure on $V^{1,2,0} \oplus \mathbb{C}$, $V^{2,0}$ the irreducible G_2-module of weight (2,0) and dimension 27.

C. HUNECKE

Numerical Invariants of Liaison

Let S be a Gorenstein local ring and $R = S/I, R' = S/T$ two quotients of S. R and R' are said to be linked (directly) if there is a regular sequence $x_1,\ldots,x_q$ in $I \cap T$, such that $(\underline{x}:I) = T$ and $(\underline{x}:T) = I$. We write $R \cup R'$. If $\exists\, R = R_0 \cup R_1 \cup \ldots \cup R_n = R'$ we say R and R' are linked and write $R \sim R'$. Set $L(R) = \{R' \mid R' \sim R\}$. We wish to give some invariants of these liaison classes.

Fix $R = S/I$. $I = (y_1,\ldots y_n)$ some generating set. If M is an R module, set $H_i(y;M) = H_i(M)$, the homology of the Koszul complex of $(y_1,\ldots,y_n) \otimes M$. If $M = S$, set $H_i(S) = H_i$. Suppose M is a module such that

1. M is perfect of dimension equal to $g = \mathrm{codim} R$,
2. $l(M \otimes R) < \infty$,
3. M satisfies the vanishing property of Serre,
4. M is rigid, i.e. $\chi(M,N) = 0$ implies $\mathrm{Tor}_i(M,N) = 0$.

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Set $k_i(y,M) = l(H_i(M)) - \binom{n-q}{i} \chi(R,M)$; and let $P_{R,M}(t) = \Sigma\, k_i(y_i M)t^i \,/\, (1+t)^m$ where $(1+t)^m$ exactly devides the polynomial in the numerator. Our main result:

Theorem. If R and R' are linked in an even number of steps, then $P_{R,M}(t) = P_{R',M}(t)$ provided M satisfies 1. - 4. above for all links involved.

Also, $t^i \mid P_{R,M}(t) \iff H_o,\dots,H_i$, are maximal Cohen-Macaulay modules over R. We are able to conclude

Corollary. If L(R) contains a complete intersection, then $P_{R,M}(t) \equiv 0$. In particular, all Koszul homology on a generating set of I is either zero or Cohen-Macaulay.

Finally we may apply this result to the vanishing of various cotangent functors. One application:

Theorem. Let R be Gorenstein and in the linkage class of a complete intersection. Here suppose $S = k[[x_1,\dots,x_n]]$. Then $T^2(R/k,R) = T_2(R/k,R) = 0$.

H. LINDEL

Projective modules over graded rings

Let A be a commutative ring, $R = \oplus R_i$ a graded ring with $R_o = A$ which is finitely generated, $R = A[t_1,\dots,t_n]$. Then the following generalizations of "Quillen's patching theorem" holds: Let M be a finitely presented R-module such that for every $m \in Max(A)$ $M_m = A_m \otimes_A M$ is extended from A_m: Then M is extended from A.

For example this theorem implies the following result of A.C.F. Vorst : If R is a discrete Hodge algebra, i.e. $R = A[T_1,\ldots,T_n]/I$. I generated by monomials and if f.g. projective $A[X_1,\ldots X_n]$ - modules are extended from A, then all f.g. projective R-modules are extended from A. Another application is the following result. Let I be an ideal in A, T an indeterminant and R = A[IT] the Rees ring over A with respect to I. Suppose that f.g. projective $A[X_1,\ldots X_n]$-modules are extended from A. If there exists an $h \in A$ with (h,I) = A such that for a f.g. projective R module P, P_h is extended from A_h, then P is extended from A.

M. MILLER

Resolutions of Gorenstein algebras and their behaviour under linkage

(joint work with A. Kustin)

In the classical situation in which a "structure theorem" is available, as e.g. for codimension two Cohen-Macaulay algebras or codimension three Gorenstein algebras, one can explicitly determine the singular locus, Sing(R/I). Our philosophy is to demonstrate that one can control (R_i) by starting with a regular complete intersection and forming "suitably" generic links. In the context of Gorenstein algebras one can maintain (R_6), which is best possible, under "general double link". One proves this by showing that both rigidity and complete intersection locus can be transported across such a general link. Indeed a some stronger condition (LG_i): $\mu(I_P) \leq \max\{g, \ \text{ht } P - i\}$ for all $P \supset I$, can be transported for $i \leq 4$. Finally, one can show that $(R/I)_P$ is regular if it is both rigid and a complete intersection, by examing the DG-algebra structure on a minimal free resolution $\mathbb{F}$ of R/I; namely $(R/I)_P$ is a c.i. if and only if P does not contain $I + F_i^{\text{grade } I}$.

U. ORBANZ

Flat morphism by blowing-up

(joint work with L. Robbiano)

If $\pi : X' \longrightarrow X$ is a blowing-up with center $Y \subset X$, let $D = \pi^{-1}(Y)$ be the exceptional divisor. We give some results in connection with flatness of the induced morphism $\pi' : D \longrightarrow Y$. Locally we have the following situation: R is any local ring, $I \subset R$ any ideal, and $R \longrightarrow R_1$ is a (local) homomorphism obtained by blowing-up R with center I. Then R normally flat along $I \Longrightarrow R/I \rightarrow R_1/IR_1$ flat for all $R_1 \Longrightarrow \text{ht}(I) = \ell(I)$ (the analytic spread) (note that $\text{ht}(I) = \ell(I) \Longleftrightarrow e(R) = e(R_I)$ if R/I is regular and R quasi-unmixed). Conditions are given to reverse the above implications. The flatness of π' has nice geometric properties: It is an open condition, of course, and it has a transitivity property like normal flatness. The proof of transitivity is based on a numerical characterization of the flatness of π' using generalized Hilbert functions. The main result is: If π' is flat, R/I regular, then $H^{(s)}[R_1] \leq H^{(0)}[R]$, where H is the Hilbert function and s = residual transcendence degree of R_1/R. Similar considerations give the following result: If $R/I \to R_1/IR_1$ is flat, I an ideal of the principal class such that R/I is Cohen-Macaulay and $e(R/I) \leq \dim R/I + 1$, and if R_1 is Cohen-Macaulay outside $M(R_1)$, then R_1 is actually Cohen-Macaulay (some technical assumptions on R are suppressed here).

R.Y. SHARP

Generalized fractions and the Monomial conjecture

After a review of generalized fractions, the talk will concentrate on a d-dimensional commutative Noetherian local ring A (with identity) (where $d \geq 1$). We set

$$U = \{(y_1,\ldots,y_d,1) \in A^{d+1} : y_1,\ldots,y_d \text{ is a system of parameter for } A\},$$

a triangular subset of A^{d+1}. Let $x_1,\ldots,x_d$ be a system of parameters for A. The talk will present the result that the Monomial conjecture holds for $x_1,\ldots,x_d$ (that is, for each $j \geq 0$,

$$x_1^j \ldots x_d^j \notin Ax_1^{j+1} + \ldots + Ax_d^{j+1})$$

if and only if $\frac{1}{(x_1,\ldots,x_d,1)} \neq 0$ in the module of generalized fractions $U^{-(d+1)}A$, and will use this to derive the following

Theorem. Suppose that d (= dim A) $\geq$ 2 and the Monomial conjecture is known to be true for all local rings of dimension d - 1.

Then there is a positive integer t (depending on A) such that, for all systems of parameters $x_1,\dots,x_d$ of A, the Monomial conjecture holds for $x_1^t, x_2, x_3, \dots, x_d$.

J. STROOKER

Pre-regular modules

(joint work with J. Bartijn)

Let A be a noetherian local ring with maximal ideal m and residue class field k,d = dim A. If $\underline{x} = x_1,\dots,x_d$ is a system of parameters (s.o.p.), we call a module M $\underline{x}$-pre-regular provided

(i) $(x_1,\dots,x_d)\,M \neq M$,

(ii) all maps $M/(x_1,\dots,x_d)M \xrightarrow{x_1^t\cdots x_d^t} M\,/\,(x_1^{t+1},\dots,x_d^{t+1})\,M$ are injective, $t \geq 1$.

The name is justified by the theorem.

Theorem. If M is $\underline{x}$- pre-regular for some system of parameters, then

(i) Its m-adic completion $\hat{M}$ is regular for every s.o.p.

(ii) $H^d_{\mathfrak{m}}(M)^{\vee}$ is regular for every s.o.p.

Here $H^d_{\mathfrak{m}}$ is the d-th local cohomology functor and $-^{\vee}$ is the Gabriel-Matlis duality.

For finitely generated M, pre-regular implies regular, but not in general.

In case A is a domain of equal characteristic, one can always construct, for a given s.o.p. $\underline{x}$, an $\underline{x}$-pre-regular module M, which has flat dimension 1. Then both $\hat{M}$ and $H^d(M)^\vee$ are big Cohen-Macaulay modules for every s.o.p. in the sense of Hochster. In case A is not Cohen-Macaulay, both of these modules have infinite flat dimension. Moreover M is not regular for any s.o.p.

In the proof of these results one uses an extension to infinitely generated modules of a classical homological identity of Auslander and Buchsbaum.

B. ULRICH

Homological properties which are invariant under linkage

(joint work with R.O. Buchweitz)

Let I and J be two perfect ideals of grade g in a local Gorenstein ring P which lies in the same linkage class. Set R = P/I, S = P/J, and denote the canonical modules by ω_R, ω_S. By constructing complexes which are homotopy equivalent under direct linkage the following theorem is shown:

Theorem. For all i and all j $\neq$ o there exists an isomorphism of P-modules

$$\mathrm{Ext}^j_R\,(\mathrm{Tor}^P_i(\omega_R,R),\ \omega_R) \cong \mathrm{Ext}^j_S\,(\mathrm{Tor}^P_i(\omega_S,S),\ \omega_S)$$

applications

(i) depth $I/I^2 \otimes_R \omega_R$ = depth $J/J^2 \otimes_S \omega_S$

(ii) depth $\mathrm{Hom}_R(I/I^2, R)$ = depth $\mathrm{Hom}_S(J/J^2; S)$

(iii) if for some i and some $q \in \mathrm{Spec}\ R$, $\mathrm{Tor}_i^{P_q}(\omega_{R_q}, R_q)$ is not Cohen-Macaulay, then $J \subseteq q$.

Let $P = k\,[|\,x_1,\dots,x_m\,|]$ and $d = m - g$.

Corollary. If there exists $n \geq o$ such that for all $i \geq 2$, $\dim T_i({}^R/_k, \omega_R) \leq d - n - 3 + i$ and $\dim T_i({}^S/_k, w_S) \leq d - n - 3 + i$, then

$$T^j({}^R/_k, R) \cong T^j({}^S/_k, S) \text{ for } 2 \leq j \leq n + 2 .$$

Corollary. Let R be a Gorenstein ring in the linkage class of a complete intersection, and $n \geq -1$ such that R_q is a complete intersection whenever $\dim R_q \leq n$, then

a) $T^j({}^R/_k, R) = 0$ for $2 \leq j \leq n + 3$

b) if for some $q \in \operatorname{Spec} R$ with $\dim R_q = n + 1$, I_q is minimally generated by $g + 2$ elements, then $T^{n+4}({}^R/_k, R) \neq 0$.

W.V. VASCONCELOS

Algebras of Linear Type

(jointly with J. Herzog and A. Simis)

For a finitely generated module E over the Noetherian domain R, one considers comparisons in the sequence of morphisms of R-algebras

$$S(E) \to B(E) \to C(E) \to D(E)$$

where S(E) = symmetric algebra of E, B(E) = S(E)/R-torsion, and for R = normal, C(E) = integral closure of B(E) and D(E) = graded bi-dual of S(E).

1. Fitting conditions: A first level of comparisons is expressed in terms of the following sliding height conditions ($R^m \xrightarrow{\varphi} R^n \longrightarrow E \longrightarrow 0$)

Definition. $(F_k) \Leftrightarrow ht(I_t(\varphi) = (t\text{-sized minors of } (\varphi)) \geq$
$\geq rk(\varphi) - t + 1 + k, \; 1 \leq k \leq rk(\varphi)$

For R Cohen-Macaulay, one has

1.1 $S(E) = B(E) \Rightarrow (F_1)$

1.2 $S(E) = D(E) \Rightarrow (F_2)$

1.3 $(F_2) \Rightarrow C(E) = D(E)$, with converse if $S(E) = B(E)$.

2. Approximation complexes. Main tool are some subcomplexes of

$$\mathcal{L} = \wedge R^n \otimes S(R^n) \otimes S(E) \; : \; (\tilde{S} = S(R^n))$$

$\mathcal{Z}(E) : 0 \to Z_\ell \otimes \tilde{S}[-\ell] \to \ldots \to Z_1 \otimes \tilde{S}[-\ell] \to \tilde{S} \to 0 \; (\ell = n-rk(E))$.
Since $H_0(\mathcal{Z}(E)) = S(E)$, often properties of $S(E)$ can be read off $Z(E)$ as long as the complex is acyclic. There is a number of such criteria.

J.-L. VERDIER

Fibre's vectoriels sur un germe de surface pointé

I) Soient $V \simeq \mathbb{C}^2$, G un sous-groupe fini de $Sl(V)$, $S = V/G$.
La surface S possede un seul point singulier noté 0 et S désigne dans la suite le germe en 0 de cette surface. La correspondance de McKay etablit des bijections entre les trois ensembles finis suivants

(i) Les classes d'isomorphismes de fibrés vectoriels indécomposables sur S - {O}.

(ii) Les sommets du diagramme de Dynkin associé à S.

(iii) Les représentations irreductibles de G.

II) Les résultats qui suivent ont été obtenus en collaboration avec M. Artin.

Les surfaces singulières du type S = V/G ont une singularité qui est un point double rationnel. En caractéristique p > o, les points doubles rationnels ont été classifier's par M. Artin. Ils ne correspondent plus à des quotients du type V/G. Cependant il existe toujours une correspondance bijective entre les ensembles i) et ii).

Les germes de surface singuliere (en caracteristique zéro) considerées en I) possede la proprieté que l'ensemble des classes d'isomorphismes de fibrés vectoriels indécomposables sur S - {O} est fini. On peut montrer que les surfaces normales qui possèdent cette proprieté de finitude, sont necèssairement des singularités quotient, i.e. du type V/G où G est un sous-groupe fini de GL(V). Hélène Esnault, M. Auslander, J. Herzog ont proposés des démonstrations très simples de ce dernier résultat.

K. WATANABE

On filtered rings and filtered blow-ups

Let (A,m) be a Noetherian local ring and $F^\cdot$ be a filtration on A satisfying the conditions (1) $G(A) = \bigoplus_{n\geq 0} F^n(A)/F^{n+1}(A)$ is a finitely generated graded ring over $A/F^1(A)$. (2) depth $G(A) > 0$. The filtration on A induces a filtration on the local cohomology module $H^i_m(A)$ and we define the invariant a(A) by $a(A) = \max\{n \mid F^n(H^d_m(A) \neq 0\})$, (d = dim A). Compared to the invariant $a(G(A))$ (def $= \max\{n \mid H^d_{G+}(G(A))_n \neq 0\}$), $a(A) \leq a(G(A))$ and if $H^{d-1}_{G+}(G(A)) = 0$, $a(A) = a(G(A))$. We put $A^{\natural} = \underset{n \geq 0}{\oplus} F^n(A)\, t^n \subset A[t]$ and $Y' = \mathrm{Proj}(A^{\natural})$. Then we have some criteria for rational singularities.

<u>Theorem.</u> (i) If A is a pseudo-rational local ring (ps-rat. = rat.sing. under the situation we have Grauert-Riemenschneider vanishing theorem), $a(A) < 0$.

(ii) If A is C-M and Y' has only rational singularities, A is a rat. sing. if and only $a(A) < 0$.

Next, we consider the condition for A to have a good filtration if d = 2. A is normal and if A has a filtration s.t. G(A) is normal, then A has a "star-shaped resolution" and conversely, if A has a star-shaped resolution and if $p_g(A) = p_g(R)$, where R is the graded ring with the same type of filtration, then $G(A) \cong R$, with the filtration $F^n(A) = f_*(\mathcal{O}_X(-nC))$, where $f : X \to \mathrm{Spec}(A)$ is the resolution and C is the central curve of the star-shaped resolution.

We can say something about the exceptional curves of the resolution of A if Proj(G(A)) has only singularities which are normal crossing. For example, if $G(A) \cong k[\Delta]$, where Δ is a triangulation of a circle, then A is a cusp singularity.

Berichterstatter: R. Waldi

Als Preprint haben ausgelegen (außer den Preprints, über deren Inhalt in Vorträgen berichtet wurde):

J. BARTIJN, J.R. STROOKER : Modifications Monomiales

W. BRUNS: The Canonical Module of a Determinantal Ring

W. BRUNS: Divisors on varieties of complexes

E.D. DAVIS, A.V. GERAMITA, P. MAROSCIA: Perfect Homogeneous Ideals: Dubreil's Theorems Revisited

R.M. FOSSUM: Invariants of formal group law actions

G.-M. GREUEL, J. STEENBRINK: On the Topology of Smoothable Singularities

G.-M. GREUEL, B. MARTIN, G. PFISTER: Numerische Charakterisierung quasihomogener Kurvensingularitäten

M. HERRMANN, Sh. IKEDA: Remarks on Lifting of Cohen-Macaulay property

J. HERZOG, M. KÜHL: On the Bettinumbers of Finite Pure and Linear Resolutions

M. HOCHSTER: Canonical Elements in Local Cohomology Modules and the Direct Summand Conjecture

C. HUNECKE, B. ULRICH: Divisor Class Groups and Deformations

A.R. KUSTIN, M. MILLER: Deformation and Linkage of Gorenstein Algebras

B. ULRICH: Gorenstein Rings as Specializations of Unique Factorization Domains

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