MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 23/1983

Dynamische Systeme

30.5. - 3.6. 1983

Die Tagung fand unter der Leitung von Herrn J. Moser (ETH-Zürich) und Herrn E. Zehnder (RUB-Bochum) statt. Im Mittelpunkt standen die folgenden Themen:

1. Periodische Bahnen
   Die Verbindung funktional-analytischer und topologischer Methoden hat neue Resultate über die Existenz periodischer Bahnen, sowie anderer Randwertaufgaben für die Theorien Hamiltonischer Systeme geliefert.

2. Iteration von Abbildungen
   Häufig lässt sich das Studium dynamischer Systeme auf die Betrachtungen von Iterationen von Abbildungen reduzieren. In der letzten Zeit haben sich Mathematiker wie auch theoretische Physiker für Iterationen inhaltstreuer oder konformer Abbildungen interessiert, wobei numerische Experimente eine grosse Rolle spielten. Neue Resultate über konforme Abbildungen, die durch rationale Funktionen gegeben sind, sind gefunden worden. Auch neue Methoden, die durch Ideen aus der statistischen Mechanik angeregt wurden, sind in dieses Gebiet eingeflossen und bei Intervallabbildungen mit Erfolg angewendet worden. Auch Ideen aus der Festkörperphysik haben ihren Niederschlag in der Matherschen Theorie gefunden.

3. Fastperiodische Potentiale

Die Spektraltheorie für eindimensionale Potentiale spielt in vielen Anwendungen eine Rolle. Im Zusammenhang mit der Invers Spektraltheorie sind in letzter Zeit fastperiodische Potentiale untersucht worden. Man interessiert sich für die Natur des Spektrums, die äusserst kompliziert werden kann, sowie für die Bestimmung eines Potentials bei gegebenem Spektrum.

Darüber hinaus wurden Themen aus der Differentialgeometrie,der Stabilitätstheorie, der Bifurkationstheorie und anderen Gebieten behandelt. Wegen der oben erwähnten Zusammenhänge mit der Physik waren auch theoretische Physiker vertreten, wobei auch Fragen über "confinement im Tokamak" zur Sprache kamen. Einige der Vorträge betrafen numerische Untersuche. Die Interessengebiete der Teilnehmer waren relativ breit gestreut und die Diskussion lebhaft. Die eingeladenen Teilnehmer kamen aus 6 verschiedenen Ländern; die Gäste aus Polen und Russland sind leider nicht erschienen.

Vortragsauszüge

V. BANGERT:

Geodesics on noncompact manifolds

Let M be a complete noncompact Riemannian manifold. The talk gave a survey on results on the following questions: Do there (or do there not) exist geodesics of the following types on M: closed geodesics, bounded geodesics, oscillating geodesics, in both directions divergent geodesics? The case of negative curvature was not treated. While I did not hesitate to make topological assumptions on M, I tried to keep the (sometimes necessary) geometric assumptions down to a minimum. One of the presented results says: Theorem: Assume M is homeomorphic to the plane $\mathbb{R}^2$. Then there exists a divergent geodesic $c : \mathbb{R} \to M$. This result has an application in Wojtkowski's work on the existence of oscillating geodesics.

H. BERESTYCKI:

Forced vibrations of some Hamiltonian systems

(joint work with A. Bahri) In this talk we report on the following result. Let $V \in C^2(\mathbb{R}^N, \mathbb{R})$ satisfy the assumption $0 < V(x) \leq \theta V'(x)\cdot x$, $x \in \mathbb{R}^N$, $|x| \geq R_0$ for some $R_0 > 0$, and where $0 < \theta < 1/2$. Theorem: For any given $f \in L^2_{loc}(\mathbb{R}, \mathbb{R}^N)$ which is T-periodic, there exist infinitely many T-periodic solutions (forced oscillations) of the system $\ddot{x}(t) + \operatorname{grad} V(x(t)) = f(t)$. The method of proof rests on constructing critical values for the "autonomous" functional $I^*(x) = \frac{1}{2}\int_0^T |\dot{x}|^2 - \int_0^T V(x(t))$ via a mini-max formula (Liusternik-Schnirelman type theory) such that they persist for functionals $I(x)$ "near" to I^*, but not necessarily symmetric (i.e. S^1-invariant). The solutions we obtain have arbitrarily large amplitude (i.e. $\|u\|_{L^\infty}$). We obtain related results for more general systems $\ddot{x}(t) + V_x'(t,x) = 0$ or $\dot{z} = JH'(z) + f(t)$ where $z : \mathbb{R} \to \mathbb{R}^{2N}$, $H \in C^2(\mathbb{R}^{2N}, \mathbb{R})$ and $f : \mathbb{R} \to \mathbb{R}^{2N}$ is given T-periodic.

H.W. BROER:

Subordinate Sil'nikov bifurcations near some singularities of low codimension (joint work with G. Vegter). On $\mathbb{R}^3$ consider a vectorfield with the origin as a singular point with eigenvalues o and $\pm i\alpha$ ($\alpha > o$). This singularity has codimension 2, so consider generic 2-parameter unfoldings of it. Densely in a C^2-open subclass of such unfoldings there exist subordinate codimension 1 bifurcations of Sil'nikov. In this Sil'nikov bifurcation there is a homoclinic orbit of a saddle point with non-real eigenvalues. This orbit induces a dynamical complexity comparable to the Smale horseshoe. The Sil'nikov bifurcation moreover is C^1-persistent. Its occurrence as a subordinate bifurcation in our local problem, however, is a flat phenomenon: due to 'formal integrability' of the central singularity, close to it, it can be cancelled completely with an infinitely flat perturbation. Nevertheless there is some persistence of this Sil'nikov phenomenon further away from the central singularity. This confirms an earlier statement of Guckenheimer. Similar results hold in the conservative case.

A. CHENCINER:

A dissipative version of the Poincaré-Birkhoff geometric theorem

The proof of Poincaré's last geometric theorem given by Birkhoff in 1925 does not use the preservation of area but only the purely topological "intersection property" (see also the recent work of P. Carter). We point out that the same proof gives a theorem concerning the existence of fixed points in 1-parameter families of homeomorphisms of the annulus which are distortions and satisfy one half of the intersection property for each extreme value of the parameter. As in Rüssmann's proof of the invariant curve theorem (K.A.M.), preservation of area appears here as a codimension one condition. This result has been used to prove the existence of periodic orbits of "high" periods not a priori belonging to an invariant curve in Hopf bifurcations of codimension greater than one.

M. CHAPERON:

Lagrangian intersections, solution of a problem of Arnold

Theorem: Let λ denote the Liouville form of the cotangent bundle $X = T^*T^n$, and let $j : M = T^n \to X$ be a (smooth) embedding such that (i) $j^* \lambda$ is exact; (ii) there exists a smooth path j_t, $0 \leq t \leq 1$, of embeddings $M \to X$ with $j_t^* d\lambda = 0$, $0 \leq t \leq 1$, $j_1 = j$ and j_0 is the null 1-form 0_M on M. Then $0_M(M) \cap j(M)$ contains at least n+1 points, and at last 2^n if all of them are transversal intersection points.

This was a conjecture of V.I. Arnold (1965); it implies the Conley-Zehnder theorem. The proof follows their ideas.

C. CONLEY AND E. ZEHNDER

Proof of a conjecture of V.I. Arnold

The conjecture is the following: A symplectic diffeomorphism on $T^2 = \mathbb{R}^2/\mathbb{Z}^2$, which preserves the center of gravity, possesses at least 3 fixed points. (at least 4 if all the fixed points are nondegenerate).The proof is based on the variational principle for periodic solutions of a timedependent Hamiltonian equation on the torus and on topological arguments for continuous flows.

W. CRAIG:

Normal forms for matrices near diagonal

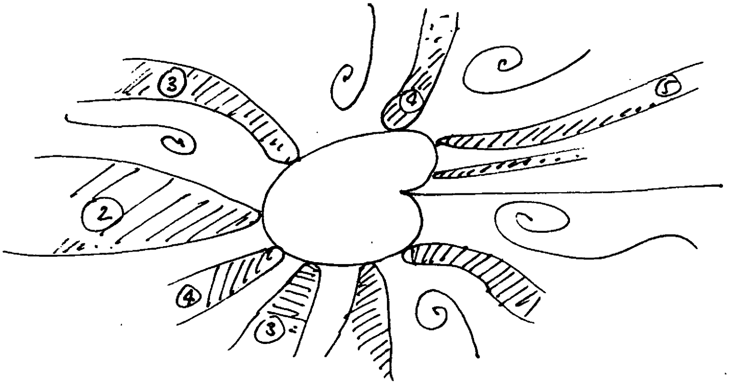
In a class of linear problems arising in PDE and for mathematical physics, small divisors in the form of a dense set of eigenvalues are encountered. In certain cases techniques similar to ones developed by Kolmogorov-Arnol'd and Moser can be adapted to obtain convergent perturbation expansions. Examples include the discrete Schrödinger operator with an almost periodic potential, and the Schrödinger equation with a spatially periodic potential plus a time periodic electric field. In both cases the normal form is an infinite diagonal matrix, to which the perturbed problem is transformed. Small divisors are encountered in the construction of this transformation, and are overcome by the quadratically convergent iteration scheme.

R. DEVANEY:

Dynamics of $\lambda \cdot \exp(z)$

Consider the map $f_\lambda(z) = \lambda \exp(z)$. For $\lambda \in \mathbb{R}$, $\lambda > 1/e$, the Julia set of f_λ is the entire complex plane. One may study the dynamics of λe^z via symolic dynamics on the fundamental domains. Modulo a growth condition, all iteneraries are possible. Moreover, there exists a unique periodic point corredponding to each repeating itinerary, and the set of points corresponding to a given sequence is a continuous curve in $\mathbb{C}$. For other λ-values, the dynamics of λ exp z are remarkably different. For $o < \lambda \leq \frac{1}{e}$, there is an open and dense basin of attraction of a fixed point, and the Julia set disintegrates into a

Cantor set of curves. Moreover, the bifurcation diagram in the λ plane looks topologically like:



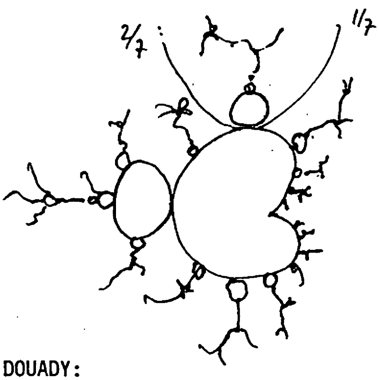
where  indicate parameter values with Julia set = $\mathbb{C}$, and  indicates a sink of period n. As a corollary, exp(z) is not structurally stable.

A. DOUADY AND J. HUBBARD:

<u>Dynamics of $P_c : z \to z^2 + c$.</u>

Let $K_c = \{z \mid P_c^n(z) \not\to \infty\}$. Then $o \in K_c \Rightarrow K_c$ is connected. $o \notin K_c \Rightarrow K_c$ is a Cantor Set. The Mandelbrot set is $M = \{c \mid o \in K_c\}$. For each c, let ϕ_c conjugate P_c to $P_o : z \to z^2$ at nbh. ∞. If $c \in M$, ϕ_c provides the conformal mapping: $\mathbb{C} - K_c \to \bar{D}$. For $c \notin M$, $\phi_c(z)$ is defined if $\eta(z) > \eta(o)$, where η is the Green function of ∞ in $\mathbb{C} - K_c$. In particular $\phi(c) = \phi_c(c)$ is defined. The map $\phi : \mathbb{C} - M \to \mathbb{C} - \bar{D}$ is the conformal mapping. For $\theta \in \mathbb{R}/\mathbb{Z}$, $R_\theta(M) = \phi^{-1}(\{re^{2\pi i\theta}\}_{r > 1})$ is the external ray of M of angle θ. It is not known that M is locally connected (this would imply generic hyperbolicity for instance), but one can prove that each $R_\theta(M)$ with $\theta \in \mathbb{Q}/\mathbb{Z}$ has a limit in M. The lecture was about the proof of the following theorem which is the main step in this direction for rationals with odd denominators.

<u>Theorem:</u> Let c be such that P_c has a rational indifferent periodic point α: $f^k(\alpha) = \alpha$, $(f^k)'(\alpha) = \rho = e^{2\pi i p/q}$. Say $P_c{}^{kqn}(c) \to \alpha$. In K_c, the point α has a finite number of external arguments of the form $\pi/2^{kq}-1$. Let θ and θ' be those arguments which are adjacent to the limit direction of the $P_c{}^{nkq}(c)$. Then θ and θ' are the external arguments of c in M.



The Mandelbrot set appears in lots of problems sometimes with its external rays (ex: Newton method for polynomials of degree 3).

R. DOUADY:

<u>Example for topological instability in dim 4</u>

Let $F_o : \mathbb{R}^n \to \mathbb{R}^n$ be a symplectic C^∞-diffeomorphism given in symplectic polar coordinates by: $(t,r,\theta,\rho) \to (t+\ell(r),r,\theta+\lambda(\rho),\rho)$, where ℓ and λ are C^∞-functions $\mathbb{R}_+ \to \mathbb{T}^1$, with $\ell'(o) \neq o$, $\lambda'(o) \neq o$. We give an example of a symplectic C^∞ map $F : \mathbb{R}^n \to \mathbb{R}^n$ which is C^∞-tangent to F_o at the origin and such that there exists a $z \neq o$ so that $o \in \overline{\{F^n(z), n \geq o\}}$. To find such a z, we prove that F satisfies the "property" (T)": There exists a sequence (T_k) of closed minimal invariant sets where, either T_k is a hyperbolic periodic orbit with $\dim W^u = \dim W^s = 2$, or T_k is a finite union of d_k circles permuted by F such that F^{d_k} restricted to one of these circles is top. conjugated to an irrational rotation, and $W^u(T_k)$ and $W^s(T_k)$ have also dim 2. The last condition is that $W^u(T_k)$ intersects $W^s(T_{k+1})$ transversaly in at last one point.

J.J. DUISTERMAAT:

Non-integrability of the 1 : 1 : 2 resonance

A Hamiltonian system, defined by the smooth function $F(q_1,\dots,q_n,p_1,\dots,p_n)$ is called formally integrable at the equilibrium point $o \in \mathbb{R}^{2n}$, if there exist formal power series $\hat{f}_i(q_1,\dots,q_n,p_1,\dots,p_n)$, $i = 1,\dots,n$, such that:

a) $\{\hat{f}_i, \hat{f}_j\} = o$ for all i,j.

b) The $\hat{f}_i$ are functionally independent, and

c) $\hat{F}$, the Taylor expansion of F at o, is a function of $\hat{f}_1,\dots,\hat{f}_n$.

Write $\hat{F} = \sum_{k>2} F^{(k)}$ where $F^{(k)}$ is homogeneous of degree k. Assume that $F^{(2)}$ is positive definite. Then, by a linear symplectic transformation one can arrange that $F^{(2)} = \sum_{i=1}^{n} \omega_i(q_i^2 + p_i^2)$ with $\omega_i > o$. The Birkhoff normal form theorem says that by a formal canonical change of coordinates (leaving $F^{(2)}$ fixed) one can also arrange that $\{F^{(2)}, \hat{F}\} = o$. If there is no resonance, that is $\dim_{\mathbb{Q}} \sum_i \mathbb{Q}\omega_i = n$, then $\hat{F}$ is a function of the $\rho_i = q_i^2 + p_i^2$, so the system is formally integrable. If there is a simple resonance, $\dim_{\mathbb{Q}} \sum_i \mathbb{Q}\omega_i = n-1$, then there are linearly independent $\nu^{(1)},\dots,\nu^{(n-1)}$ in $\mathbb{R}^n$ such that the $f_k = \sum_{j=1}^{n} \nu_i^{(k)} \rho_i$, $k = 1,\dots,n-1$ Poisson commute with $\hat{F}$. So in this case the system is formally integrable again, but with an entirely different fibration into invariant tori. So for non formally integrability we need $n \geq 3$, for $n=3$ all the ω_i must be a common multiple of integers. We take $\omega_1 : \omega_2 : \omega_3 = 1 : 1 : 2$.

Theorem 1. $F^{(3)}$ can be brought in the form $q_3\{\beta_1(q_1^2 - p_1^2) + \beta_2(q_2^2 - p_2^2)\} + 2p_3\{\beta_1 q_1 p_1 + \beta_2 q_2 p_2\}$ with $\beta_1 \geq \beta_2 \geq o$. If $\beta_1 > \beta_2 > o$ then the system is not formally integrable. Homoclinic behaviour with Smale horseshoes and an angle between the stable and unstable manifold which decays of linear order if one approaches the origin, if $\beta_1 = \beta_2 > o$ (then the system of $F^{(2)} + F^{(3)}$ is integrable) and $F^{(4)}$ is chosen in a suitable non-void open subset in the space of polynomials of degree 4.

J.P. ECKMANN AND H. EPSTEIN:

Asymptotics of high order iterations of maps of the interval

Under certain conditions, high order iterates of a map of the interval approach a quadratic function. In this work in collaboration with P. Wittwer, advantage is taken of this observation to give a simple proof of the Feigenbaum theory of p-tupling cascades when p becomes very large, together with an explicit asymptotic estimate (exact in the leading order) of the relevant constants. The operator $N = N_p$ is defined by

$$(Nf)(x) = \frac{1}{\lambda} f^p(\lambda x), \ \lambda = f^p(0)$$

on a set of even functions f analytic in $\{z \in \mathbb{C} : |z| < p\}$ whose restrictions to $[-1,1]$ have the property: $\lambda_f < o$, $J_o = [\lambda_f, -\lambda_f]$, $J_k = f(J_{k-1})$, $k = i,\ldots,p$; $J_2 < J_3 < \ldots < J_p \subset J_o$; $f''(x) < o$. It is proved that, for sufficiently large p, N_p has a fixed point g_p verifying $|g_p(z) - \psi(z)| <$ Const. 4^{-p} for $|z| < p$, $\psi(z) = 1-2z^2$. DN_p has, at this point an unstable direction with eigenvalue $\delta_p \sim \frac{128}{3\pi^2} 16^{2-p}$ and $\lambda_p \sim -\frac{\pi}{8} 4^{2-p}$.

H. ELIASSON:

The Morse-Lemma for Poisson commuting functions

We describe a Morse-Lemma for Poisson commuting functions. Given a set of C^∞-functions $(h_1,\ldots,h_k)$ defined in a neighborhood of $0 \in \mathbb{R}^{2n}$, such that $h_j(o) = o$ and $dh_j(o) = o$ for all j. We assume that

i) $\{h_i, h_j\} = o, \forall i,j$, with the Poisson bracket defined by the standard symplectic 2-form $\Omega_o = \Sigma\, d\,x_j \wedge d\,y_j$ on $\mathbb{R}^{2n}$,

ii) $h_j = \sum_{i=1}^{n} b_{ij}\, q_j$ + higher order terms with $q_j = x_j^2 + y_j^2, \forall j$, and the matrix $B = (b_{ij})$ satisfying a strong non-degeneracy condition: each $k \times k$-minor of B is non-singular.

Theorem: Under the above assumptions, there exist a local C^∞-diffeo. ϕ of $\mathbb{R}^{2n}$, $\phi(o) = o$, $d\,\phi(o) = id$, and k functions $\psi_1,\ldots,\psi_k$, defined in a neighborhood of $0 \in \mathbb{R}^n$, such that $h_j \circ \phi = \psi_j(q_1,\ldots q_n)$; $\forall$ j. Moreover, when $k = n$ we can assume ϕ to be symplectic. In this case the functions ψ_j are uniquely determined.

The work of Siegel and others suggest very strongly that we cannot expect ϕ to be symplectic when $k < n$.

J.P. FRANCOISE:

Analytic invariants for germs of vectorfields

The group of germs of analytical diffeomorphism of $(\mathbb{C}^n,o)$ into itself acts on the space of germs of analytical vectorfields of $(\mathbb{C}^n,o)$ which fix o. We consider functions which are constant on the orbits of this action and depend analytically on the coefficients of the Taylor expansions of the vector fields. We say that we have a complete system of analytic invariants if we have a family of such functions which allows to seperate the orbits. We define the reduced vectorfield and prove that a complete system of analytic invariants cannot exist in the neighborhood of a vectorfield whose reduced vectorfield is 0. This suggest a new interpretation of a result of C.L. Siegel about Hamiltonian vectorfields near a Birkhoff normal form.

M. GERBER:

Real Analytic Models of Pseudo-Anosov Mappings

As a special case of my work on Pseudo-Anosov mappings we obtain:

Theorem: There exists a real analytic diffeomorphism h of $D^2 = \{(u_1,u_2): u_1^2 + u_2^2 \leq 1\}$ such that h is Bernoulli with respect to $\frac{1}{\pi}\, du_1\, du_2$.

D.L. GOROFF:

Hyperbolic invariant sets for twist maps

The purpose of this talk is to observe that the arguments of Aubry, La Daeron and André give examples of hyperbolic invariant Cantor sets in area preserving monotone twist maps. This gives a positive answer to Katok's question about the existence of positive Lyapunov exponents.

R. HALL:

A Denjoy example in the annulus

Let $\mathbb{T} = \mathbb{R}/\mathbb{Z}$ be the circle, d the usual metric on $\mathbb{T}$ and let $A = [o,1] \times \mathbb{T}$ be the annulus. For $f : \mathbb{T} \to \mathbb{T}$ let $W^s(\theta) = \{y : d(f^n(\theta), f^n(y)) \to o \text{ as } n \to \infty\}$ and $w(\theta)$ be the usual ω-limit set. Denjoy's Theorem says that if $f : \mathbb{T} \to \mathbb{T}$ is a C^2-diffeomorphism and $\theta \in$ interior $W^s(\theta)$ then $\omega(\theta)$ is periodic (i.e. every point is periodic). There exist C^1-diffeomorphisms $f : \mathbb{T} \to \mathbb{T}$ which have points $\theta \in$ interior $W^s(\theta)$ but f has no periodic points: such maps are called Denjoy maps. In this talk we outlined the construction of a C^∞-diffeomorphism $F : A \overset{\text{into}}{\to} A$ such that $\omega(A) = C$ is a (Lipschitz) circle and $F|_C$ is a Denjoy map.

R. JOHNSON:

Almost periodic linear systems

Consider the almost periodic differential equation

$$(*)_\lambda \qquad \bar{x}^{1} = \begin{pmatrix} b(x) & f(x)+e(x) \\ -f(x)+e(x) & -b(x) \end{pmatrix} \bar{x} + \lambda \begin{pmatrix} 0 & +1 \\ -1 & 0 \end{pmatrix} \bar{x}$$

where $\bar{x}$ is a complex 2-vector, b,e,f are real valued almost periodic functions and λ is a complex parameter. We consider a holomorphic function $w(\lambda)$ ($\mathrm{Im}\,\lambda > o$) whose real part is a Lyapunov number and whose complex part measure rotation for appropriate solutions of $(*)$. The function $-\hat{\beta}(\lambda) + i\alpha(\lambda) = \lim_{\varepsilon \to 0} w(\lambda + i\varepsilon)$ for $\lambda \in \mathbb{R}$ has the following properties.
(i) If $\bar{x} = \binom{x_1}{x_2} \in \mathbb{R}^2$ and $\theta = \arg(x_1 + ix_2)$, then $\alpha(\lambda) = -\lim_{T\to\infty} \frac{\theta(T)}{T}$ is the rotation number. (ii) $\hat{\beta}(\lambda)$ is (roughly speaking) the maximal Lyapunov number for $(*)_\lambda$. The beginnings of an extension to more general (i.e. higher dimensional) systems $(*)_\lambda$ were discussed.

A. KATOK:

Rigidity and Stability for Cocycles over Dynamical Systems

We consider the so called cohomological equation over a discrete-time dynamical system $f : X \to X$:

$$h(x) = \phi(f(x)) - \phi(x) \qquad (1)$$

where h is a given function from a fixed class H (e.g. continuous, Hölder, C^r, C^∞, real analytic) and we look for a solution ϕ from another fixed class Φ! Corresponding equation for a continuous time system is $h(x) = (D\phi)(x)$ where D is the differential operator generated by the flow. We say that the space H is Φ-rigid (correspondingly Φ-stable) if for any $h \in H$ there is a constant h_0 such that for $h - h_0$ equation (1) has a solution $\phi \in \Phi$ (correspondingly the set of all $h \in H$ for which (1) can be solved in Φ is closed in H). Let μ be an f-invariant measure. The space H is called Φ-effective (with respect to μ) if for every $h \in H$ the existence of a measurable solution ϕ of (1) implies that $\phi \in \Phi$. The only known case of C^∞ rigidicity appears for toral rotations with not too well rationally approximable rotation numbers. We conjecture that this is the only case. We show that neither stability nor effectiveness ever take place for continuous functions. This remains the case for C^∞ functions if f admits abnormally fast periodic approximation. Known cases of stability include Anosov systems (C^1case-Lipschitz-theory, C^∞ case for geodesic flows only - Guillemin and Kazdan), where the solvability of (1) follows from the vanishing of the sums of values over all periodic orbits, and several strictly ergodic algebraic systems, including affine maps of the torus and horocycle flows on surfaces of constant negative curvature. In the last two cases the solvability of (1) is equivalent to vanishing of infinitely many invariant distributions which are not measures. Reference: A. Katok, Constructions in Ergodic Theory, to appear in Progress in Mathematics, Birkhäuser, 1983.

J. MARTINET:

Some properties of resonant differential equations

We consider analytic germs of resonant differential equations in $\mathbb{C}^2$:

$$(1) \qquad \omega = y(p+\dots)dx + x(q+\dots)dy = 0$$

when p and q are positive integers. We prove that the classification of such equations, through analytic diffeomorphisms, is equivalent to the classification, up to analytic conjugacy, of local diffeomorphisms of the complex line $\mathbb{C}$, with linear part $z \to e^{2i\pi p/q}z$. The relation between the two problems is mady by means of the holonomy of the separatices of (1). Moreover, we describe the "Moduli space" of equivalence classes as a set of one-dimensional non Hausdorff, complex manifolds, which are the leaf-spaces (or orbit spaces) of these differential equations (or diffeomorphisms).

R. MACKAY:

Selfsimilarity of invariant circles

1. Breakup of invariant circles for area preserving twist maps

Numerical work using the criteria of Greene and Mather suggests that given ω Diophantine, the boundary between systems with a smooth circle of rotation number ω and those with no circle of rotation number ω is a codimension 1 surface. In the case of noble frequencies, self-similarity of systems on the critical surface suggests it is the stable manifold of a certain fixed point of a renormalization operator in the space of commuting pairs of a.p. map.

2. Boundary of Siegel domains

Numerical work indicates self similarity of the boundary of Siegel domains of arbitrary rotation number. This suggests there is an attracting 2-D set under a renormalization operator on which the motion is equivalent to a shift on doubly infinite continued fractions.

J. MATHER:

Invariant circles for area preserving twist homeomorphisms

A twist diffeomorphism of the annulus is one which satisfies $f(x,y) = (x',y')$ if and only if $y = h_1(x,x') = \partial h(x,x')/\partial x$ and $y' = -h_2(x,x') = -\partial h(x,x')/\partial x'$, where h is an auxillary function called, in classical mechanics, a generating function. G.D. Birkhoff showed that an invariant circle for such a diffeomorphism which goes around the annulus is the graph of a Lipschitz function. Aubry, et. al., showed, that for an orbit $\{\ldots,(x_i,y_i),\ldots\}$ on an invariant circle, $W_{m,n}(x) = \sum_{i=m}^{n-1} h(x_i,x_{i+1})$ is maximal for variations of x subject to x_m and x_n fixed. This is valid for all integers $m < n$. Let $\Delta W_{p,q}$ be the difference of the actions of a Birkhoff max orbit and a Birkhoff minimax orbit of type p/q. Then, for ω irrational, $\Delta W_\omega = \lim_{p/q \to w} \Delta W_{p,q}$ exists and there is an invariant circle of rotation number ω if and only if $\Delta W_\omega = 0$.

J. PALIS:

Rigidity of centralizers of diffeomorphisms

Several results showing that among Axiom A C^∞ diffeomorphisms of a compact manifold, generically the centralizer is trivial (just powers of the map). The same is true for an open dense set when dim M = 2, in higher dimensions, when there is a periodic attractor. Finally, the maps of an open dense set have no roots of any order. Questions of the same nature seem to be open for volume preserving or symplectic transformations.

H.-O. PEITGEN:

Homoclinic bifurcations and spurions solutions of nonlinear eigenvalue problems

It is a striking fact in the numerical analysis of nonlinear elliptic boundary value problems that numerical approximations typically generate spurions solutions. These are solutions, which are perfect solutions of the approximation scheme but are by no means approximate solutions to the given boundary value problem. For infinite difference approximations it is possible to associate with the numerical scheme a discrete time dynamical systems which is parametrized by the meshsize of the discretization. The boundary value conditions are reflected in this setting by orbits which are consistent with intersection properties of certain natural submanifolds. We discuss particular nonlinearities, which generate a hyperbolic structure for the dynamical system and observe that as we change the meshsize, the homoclinic structure undergoes an odd-type bifurcation, which in turn by means of the λ-Lemma gives rise to a bifurcation of spurious solutions. For proofs we use in an essential way the underlying involution structure, a topological index (which is the intersection number) for transverse homoclinic points together with a very special model for the nonlinearity (close to a PL-function). By these means we obtain an infinite sequence of bifurcations of spurious solutions as the meshsize goes to zero.

I. PERCIVAL:

Motions across Cantor-sets

Invariant Cantor sets of Aubry or Mather form barriers to iterates of an area preserving twist map. The barrier can be penetrated at the gaps. A single iterate of the map interchanges an area in the plane equal to Mather's ΔW, with a given definition of curves joining the ends of the gaps. The theory has applications to particle confinement in Tokomaks.

J. PÖSCHEL:

On the spectrum of Schrödinger operators

We consider the stationary Schrödinger (*) $Ly = -y'' + q(x) = \lambda y$ on the real line, where q in quasiperiodic with basic frequencies $\omega = (\omega_1,\dots,\omega_d)$. We suppose that ω is diophantine, and $q \in Q^a(\omega)$, that is q extens to an analytic function on its hull. The spectral gaps of L are precisely the intervals of constancy of the rotation number $\alpha(\lambda)$ of q, and there, $\alpha(\lambda) = \frac{1}{2}(j,\omega)$ for some $j \in Z^d$ ("gap labelling"). We show: If $\mu = \frac{1}{2}(k,\omega)$ is sufficiently large and badly approximable by all the other resonances $\frac{1}{2}(j,\omega)$, $j \neq k$, then the gap $[\lambda_-,\lambda_+] = \alpha^{-1}(\mu)$ is generically open, and (*) has Floquet solutions $e^{i\mu x}(\chi_1 + x\chi_2)$, $e^{i\mu x}\chi_2$, where $\chi_{1,2} \in Q^a(\omega)$, for $\lambda = \lambda_\pm$. If $\lambda_- = \lambda_+$, then all solutions are of the form $e^{i\mu x}\chi$, $\chi \in Q^a(\omega)$. This complements a result of Dinaburg and Sinai. For λ in the resolvent set there always exist Floquet solutions $e^{i\mu x}\chi_1$, $e^{+i\mu x}\chi_2$.

K. RYBAKOWSKI:

On the homotopy-Index and solutions of parabolic equations

Let $\Omega \subset \mathbb{R}^n$ be a smooth bounded domain and $f : \bar\Omega \times \mathbb{R} \to \mathbb{R}$ be continuous, locally Lipschitzian in $s \in \mathbb{R}$, uniformly for $\xi \in \bar\Omega$. Consider the following parabolic BVP:

$$(1)\quad \begin{aligned} &\frac{\partial u(t,x)}{\partial t} + A(x,D)\, u(t,x) = f(x,u(t,x)) && x \in \Omega \\ &B_i(x,D)\, u(t,x) = o, \quad i = 1,\dots,m && x \in \partial\Omega\,. \end{aligned}$$

Here A(x,D) is a uniformly strongly elliptic linear differential operator of order 2m, $m \geq 1$, and $B(x,D) = (B_1(x,D),\dots,B_m(x,D))$ are boundary operators such that (A(.,D), B(.,D)) is formally self-adjoint and satisfies the hypotheses of the Agmon, Douglis,Nirenberg theory. We prove existence of equilibriua and heteroclinic orbits of (1) under various hypotheses on f, comprising both the "nonresonance" and the "resonance" of f at zero. The main tool is an extension of Conley's index theory to non compact spaces. Our results extend earlier results of Amann - Zehnder (who were the first apply the Conley index to elliptic BVP), Peitgen - Schmitt and this author.

J. YOCCOZ:

Conjugation of diffeomorphisms of the circle with diophantine rotation numbers

Some new results are presented:

Prop. 1: Let f a C^∞-homeomorphism of $\mathbb{T}^1$ with no periodic orbits and non-flat critical points. Then f is topologically conjugate to a rotation.

Th. 1: Let $f \in \mathrm{Diff}_+^k(\mathbb{T}^1)$, $\rho(f) = \alpha \in \mathbb{T}^1 - \mathbb{Q}/_{\mathbb{Z}}$, s.t.

i) $\exists C, \beta \geq 0$ s.t. $\forall p/q \in \mathbb{Q}$, $|\alpha - p/q| \geq \frac{C}{q^{2+\beta}}$

ii) $k \in \mathbb{N}$, $k > 2\beta+1$, $k \geq 3$.

Then there exists $h \in \mathrm{Diff}^{k-1-\beta-\varepsilon}(\Pi^1)$ such that $f = hR_\alpha h^{-1}$.

Th. 2: In $F_\alpha = \{f \in \mathrm{Diff}_+^\infty(\Pi^1), \rho(f)=\alpha\}$, those which are smoothly conjugated to R_α are dense in the C^∞ topology (for any given irrational α).

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