MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 22/1983

Differentialgeometrie im Großen

22.05. bis 28.05.1983

Die Tagung wurde von S.S. Chern (Berkeley) und W. Klingenberg (Bonn) organisiert, und fand unter der Leitung von Herrn Prof. Klingenberg statt. Die 44 Teilnehmer kamen aus verschiedenen Ländern und vertraten einen breiten Themenkreis der Differentialgeometrie.

Die Vorträge präsentierten eine entsprechende Themenvielfalt. Das Programm bestand aus 3 Vormittagsvorträgen und 4 halbstündigen Kurzvorträgen am Nachmittag. Auch so konnten nicht alle Vorträge untergebracht werden.

Trotz der Vielfalt der Themen sind einige Schwerpunkte zu nennen: Riemannsche Geometrie (D. Hulin, Chr. Croke, Z. Har'El, W. Meyer, P. Buser, W. Ziller), Geometrie eingebetteter Untermannigfaltigkeiten (R. Walter, U. Pinkall, N. Kuiper, D. Gromoll, D. Ferus, U. Simon), geschlossene Geodätische (M. Hamburg, N. Hingston, V. Bangert). Es gab einen Ergodennachmittag (P. Eberlein, M. Brin, R. Spatzier, W. Ballmann) und einen Blätterungsnachmittag (Ph. Tondeur, F. Kamber, G. Andrzejczak). Aber hiermit sind bei weitem nicht alle Themen aufgezählt, und es hat auch viele Querverbindungen gegeben.

Der größte Teil der Ergebnisse wurde in interessanter und verständlicher Weise vorgetragen. Besondere Resonanz fand folgende

Definition (DeTurck): Einen Trick, den man mindestens zweimal anwenden kann, nennt man eine Technik.

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Vortragsauszüge

Hermann Karcher:

Relativistic solar system experiments

Experiments which are presently discussed by relativists can very effectively be described without using local coordinates. The Schwarzschild geometry describes the exterior of a spherically symmetric star (or planet). Its curvature operator has an eigenbasis of decomposable 2-forms from which all eigenvectors and eigenvalues of the Jacobitensors can be obtained. Gravitational redshift is expressed in terms of timelike variations of nullgeodesics; spacelike Jacobifields tangent to the lightcones describe the various distance measurements. This gives the redshift-observed size - observed luminosity relations (of each relativistic model). There are enough Killing fields to reduce the geodesic equation to one first order equation. From this the deflection of light and the time delay of light which passes close to the sun is deduced. The Jacobi equation along the worldline of a planet on a circular orbit has constant coefficients; it gives the perihel advance predicted by relativity. Gyroscopes are described by parallel vectorfields so that relativistic effects on gyroscopes circling the sun are immediate. The rotation of the sun (being small) is treated by linearizing the Kerrmetric at zero angular momentum which gives a symmetric 2-tensorfield solving the linearized Einstein equations along the Schwarzschild geometry. The first derivative of this field gives the linearized contribution to the connection, the second derivative gives the curvature correction caused by the rotation of the sun; this allows to discuss the influence of this rotation on the above effects.

Dennis DeTurck:

Diffeomorphisms with prescribed eigenvalues

Given a diffemorphism of Riemannian manifolds $\phi: (M,g) \longrightarrow (N,h)$, one can compute the eigenvalues of the pullback of h with respect to g. These eigenvalues have physical meaning in the context of continuum mechanics. We consider the problem, given positve C^∞ functions $\lambda_1(x) \ldots \lambda_n(x)$ on M, of showing the existence of a diffeomorphism q that realizes these eigenvalues. As it turns out, this is a problem that involves a nonlinear hyperbolic system of partial differential equations. The proof of a result (that such diffeomorphisms exist provided $\lambda_i(x) \neq \lambda_j(x)$ for $i \neq j$ and all x) involes a Nash-Moser argument.

Another application of this reasoning yields the existence on any C^∞ three-dimensional Riemannian manifold of an atlas of C^∞ coordinate charts such that all the coordinate systems are triply orthogonal (i.e., the metric tensor is diagonal at all points). Both these results had been known on the analytic categoy - our contribution is the extension to C^∞. (Joint work with Deane Yang)

Dominique Hulin:

Pinching and Betti numbers

Starting from the "sphere and rigidity theorems" (Berger), restricting ourself to the real cohomology of the manifold, and then considering the first nontrivial dimension, we prove the following semicontinuity theorem:

Theorem: There exists an $\varepsilon > 0$ such that if (M,g) is a four dimensional connected and k-pinched riemannian manifold with $k > \frac{1}{4} - \varepsilon$, then $b_2(M) \leq 1$; one can take $\varepsilon = 2.5 \cdot 10^{-4}$.

The main tools that are used in the proof of this theorem are the Weitzenböck formula for harmonic 2-forms and a Sobolev inequality by Elias.

Christopher Croke:

A sharp four dimensional isoperimetric inequality

Consider the following conjecture

Conjecture: Let M be a compact subdomain of a complete simply connected riemannian manifold of nonpositive curvature. Then

$$\mathrm{Vol}(\partial M)^n \geq n^{n-1}\, \alpha(n-1)\, \mathrm{Vol}(M)^{n-1}$$

with equality holding if and only if M is isometric to a flat ball ($\alpha(n)$ represents the volume of the unit n-sphere).

The conjecture was proved for $n = 2$ by Beckenbach and Rado in 1933. In the talk we show that the conjecture is true for $n = 4$. In fact we derive the best known constants $c(n)$ for all n such that

$$\mathrm{Vol}\,(\partial M)^n \geq c(n)\ \mathrm{Vol}(M)^{n-1}$$

although $c(n) = n^{n-1}\alpha(n-1)$ only for $n = 4$.

Matthias Hamburg:

Bifurcations of closed geodesics

Consider a manifold M and let $G(M;I)$ denote the set of all one-parmeter families g_μ of metrics on M, parametrized by $\mu \in I$, $I \in \{\mathbb{R}, \mathbb{R}/\mathbb{Z}\}$. Let c_0 be a given closed geodesic of g_0. Under some regularity conditions, we have a family c_μ of closed geodesics of g_μ passing through c_0. But there are some unavoidable cases, where this will not be the case. Such geodesics are bifurcational. To classify all the typical bifurcations, we consider the Poincaré maps P_μ, which will be symplectic mappings. Then we give a complete classification of the bifurcations of periodic points of symplectic mappings P_μ, depending on a 1-diemnsional parameter. This classification will be only up to nondegeneracy conditions. Next we prove a local perturbation theorem, asserting that by a small perturbation of the metric around c_0, we can achieve that P_μ satisfies the nondegeneracy conditions needed above. Finally, we prove the main

Theorem: there is a residual set $G'(M;I) \subset G(M;I)$ such that for any $g_\mu \in G'(M;I)$ and any closed geodesic c_0 of a g_{μ_0}, c_0 lies either on a regular family or on a bifurcation family, which corresponds to a nondegenerate bifurcation of P_μ.

Thus, given a generic curve of metrics on M, we can describe all the unavoidable local bifurcation phenomena.

Hans-Christoph Im Hof:

Discrete Reflection Groups in Hyperbolic Geometry

A discrete reflection group in hyperbolic space H^n is a discrete group of isometries generated by the reflections w.r.t. finitely many hyperplanes of H^n. Such a group gives rise to a fundamental polyhedron in H^n whose dihedral angles are natural, i.e., of the form $\frac{\pi}{p}$, $p \in \mathbb{N}$, $p \geq 2$. Conversely, a polyhedron with natural angles generates a discrete reflection group.

In spherical and euclidean geometry, discrete reflection groups have been classified by Coxeter, but in hyperbolic geometry such a classification is out of sight. In this talk, I discuss a construction which gives rise to numerous new examples of hyperbolic finite volume polyhedra with natural angles.

We start with an orthoscheme (i.e., a simplex $(P_o,\dots,P_n)$ such that span $(P_o,\dots,P_i) \perp$ span $(P_i,\dots,P_n)$ for all i).
If P_o and P_n lie inside or on the quadric defining the hyperbolic structure, then this orthoscheme has finite volume. If P_o (or P_n, or both) lie outside the quadric, we introduce the polar hyperplane of P_o (or P_n, or both) and so we get a truncated (or doubly truncated) orthoscheme of finite volume. In good cases the polyhedra so obtained have natural dihedral angles. The classification of the good cases yields: continuous families for $n = 2$, infinite families for $n = 3$, finitely many cases for $4 \leq n \leq 9$, nothing at all for $n \geq 10$ (the calssification for $n = 4$ has not yet been completed).

The trigonometry involved is related to the pentagramma mirificum.

Zvi Har'El:

Eigenvalue expansions and volume functions

The talk is concerned with the functions u_k, v_k, appearing as coefficients in Minakshisundaram-Pleijel asymptotic expansion for the eigenvalues of the laplacian

$$U(t,\cdot) = \sum e^{-\lambda_i t} \phi_i^2 \sim (4\pi t)^{-\frac{n}{2}} \{u_o + tu_1 + t^2 u_2 + \dots\} \quad (t \to o)$$

and on the Taylor power series representing the volume function of geodesic balls,

$$V(t,\cdot) = \frac{(\pi t^2)^{n/2}}{(n/2)!} \{v_0 - \frac{t^2}{n+2} v_1 + \frac{t^4}{(n+2)(n+4)} v_2 - \dots\}.$$

We explain the universality of these coefficients, and the independence of dimension n as a result of the multiplicative properties of the heat kernel U, and of a certain tranform of the surface area function of geodesic spheres,

$$\hat{S}(s,\cdot) = \int_0^\infty e^{-\frac{s^2t^2}{2}} \frac{\partial V}{\partial t} dt = \int_{\xi \in M_m} e^{-\frac{s^2|\xi|^2}{2}} (\exp^* \mathrm{vol})_\xi =$$
$$= (2\pi)^{n/2} s^{-n} \{v_0 - s^{-2} v_1 + s^{-4} v_2 - s^{-6} v_3 + \dots\}.$$

We then present a partial solution to the problem of determining coefficients of arbitrary order by computing a dominant part of their expression in terms of basic curvature invariants (τ = scalar curvature, ρ = Ricci curvature).

$$u_k = \frac{1}{6^k k!} (\tau^k - \frac{k(k-1)}{5} |\rho|^2 \tau^{k-2} + \dots)$$

$$v_k = \frac{1}{6^k k!} (\tau^k + \frac{4k(k-1)}{5} |\rho|^2 \tau^{k-2} + \dots)$$

Rolf Walter:

Hypersurfaces with a constant higher mean curvature

The r-th mean curvature H_r of a hypersurface M in a riemannian manifold $\tilde{M}$ is (up to a constant factor) the r-th elementary symmetric function of the principal curvatures $k_1,\dots,k_m$. There are given several characterisations of complete or compact hypersurfaces in a space of constant sectional curvature c, $\tilde{M} = \tilde{M}(c)$, which have a constant fixed $H_r \neq 0$ and fulfill the intrinsic non strict lower bound curvature condition $K \geq \max\{0,c\}$. The results generalize the classical Liebmann/Süß-theorems and also recent results of (e.g.) Cheng/Yau, Nomizu/Smyth and U. Simon. The proofs are based on certain elliptic partial differential operators generalizing the Laplace/Beltrami-operator.

Jost Eschenburg:

Free Isometric Actions on Compact Lie Groups and Manifolds of Positive Curvature

Consider the manifold $M_6 = T_2 \backslash U(3)/T_1$ where

$$T_1 = \{({}^{a}{}_{a}{}_{\bar{a}});\ a \in S^1\},\ T_2 = \{({}^{b}{}_{c}{}_{1});\ b,c \in S^1\}.$$

M_6 is a manifold since $T_1 \times T_2$ is acting freely. We showed that there is a left invariant metric on $U(3)$ with respect to which $T_1 \times T_2$ acts isometrically such that the induced metric on M_6 has positive sectional curvature. M_6 is simply connected and not homotopically equivalent to any known space of positive curvature. We indicated the proof of the following uniqueness theorem: Let G be a compact simple Lie group with left invariant metric, which is right invariant w.r. to some maximal torus. Let U be a compact subgroup of $G \times G$ acting isometrically without fixed points. If G/U has $K > 0$ and even dimension, then G/U is diffeomorphic to a homogeneous space of positive curvature or to M_6.

Ulrich Pinkall:

Compact conformally flat hypersurfaces

E. Cartan proved in 1917 that every conformally flat hypersurface in $\mathbb{E}^{n+1}$, $n \geq 4$ locally (i.e. in the neighborhood of each non-umbillic point) is the envelope of a one-parameter-family of hyper-spheres. Here we consider the corresponding global problem and study compact conformally flat hypersurfaces. In particular we determine the intrinsic conformal structure of such a hypersurface: Every compact conformally flat immersed hypersurface in $\mathbb{E}^{n+1}$, $n \geq 4$ is confomorphic to a Schottky-manifold. A Schottky-manifold is a conformally flat manifold constructed in the following way: Start with a sphere S^n and cut out an even number of spherical holes $B_1,\ldots,B_k,B_1',\ldots,B_k'$. Then identify in $S^n - \bigcup_{i=1}^{k}(B_i \cup B_i')$ the boundaries $\partial B_i'$ with ∂B_i by means of Moebius transformations $f_i: S^n \longrightarrow S^n$.

Patrick Eberlein/Misha Brin/Ralf Spatzier/Werner Ballmann:

Compact Manifolds of Nonpositive Curvature

Problem: To classify compact manifolds of nonpositive sectional curvature in some reasonable sense.

Question: Given M compact with $K \le 0$ and no Euclidean factor, can one find a finite cover M' that splits as a Riemannian product of locally symmetric spaces and spaces with "rank" = 1? (There is much evidence that the answer to this question is yes.)

Rank of a manifold: Recall that if Y is a parallel Jacobi vector field perpendicular to a geodesic γ in any Riemannian manifold N, then $K(Y,\gamma')(t) \equiv 0$. For a unit vector v tangent to a compact manifold M with $K \le 0$ we define $r(v)$ = dimension of the space of parallel Jacobi vector fields along γ_v. Then define $r(M)$ = rank of M = $\inf_{v \in SM} r(v)$. Moreover call v regular if $r(v) = r(M)$. Note that this definition of rank agrees with the usual definiton for locally symmetric spaces. Also $r(M_1 \times M_2) = r(M_1)+r(M_2)$. Spaces of rank 1 behave geometrically like spaces with $K < 0$ but they may look very flat. For example, any compact surface of genus $g \ge 2$ and any metric with $K \le 0$ is rank 1 in the sense defined above.

Results: (I) Let $\{g^t\}$ denote the geodesic flow on SM, M compact with $K \le 0$. Then $\{g^t\}$ is mixing <=> ergodic <=> it has a dense orbit in SM <=> M has rank 1. If M has these properties and M^* is homotopically equivalent, then M^* also has these properties.

(II) If M has rank $k \ge 2$, then every $v \in SM$ is tangent to at least one k-flat in M(=isometrically immersed, totally geodesic copy of $\mathbb{R}^k$). If v is regular then v is tangent to exactly one k-flat. Remark: These flats even have Weyl chambers. The conjecture is that any such M must be a product manifold (or finitely covered by one) or a locally symmetric space.

(III) Every immersed k-flat is a limit of immersed k-tori in the case $k \ge 2$ (cf. the analogous situaton in locally symmetric spaces). In particular $\pi_1(M)$ contains many different free abelian groups of rank k.

Corollaries: 1) Every compact M with nonpositive sectional curvature has its closed geodesics dense in the space of all geodesics (no assumption on rank).

2) If $N(t)$ = the number of free homotopy classes of closed curves that contain a closed geodesic of length $\le t$, then $N(t)$ grows exponenti-

ally in t.

3) There exists an open $\{g^t\}$ - invariant subset $U \subseteq SM$ on which there exist $k-1$ continuous first integrals if $k \geq 2$. In particular $\{g^t\}$ does not have a dense orbit if $k \geq 2$ (cf. I above).

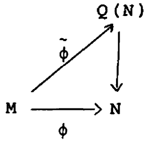
James Eells:

Twistor constructions of harmonic maps

1^{st} order constructions. Let N be an oriented Riemannian n-manifold and $Q(N) \overset{\pi}{\to} N$ the Grassmannian of oriented 2-planes. The fibre is Kählerian: the complex quadric Q_{n-2}. We define the vector subbundle $\Pi \subset TQ(N)$ as follows: Each $q \in Q(N)$ is an or'd Euclidean plane in $T_{\pi(q)}N$, and therefore a complex line L_q. The space Π_q is the subspace of $T_qQ(N)$ spanned by the lift of L_q to the horizontal subspace, and the vertical $T_q^VQ(N)$. Those components have complex structures J_q^H and J_q^V. Define complex structures on Π:

$$J_1 = \begin{cases} J^H & \text{on } T^HQ(N) \\ J^V & \text{on } T^VQ(N) \end{cases} ; \quad J_2 = \begin{cases} J^H & \text{on } T^HQ(N) \\ -J^V & \text{on } T^VQ(N) \end{cases} .$$

Theorem (Eells-Salamon CR Paris 1983) *The correspondence* $\phi \rightsquigarrow \tilde{\phi}$ (Gauss lift of ϕ



) *is a bijection between conformal harmonic maps and J_2-holomorphic maps* $\tilde{\phi}$. (Exclude ϕ constant and $\tilde{\phi}$ vertical).

Examples. $N = S^n$ with $n = 3$ (Lawson's examples)
$n = 4,6$ (Bryant's examples).

$N = \mathbb{R}P^n$ (Calabi examples)
$N = \mathbb{C}P^n$ (Eells-Wood examples)

Nicolaas Kuiper:

The total (absolute) curvature of knotted surfaces

The total absolute curvature of a compact smooth submanifold imbedding $f: M^n \to \mathbb{R}^N$ can be defined by

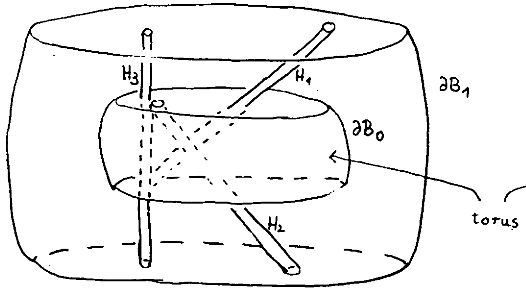
$$\tau(f) = \mathcal{E}_L \mu(\pi_L \circ f) \qquad (1)$$

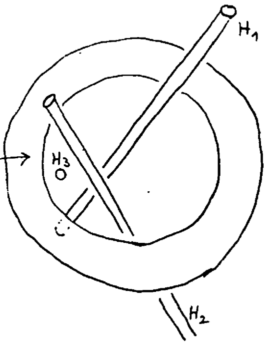
where π_L is orthogonal projection into a line L, μ is the number of nondegenerate critical points, and $\mathcal{E}_L$ is the expectation or mean over all lines through $O \in \mathbb{R}^N$.

For closed curves: $\tau(f) = \int |p \, ds|/\pi$.

For closed surfaces in $\mathbb{R}^3$: $\tau(f) = \int |K \, d\gamma|/2\pi$.

By Morse theory $\tau(f) \geq \beta$ = sum of the Betti numbers of M.
For knotted curves $\tau(f) \geq 4$ (Milnor-Fary) and even $\tau(f) > 4$ (Milnor) (by Langevin-Rosenberg-Meeks-Morton).
For knotted surfaces $\tau(f) \geq \beta+4$, $\beta = 2+2\cdot\text{genus}$. (2)
W. Meeks and myself examined the possibility $\tau(f) = \beta+4$ for knotted surfaces in $\mathbb{R}^3$. For the torus and the surface of genus 2 this is not possible, but for genus ≥ 3 there are examples:
here is one for $g = 3$





H_1, H_2, H_3 connecting $K \leq 0$-tubes.
$\partial B_2, \partial B_1$ convex surfaces.

Wolfgang T. Meyer:

Examples of complete manifolds with positive Ricci curvature

We study the geometry and topology of certain real algebraic varieties and construct a series of manifolds $V_- = V_-(\ell,p,q)$, $V_0 = \partial V_-(\ell,p,q)$ with the following properties: For integers $\ell \geq 3$, $p > q > 0$, $p-q$ sufficiently large but bounded and $p+q$ sufficiently large, V_0 is a closed $(q-1)$-connected manifold with negative Euler number $\chi = 2(2-\ell)$ for p,q odd and positive Ricci curvature. The interior of V_- is an open $(q-1)$-connected manifold admitting a complete metric with positive Ricci curvature not having the homotopy type of any closed manifold. In particular V_- does not admit a metric of nonnegative sectional curvature. Similar examples of closed manifolds are expected, but not known to exist. Candidates are manifolds of type V_0 above and certain complete intersections as for example the cubic in $\mathbb{C}P^4$ having $c_1 > 0$ and $\chi < 0$.

This is joint work with D. Gromoll.

Detlef Gromoll:

Parallel Gauss maps and rigidity aspects of minimal submanifolds

We reported on joint work with M. DAJCZER. Starting point is the observation that hypersurfaces in euclidean spaces (and spheres) with constant relative nullity have a representation by the inverse of the Gauss map on the normal bundle of its image ('Gauss parametrization'). This has many interesting applications. A main result is: Any complete minimal hypersurface M^n in $\mathbb{R}^{n+1}$ is rigid as minimal submanifold in $\mathbb{R}^{n+p}$ for any $p \geq 1$, provided $n \geq 4$ and M^n has no euclidean factors $\mathbb{R}^{n-2}$ or $\mathbb{R}^{n-3}$. Local rigidity as minimal hypersurfaces can be completely described in terms of 'associated families' of minimal immersions and 'super-minimality' of their Gauss image, which we define in general for certain ('circular') Kaehler manifolds in real spaces. Such Kaehler manifolds play a surprising role also in the congruence problem for isometric submanifolds with parallel Gauss maps which we analyze fairly completely. There are various other results in this context. For example, all Kaehler hypersurfaces of real euclidean space can be classified essentially in terms of super-minimal surfaces in spheres.

Nancy Hingston:

Isometry-invariant geodesics on S^2

Let M be a compact simply connected Riemannian manifold and $A: M \to M$ an isometry. A geodesic $c: \mathbb{R} \to M$ is called isometry invariant if $c(t+1) = Ac(t)$. Such geodesics were first studied by K. Grove. We prove:

> Let $A: S^2 \to S^2$ be an orientation preserving diffeomorphism of finite order $\neq 2$. Then for a generic A-invariant metric on S^2 there will be infinitely many A-invariant geodesics.

Note that if we take the standard metric on S^2 and if A is a rotation, then there is only 1 nontrivial A-invariant geodesic: the equator.

The proof consists of two steps. First the Birkhoff-Lewis fixed point theorem implies the existence of infinitely many for a generic metric with an elliptic A-invariant geodesic. Next we use the equivariant Morse theory to conclude that if all A-invariant geodesics are hyperbolic, then there must be infinitely many.

William H. Meeks III:

Classification of finite group actions on compact 3-dimensional manifolds

(Joint work with S.T. Yau, Peter Scott, Leon Simons)

The problem is related to a conjecture of Thurston which states that every prime compact 3-manifold admits a geometric structure and that finite group actions are conjugate to a group of isometries of the geometric structure. This is a rather loose statement in that the manifold may break up into isometric pieces but the statement gives the idea. Methods of proof that deal with this question use minimal surface theory and hyperbolic geometry. Particular questions which are solved are

(1) Finite group actions on $\mathbb{R}^3$ are conjugate to linear actions

(2) if M has a geometric structure and M is not based on S^3 or H^3, then every finite group action is "geometric".

Ernst Ruh:

An integrability condition for simple Lie groups

(Joint work with Min-Oo)

The Maurer-Cartan equation, $d\bar{\omega}+[\omega,\omega] = 0$, is the well-known integrability condition for a local Lie group structure. Pinching theorems deal with the following question: What can be said if the integrability condition $\Omega = d\omega+[\omega,\omega] = 0$ is satisfied only up to a certain degree, i.e., $\|\Omega\|$ is small in a suitable norm? Since the definition of ω requires a global parallelism of the manifold

M on which it is defined, its assumption is rather restrictive. Another possibility to define a Lie algebra structure in every tangent space of M is simply to define a tensor field $T: TM \otimes TM \to TM$ which restricts to a Lie algebra bracket in each tangent space. In 1965 Nomizu asked for the integrability condition for T. In case the Lie algebra $\mathfrak{g}$ used as model is simple the answer is as follows: T defines a pseudo-riemannian metric $\langle , \rangle$. Let D denote ist Levi-Civita connection, and define $dT(X,Y,Z) = (D_X T)(Y,Z) + (D_Y T)(Z,X) + (D_Z T)(X,Y)$.

<u>Theorem 1</u>. If $\mathfrak{g}$ is simple, rank $\mathfrak{g} \geq 2$, then $dT = 0$ if and only if either M is locally isometric to the Lie group G with Lie algebra $\mathfrak{g}$, or M is flat.

The proof relies on the following version of Berger's theorem on holonomy groups.

<u>Theorem 2</u>. If $\mathfrak{g}$ is simple, rank $\mathfrak{g} \geq 2$, and $\beta: \mathfrak{g} \wedge \mathfrak{g} \to \mathfrak{g}$ satisfies $[X,\beta(Y,Z)] + [Y,\beta(Z,X)] + [Z,\beta(X,Y)] = 0$ (Bianchi equation), then $\beta = \lambda[\,,\,]$, where $\lambda \in \mathbb{R}$ and $[\,,\,]$ = Lie bracket of $\mathfrak{g}$.

In case $\mathfrak{g}$ is compact and simple, Theorem 2 is a special case of Simons' result on holonomy systems.

Karsten Grove:
<u>Group actions, Morse theory and the double mapping cyclinder</u>

A 1-connected manifold M is called <u>Q-elliptic</u> provided $\dim \pi_*(M) \otimes \mathbb{Q} < \infty$. The following gluing construction leads to a new class of $\mathbb{Q}$-elliptic manifolds:

Theorem (Grove-Halperin) If M can be written as $M = D_0(A) \cup_S D_1(B)$, where $D_0(A)$ and $D_1(B)$ are disc bundles with common sphere bundle S, then M is $\mathbb{Q}$-elliptic iff S is $\mathbb{Q}$-elliptic.

Corollary All 1-connected manifolds of cohomogeniety 1 are $\mathbb{Q}$-elliptic.

A proof of this can be given by applying Morse theory to the relative loop space $\Omega(M,S)$. It is possible to give a complete list of the possible $\mathbb{Q}$-homotopy types of $\Omega(M,S)$ that occur in the more general setting of double mapping cylinders $M = {}^{S\times I}/f_0,f_1$ with the (rational) homotopy fibers of $f_0: S \to A$, $f_1: S \to B$ being spheres (for general CW-type spaces). An amusing consequence of the above is that the complement of a non-$\mathbb{Q}$-elliptic submanifold N of a $\mathbb{Q}$-elliptic manifold M does not admit a structure of a disc bundle, in particular it cannot admit a complete metric of nonnegative sectional curvature.

Victor Bangert:

Closed geodesics on manifolds with $\pi_1 = \mathbb{Z}$

Let M be a compact Riemannian manifold with fundamental group $\pi_1(M) \cong \mathbb{Z}$. We prove that the number $n(\ell)$ of geometrically distinct closed geodesics of length $\leq \ell$ grows like the prime numbers, so

$$\liminf_{\ell\to\infty}\left(n(\ell)\frac{\log \ell}{\ell}\right) > 0.$$

The proof consists in reducing the problem first to a topological and then to an algebraic problem which finally can be solved. This is joint work with N. Hingston.

Peter Buser:

Almost negative curvature on S^3

This is an example by Gromov of a smooth Riemannian metric on the 3-sphere with diameter = 1 and with the positive upper curvature bound arbitrarily close to zero. The example is obtained by replacing small tubular neighbourhoods of great circles on the standard S^3 with suitable copies of $\mathring{S}^2 \times S^1$, where $\mathring{S}^2$ is the two-sphere with a small circular disc removed. The procedure is uniquely restricted to three-manifolds. It is not clear, whether such metrics also exist on S^n for $n \geq 4$.

Wolfgang Ziller:

Pinching Theorems for the Diameter

A report was given on some recent results of D. Brittain. Let Ric be the average sectional curvature. Then we have:

Theorem A: There exists an $\varepsilon(n, \max K, \mathrm{vol}) > 0$ such that M^n with $\mathrm{Ric} \geq 1$, $d(M) \geq \pi-\varepsilon$ is homeomorphic to S^n.

Theorem B: There exists an $\varepsilon(n, \max K) > 0$ such that M^n with $K \geq 1$ and $d(M) \geq \pi-\varepsilon$ is diffeomorphic to S^n.

These results were motivated by:

Bonnet-Myers: $\mathrm{Ric} \geq 1 \Rightarrow d(M) \leq \pi$

Cheng (75) : $\mathrm{Ric} \geq 1$, $d(M) = \pi \Rightarrow M$ isometric to $S^n (K \equiv 1)$

Grove-Shiohama: (75), $K \geq 1$, $d(M) > \pi/2 \Rightarrow M$ homeomorphic to S^n

Itokawa-Shiohama: (82), ex. $\varepsilon(n, \min K) > 0$ s.t. $\mathrm{Ric} \geq 1$,

$\mathrm{vol}(M^n) \geq \mathrm{vol}(S^n)(1-\varepsilon) \Rightarrow M$ homeomorphic to S^n.

Philippe Tondeur:

Foliations and metrics

A foliation $\mathcal{F}$ on a Riemannian manifold M is called harmonic, if all leaves of $\mathcal{F}$ are minimal submanifolds of M. The terminology is motivated by the fact that this geometric property is characterized by the harmonicity of the projection $\pi: TM \to Q$ to the normal bundle Q, viewed as a Q-valued 1-form. Riemannian foliations of this type on a compact and oriented manifold are further characterized as the critical points of an energy functional on the space of foliations. In this talk we discuss examples and geometric properties of such foliations (joint work with F. Kamber).

Franz Kamber:

Duality theorems for Riemannian foliations

(joint work with Ph. Tondeur)

About 25 years ago, B.L. Reinhart (AJM 1959) asserted that the basic complex $\Omega_B^{\bullet}(F)$ of a Riemannian foliation F on a closed manifold with oriented normal bundle satisfies Poincaré duality, i.e., $H^r(\Omega_B(F)) \cong H^{q-r}(\Omega_B(F))$, $q = \mathrm{codim}\, F$. In 1981, J. Carrière produced a counterexample to this assertion, namely a Riemannian flow F transverse to the fibre of a fibration $\mathbb{T}^2 \to M^3 \to S^1$ satisfying $H^0(\Omega_B(F)) \cong \mathbb{R}$, $H^2(\Omega_B(F)) = 0$, $g = 2$. On the other hand, the flow F in the above example is not *geometrically taut*, i.e., M^3 does not admit a metric for which the leaves of F become minimally immersed submanifolds of M^3 (the flow is not geodesible). It turns out that *tautness* of F is precisely the condition needed to establish Poincaré duality for $H(\Omega_B(F))$.

More generally we say that a foliation F is *geometrically tense* if there exists a Riemannian metric on M for which the leaves of F become submanifolds of constant (= parallel) mean curvature, i.e., $\kappa = \mathrm{Tr}W \in \Omega^1_B(F)$, W the Weingarten operator. The mean curvature form κ is then a closed basic 1-form and one obtains two pairs of mutually adjoint operators $(d_B, \bar{d}^*_\kappa)$ and $(\bar{d}_\kappa, d^*_B)$ relative to a suitable metric on $\Omega^\cdot_B(F)$ (d_B = ext. diff. in $\Omega^\cdot_B$, $\bar{d}_\kappa = d_B - \kappa\wedge$, $\bar{*}$ = star operator in $\Omega^\cdot_B$ defined by the transverse Riemannian metric). Using the transversally elliptic operator $\Delta_B = d_B\bar{d}^*_\kappa + \bar{d}^*_\kappa d_B$, we can then prove the following theorem(s).

<u>Theorem</u>: Let F be a Riemannian foliation with oriented normal bundle on a closed oriented manifold M^n (q = codim F). Then the following statements are equivalent:

(i) F is *tense (resp.* taut);

(ii) there exists a bundle-like metric on M for which $\bar{d}^*_\kappa \nu \in \Omega^{q-1}_B(F)$ (*resp.* $\Delta_B \nu = 0$), where $\nu \in \Omega^q_B(F)$ is the invariant transversal volume form of F;

(iii) there exists a volume form $\omega_0 \in \Gamma\Lambda^p L(F)$ (p = dim F) representing the induced orientation on F such that the pairing $\Psi(\alpha,\beta) = \int_M (\alpha\wedge\beta)\wedge\omega_0$, $\alpha \in \Omega^r_B$, $\beta \in \Omega^{q-r}_B$ induces a non-degenerate pairing

$$\Psi_*: H^r(\Omega_B^\cdot, d_B) \otimes H^{q-r}(\Omega_B^\cdot, d_\kappa^-) \to \mathbb{R},\ r = 0,\dots,q;$$

i.e., the basic cohomology $H(\Omega_B^\cdot, d_B)$ is dual to the basic *twisted* cohomology $H^\cdot(\Omega_B^\cdot, d_\kappa^-)$ (*resp.* $H(\Omega_B^\cdot, d_B)$ satisfies *Poincaré duality).*

(Observe that $d_\kappa^- = d_B$ in the taut case!)

Several applications of this theorem were discussed:

(a) $H^0(\Omega_B, d_\kappa^-)$ consists of solutions of the 1^{st} order PDE

$d_B f = \kappa f$. Hence either $H^0(\Omega_B, d_\kappa^-) = 0$ or $H^0(\Omega_B, d_\kappa^-) =$

$= \{f \in \Omega_B \mid \kappa = d_B \log f\} \cong \mathbb{R}$.

For a tense R-foliation one has therefore:

F taut $\Longleftrightarrow$ $H^q(\Omega_B, d_B) \cong \mathbb{R}$ $\Longleftrightarrow$ $[\kappa] = 0 \in H^1(\Omega_B, d_B)$

relative to a suitable metric on M.

(b) Foliations F with *compact* leaves are taut $\Longleftrightarrow$ F locally stable $\Longleftrightarrow$ F R-foliation (Rummler).

Thus the base space B (= leaf space) of a locally stable compact foliation satisfies Poincaré duality in the deRham cohomology.

(c) Foliation cycles of a taut R-foliation are homologically unique up to a constant positive factor. This applies in particular to the foliation cycles defined by compact leaves.

A conjecture was made concerning a transversal signature Theorem on taut R-foliations.

Grzegorz Andrzejczak:

Relations between transverse structure of a foliation and its characteristic classes

Characteristic classes of a foliation admit natural lifts to cohomology groups of the classifying space of the corresponding (transverse) holonomy groupoïd. The lifts depend on the transverse structure of the foliations and not on the foliation itself. This is, in short, the "philosophy" of connections between characteristic classes and the transverse structure.

Theorem 1. If a codimension q foliation F on X admits a transverse k-field $(X_1,\dots,X_k)$ of infinitesimal automorphisms such that F and the fields span a codimension $(q-k)$ foliation, say F', then the characteristic homomorphism α_F of F admits a factorization

$$H(WO_q) \to H(WO_{q-k}) \xrightarrow{\alpha_{F'}} H^*(X).$$

Corollary. α_F annihilates $\mathrm{Ker}(H(WO_q) \to H(WO_{q-k}))$.

(Theorem 1 has been proved independently by Cordero and Mose).

Theorem 2. If a foliation F admits a family of submersions on $\mathbb{R}^q$ such that the transition maps have (locally) constant Jacobians, then α_F annihilates $\ker(H(WO_q) \to H(\tilde{WO}_q))$, where

$$\tilde{WO}_q := WO_q/(c_1)$$

Remark. In codimension 1 such an F is simply a transversally affine foliation; the G-V class of F is then 0.

The proof of the two theorems is based on a decomposition of characteristic classes into "elementary blocks" corresponding to the generators $c_1, \ldots, c_q$ and $y_1, y_3, \ldots$ of WO_q. In the case of theorem 1 we have $c_i = 0$ and $y_j = 0$ for $i,j > q-k$, whereas in the case of theorem 2 the condition imposed on F is simply the solution to the equation $c_1 \equiv 0$.

Remark. The equality $y_1 \equiv 0$ holds iff F admits (up to ± 1) a transverse volume form.

Jean Pradines:

Some universal factorizations of differentiable functors

Smooth groupoïds and smooth functors between them occur everywhere in global Geometry, and their algebraico-differential significance and structure are much richer than in the very special case of Lie group morphisms.

For instance smooth functors may describe such various situations as: a Lie group action, a maximum or globalizable local transformation group in the sense of Palais, a one-parameter group, a principal fibration, a cocycle, etc.

Those functors arising from a group (or groupoïd) action will be called "actors".

By describing algebraic properties of functors by means of suitable diagrams, we are able to give a very natural "smooth" version of algebraic concepts such as faithfulness, functorial equivalence, etc...., and we can then state: "every faithful functor factorizes

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through an equivalence and an actor", which we call its "<u>universal activation</u>".

The (purely diagrammatic though rather sophisticated) proof unifies all the specific constructions (Palais globalization, homogeneous spaces, principal bundles, Haefliger structures) and of course covers many new situations; it avoids any local triviality assumption.

This "universal activation" is an invariant of the class of the smooth functor up to an equivalence at the source. On the other hand, the "<u>transverse structure" of a foliation</u> may be described by the class of its smooth holonomy groupoïd up to functorial equivalences, a concept which turns out to be equivalent to the notion of equivalence recently considered by Skandalis-Haefliger (and in the special case of pseudo-groups by Van Est-Haefliger) in a maybe less natural way.

Vladimir Oliker:

<u>Hypersurfaces with prescribed Gaussian curvature</u>

Let $\varphi: E^{n+1} \to \mathbb{R}$ be a given function. Under what conditions there exists a closed hypersurface F in E^{n+1} with prescribed genus and such that the Gauss curvature $K_F(X) = \varphi(X)$, $X \in F$? This question was raised by S.T. Yau in "Problem Section", Seminar on Differential Geometry, Annals of Math. St., v. 102 (1982). The following theorem gives a partial answer to this question:

Suppose a) $\varphi(X) > 0$, $X \in E^{n+1} \setminus \{0\}$;

b) $\varphi(X) \in C^m(E^{n+1} \setminus \{0\})$, $m \geq 3$;

c) there exist two numbers R_1 and R_2, $0 < R_1 \leq 1 \leq R_2 < \infty$ such that $\varphi(X) > |X|^{-n}$ when $|X| < R_1$ and $\varphi(X) < |X|^{-n}$ when $|X| > R_2$;

d) $\frac{\partial}{\partial\rho}(\rho^n \varphi(u,\rho)) \leq 0$, $\rho \in [R_1, R_2]$, where $X = (u,\rho)$ are the spherical coordinates in E^{n+1}.

Then there exists a closed convex hypersurface F in E^{n+1} such that

i) F is a graph of a radial function $\rho(u) > 0$ over a unit sphere $S^n \subset E^{n+1}$;

ii) $\rho(u) \in C^{m+1,\alpha}(S^n)$, $\alpha \in (0,1)$, and if φ is analytic then ρ is analytic;

iii) the Gaussian curvature of F is given by $\varphi(u,\rho(u))$,

iv) F is unique up to a homothetic transformation.

The proof is based on a study of a nonlinear elliptic equation of Monge-Ampère type on S^n which the function ρ must satisfy.

Dirk Ferus:

On a conjecture of Osserman on the volume of the generalized Gauss map

For an immersed n-manifold in $\mathbb{R}^{n+p}$ one has two natural Gauss maps γ_1: unit normal bundle $\rightarrow S^{n+p-1}$, γ_2: $M \rightarrow G_p(\mathbb{R}^{n+p})$. Then for the (suitably normalized) volumes of the images of γ_1 and γ_2, denoted $\tau(f)$ (= total absolute curvature) and $\sigma(f)$, Osserman conjectured in his lecture at the Chern Symposium 1979

$$\tau(f) \leq \sigma(f)$$

for any $n \geq 2$. The conjecture is true.

Udo Simon:

Minimal submanifolds of spheres

Conjecture. Let (M,g) be a closed, 1-conn., oriented 2-manifold with curvature K, let $s \in \mathbb{N}$. Define the constants $K(s) = \frac{2}{s(s+1)}$. Let $\tilde{x}: M \to S^N(1)$ be an isometric minimal immersion. Then

$$K(s+1) \leq K \leq K(s)$$

implies $K = K(s)$ or $K(s+1) = K$, and (M,ρ) is a 2-sphere of curvature K.

Theorem. (a) The conjecture is true for $s = 1$ (Lawson $N = 4$, Simon et al. N arbitrary). (b) The conjecture is true for $s = 2$ (M. Kozlowski and U. Simon).

The proof extends a method from Coll. Math. 1979 (K. Benko, U. Simon et al.).

Theorem. Let M be closed, conn., oriented, dim $M = n$, let $\tilde{x}_t: M \to S^N(1)$ be a 1-param. family of isom. minimal immersions. Let g_t be the family of corr. metrics and $g := g_o$ be of constant curvature. Let $j: S^N(1) \to \mathbb{R}^{N+1}$ be the canonical embedding and let ξ_t be the mean curvature vector of $x_t = j \circ \tilde{x}_t$ with corresponding second fundamental forms $II(\xi_t)$, $\xi := \xi_o$. Then $\delta II(\xi) = 0$ (δ first variation) implies that the family $\tilde{x}_t$ of infinitesimal deformations is trivial. The proof uses results on deformations of Codazzi-tensors of V. Oliker and U. Simon.

Grosio Stanilov:

Decomposition of the curvature tensor in the almost Hermitian geometry and applications (*)

For the curvature tensor R in the almost Hermitian geometry the decomposition $R = \sum_{j=1}^{10} p_j(R)$ under the action of the unitary group holds. The classical Weyl's tensor is decomposed in the following way: $C(R) = C_1(R) + C_2(R) + p_3(R) + p_6(R) + p_7(R) + p_9(R) + p_{10}(R)$ (all components are orthogonal). It is given an analogue of this tensor: $C^*(R) = C_1^*(R) + C_2^*(R) + p_3(R) + p_6(R) + p_7(R) + p_8(R) + p_{10}(R)$ (all components are orthogonal). We have: $\rho \circ C(R) = 0$ and $\rho^* \circ C^*(R) = 0$. $C_1^*(R) \perp \sigma_2$, $C_1(R) \perp \sigma_1$. It is proved a theorem for the globality of the Kählerian defect $\Delta_R(p,E^4) = K_R(p,E^4) - K_R^*(p,E^4)$ in the class of QK_2-manifolds. The AHM with constant type are characterized by $p_i(R) = 0$, $i = 5,6,7,8,9,10$ and the AHM with conformal type by $p_i(R) = 0$, $i = 6,7,8,9,10$.

Berichterstatter: Matthias Hamburg

(*) Der Vortrag hat wegen Zeitmangel nicht stattgefunden.

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