MATHEMATISCHES FORSCHUNGSINSTITUT OBERWOLFACH

Tagungsbericht 25/1983

Spectral Theory of Ordinary Differential Operators

June 5 - June 11, 1983

The conference was held under the head of Professor H.D. Nießen (Essen) and Professor A. Schneider (Dortmund). The following subjects from the field of spectral theory of ordinary differential operators were primarily dealt with: expansion theorems, extensions of differential operators, asymptotic methods, criteria for the determination of the deficiency indices and the nature of the spectrum.
There has been an animated exchange of ideas not only among the participants of this conference, but also with those of the meeting: Special functions and differential expressions in the complex plane which took place simultaneously. Two common lectures were also organized which were of interest to both groups. Nevertheless, the separation into two meetings turned out to be very efficient since the setting of the problems and the practical methods are developing into different directions.

Abstracts

H.E. Benzinger

Spectral Theory of Ordinary Differential Operators

Differential operators defined by Birkhoff regular boundary conditions generate eigenfunction systems with convergence properties similar to those of Fourier series. The definition of Birkhoff regularity has no intrinsic spectral theoretic content. We show that the resolvent operators of Birkhoff regular operators with simple spectrum are <u>well-bounded</u> operators. This implies the existence of a functional calculus and an integration theory with respect to a family of projections. This can be applied to the study of semigroups generated by even order differential operators.

B.L.J. Braaksma

<u>Asymptotics and Deficiency Indices for Certain Pairs of Differential Expressions</u>

Let $M = \prod_{1}^{n} (xD - a_j) - \mu x^{\alpha} \prod_{1}^{m} (xD - b_j)$, $D = \frac{d}{dx}$, $0 \leq m < n$.

Solutions of $My = 0$ have been studied by Meijer, Kohno and Ohkohchi who solve the connection problem in the general case except for certain exceptional cases. We give an outline of the method of Meijer and show how the restrictions can be removed. Next we consider perturbations of M with the property that their solutions have the same behavior as $x \to +\infty$ as those of M. This is used to determine deficiency indices related to $L_1 y = \lambda L_2 y$ where L_1, L_2 are formally symmetric, one of them is positive and L_1, L_2 are perturbations of operators $x^{\gamma} M_1, x^{\varepsilon} M_2$ with M_1 and M_2 of the same type as M.

R.C. Brown

<u>On a Von Neumann Factorization for Some Selfadjoint Differential Operators</u>

Let L, L_o be the maximal and minimal operators induced in the space $L^2_{\omega}(a,b)$, $-\infty < a < b \leq \infty$ by the quasiderivative expression $\omega^{-1} y^{[2n]}$ where $\omega, p_o^{-1}, p_1, \ldots, p_n$ are locally integrable, $\omega^{>0}, p_o, \ldots, p_n$ are nonnegative and additionally $p_n \geq \epsilon > 0$ on $[a,b)$. Let H be $\bigtimes_{i=0}^{n} L^2(a,b)$ with the usual inner product. Define $L : L^2_{\omega}(a,b) \to H$ by $Ly = (p_o^{1/2} y^{(n)} \ldots p_n^{1/2} y)^t$. Let L'_o be its "preminimal" restriction. We compute $L^{*}\ L'^{*}_o$ and $L'^{**}_o = \bar{L}'_o : = L_o$. We show that $L^{*}_o L_o\ L^{*} L$ are selfadjoint restrictions of

L defined on cores of L_o and L and explicitly determine their structure. For example $D(L_o^{x} L_o) = \{y \in D(L) : y^{[i]}(a) = 0$

$0 \leq i < n; \int_a^b \sum_{i=0}^{n} p_i |y^{(n-i)}|^2 < 0 \quad D(f,\bar{y})(b^-) = 0, \quad \forall f \in D(L_o)\}$.

These operators are the Friedrich extensions representing the forms $\|L_o y\|^2$. $\|Ly\|^2$; from them the Dirichlet inequalities follow. The apparatus is also pertinent to the Dirichlet index problem - the statement that the index is minimal (the "limit pointness" of L_o) i.e. $\dim D(L)/D(L_o) = n$. The index can also be shown to be invariant under the class of $\underline{\underline{t}}$ bounded perturbations of the form. We also obtain that the index of a class of 4th order operators is "2" under hypothesis complementing those of T.T. Read. Finally (using a somewhat different apparatus) we sketch how the theory goes over to the negative coefficient case and discuss the "dual Dirichlet inequalities" which appear to be new - e.g. on $[1,\infty)$, $n = 1$

$$\int_1^d \left|-(p_o^{1/2}\zeta_1)' + p_1^{1/2}\zeta_2\right|^2 \geq u_o \left|\sup_{\substack{\{y : \int p_o|y'|^2 + p_1|y|^2 = 1\} \\ p_o y'(a) = 0}} [\zeta_1 p_o^{1/2} y' + \zeta_2 p_1^{1/2} y]\right|^2$$

and $u_o = \inf \operatorname{spec}\{L^{x} L\}$

A. Dijksma and H. de Snoo

Selfadjoint Extensions of Symmetric Subspaces in Pontrjagin Spaces Part I and II

A. Dijksma, part I: Let S be a symmetric subspace in the Hilbert space $(H,(\ ,\ ))$. Let A be a selfadjoint subspace in the Hilbert space $(K,[\ ,\ ])$. Then (A,K) is called a selfadjoint extension of (S,H) if $H \subset K, [\ ,\ ]|_{H\times H} = (\ ,\ )$ and $S \subset A$. All selfadjoint extensions of (S,H) can be characterized by means of generalized resolvents of S or by socalled families of Štraus subspace-extensions of S in H. In the lecture we will discuss their properties and apply the results to ordinary differential operators. The results can be found partly in McKelvey (1965), Dijksma and de Snoo

(1974), Langer and Textorius (1977). In cooperation with Langer (Dresden) de Snoo and I are working on the case where K is replaced by Pontrjagin spaces. A report of these results will be given in the part II-lecture by H. de Snoo.

H. de Snoo, part II: Let H be a Hilbertspace with inner product $(\, , \,)$ and let $S \subset S^* \subset H^2$ be a symmetric subspace. Let K_k be a Pontrjagin space with $k (0 \leq k < \infty)$ negative squares and with inner product $[\, , \,]$ and let $A = A^+$ be a selfadjoint subspace in K_k^2. Then A is a selfadjoint extension of S if H is an orthocomplemented subspace of K_k, $[\, , \,] = (\, , \,)$ on H and $S \subset A$. If the resolvent set of A is non-empty, then we can characterize such extensions in terms of operators or subspaces in H^2. The positivity conditions which are required for the case $k = 0$ now have to be replaced by conditions, that require certain associated kernels to have k negative squares. Classes of analogous kernels have been studied by M.G. Krein and H. Langer. We characterize the associated boundary conditions, when the deficiency indices of S are finite, and we show what this implies for differential operators. In part I by A. Dijksma the case $k = 0$ was described. The results in part II will appear in a joint paper with A. Dijksma and H. Langer.

M.S.P. Eastham

A Property of Matrices Arising in the Asymptotic Theory of Higher-Order Quasi-Differential Equations

Suppose that a higher-order formally self-adjoint differential expression is written in quasi-differential form as $Y' = AY$. In the asymptotic theory (as $x \to \infty$), the first step is to diagonalise $A : A = T^{-1} \Lambda T$. Then $Y = TZ$ gives $Z' = (\Lambda - T^{-1}T')Z$. In estimating the size of $T^{-1}T'$, the expression $w^t v$ (= m) is encountered, where v is an eigenvector of A, w is the vector obtained from v by writing the components in reverse order, and t denotes the transpose. It is shown that an explicit form for m can be obtained for a class of matrices A which includes those arising in quasi-derivatives, and the significance of the expression m is discussed.

W.D. Evans

Maximal Accretive Extensions of Ordinary Differential Operators

Let $\tau u = -(pu')' + qu$ on $[a,b)$, $-\infty < a < b \leq \infty$ where the coefficients p,q are real-valued and satisfy the conditions

i) $p(x) > 0$, $1/p \in L^1_{loc}[a,b)$,

ii) $q(x) \geq \delta > 0$ and $q \in L^1_{loc}[a,b)$.

The expression τ is therefore regular at a and at b is is assumed to be singular and to satisfy both the Strong-Limit-Point and the Dirichlet conditions. In other words for $\phi \in \Delta$ where

$$\Delta := \{\phi : \phi \text{ and } \phi^{[1]} := p\phi' \in AC_{loc}[a,b),\ \phi, \tau\phi \in L^2(a,b)\}$$

we have $\lim_{x\to b} \phi(x)\phi^{[1]}(x) = 0$ and $p^{1/2}\phi'$, $q^{1/2}\phi \in L^2(a,b)$. Let $T := \tau|_\Delta$ and $T_o := \tau|_{\Delta_o}$, where

$$\Delta_o := \{\psi \in \Delta : \psi(a) = \psi^{[1]}(a) = 0\}.$$

Then T_o is closed and densely defined in $L^2(a,b)$ and furthermore $T_o{}^* = T$. Also, on setting

$$D[u,v] := \int_a^b pu'\bar{v}' + qu\bar{v}, \qquad u,v \in \Delta$$

we get that

$$(T_o u,u) = D[u,u], \qquad u \in \Delta_o$$
$$\geq \delta \|u\|^2$$

and hence T_o is accretive. The subject of the talk is the characterization of all the maximal accretive extensions of T_o. This is achieved by means of the Phillips theory in which the problem is related to obtaining the maximal negative subspaces of the so-called "boundary space". The result obtained is the following

Theorem 1 An operator S is a maximal accretive extension of T_o if and only if its adjoint S^* is the restriction of T to

$$D(S^*) := \{u \in D(T) = \Delta : \bar{\beta}\, u(a) + \bar{\alpha}\, u^{[1]}(a) + 2D[u,\phi] = 0\}$$

for some $\phi \in H_1$, the completion of Δ with respect to $D^{1/2}[\cdot,\cdot]$, and $\alpha,\beta \in \mathbb{C}$ which satisfy

$$\mathrm{re}(\alpha\bar{\beta}) + D[\phi,\phi] \leq 0.$$

On using a result of R.C. Brown and A. Krall the operators S in Theorem 1 are given explicitly by

Theorem 2 For some $\lambda \in \mathbb{C}$, S is the restriction of the expression σ defined by

$$\sigma u = -\{(u - 2\lambda\phi)^{[1]} + 2\lambda \int_a^t q\phi\}' + qu$$

Gefördert durch DFG Deutsche Forschungsgemeinschaft

to the space

$$D(S) = \{u : u-2\lambda\phi \text{ and } (u-2\lambda\phi)^{[1]} + 2\lambda \int_a^t q\phi \in AC_{loc}[a,b),\ u, \sigma u \in L^2(a,b)$$

$$\text{and} \quad u(a+) = \lambda\alpha,\ (u-2\lambda\phi)^{[1]}(a+) = -\lambda\beta\}$$

There is an analogous result when $\tau u = \sum_{i=0}^{n} (-1)^{(i)} [p_{n-i} u^{(i)}]^{(i)}$ with p_{n-j} real.

W. N. Everitt

Asymptotic Form of the Titchmarsh-Weyl m-Coefficient

For the differential equation (in general this is a quasi-differential equation in the sense of Shin-Zettl)

$$-(py')' + qy = \lambda wy \quad \text{on } [a,b) \qquad (*)$$

with (i) $-\infty < a < b \leq \infty$

(ii) $p,q,w : [a,b) \to R$

(iii) $p^{-1}, q, w \in L_{loc}[a,b)$

(iv) $w \geq 0$ on $[a,b)$, and

(v) $\lambda \in C$,

let the solutions of (*) $\theta, \varphi : [a,b) \times C \to C$ be determined by the initial conditions

$\theta(a,\lambda) = 0$, $(p\theta')(a,\lambda) = 1$ $\quad \varphi(a,\lambda) = -1$, $(p\varphi')(a,\lambda) = 0$.

The Titchmarsh-Weyl m-coefficient $m = (m_+, m_-)$ has the properties

$m_{\pm} : C_{\pm} \to C_{\pm}$, $m_{\pm} \in H(C_{\pm})$ and

$$\int_a^b w|\theta + m_{\pm} \varphi|^2 = \mathrm{im}[m_{\pm}(\lambda)]/\mathrm{im}[\lambda] \qquad (\lambda \in C_{\pm}).$$

The asymptotic form of the m-coefficient, for large values of $|\lambda|$, is to be considered, in the lecture, and compared with the asymptotic form of the spectral function ρ of m in the Herglotz-Nevanlinna-Pick-Riesz integral representation.

J. Fleckinger

Eigenvalues of a Non Definite Elliptic Problem

We study here the eigenvalues and eigenfunctions of

$$\begin{cases} -\Delta u + cu = \lambda\, gu \quad \text{on} \quad \Omega \subset \mathbb{R}^n & \text{(bounded)} \\ u|_{\partial\Omega} = 0 & \end{cases}$$

c may be negative (and $-\Delta + c$ is not necessarily positive)
g changes sign.

G. Freiling

On the Completeness of the System of Eigenfunctions of Irregular Eigenvalue Problems in $L_2[0,1]$

We consider several special cases of eigenvalue problems on the segment [0,1] of the form

$$\ell(y,\lambda) = y^{(n)} + \sum_{\nu=1}^{n} p_\nu(x,\lambda)\, y^{(n-\nu)} = 0 \tag{1}$$

$$U_j(y,\lambda) = \sum_{\nu=0}^{n-1}\left[\alpha_{j\nu}(\lambda)y^{(\nu)}(0) + \beta_{j\nu}(\lambda)y^{(\nu)}(1)\right] = 0, \quad 1 \le j \le n, \tag{2}$$

where $p_\nu(x,\lambda) = \sum_{k=0}^{\nu} p_{\nu k}(x)\lambda^k$ and the coefficients $\alpha_{j\nu}(\lambda)$ and $\beta_{j\nu}(\lambda)$ are certain polynomials in λ. Extending results of Eberhard (Math. Z. 146), Shkalikov (Funkts. Anal. Prilosz. 10) and Vagabov (Sov. Mat. Dokl. 23) we show that the system of eigen- and associated functions of certain classes of irregular eigenvalue problems of type (1),(2) is complete in $L_2[0,1]$.
Examples show that the assumptions used in the proof cannot be weakened.

H. Frentzen

A Limit-Point Criterion for Symmetric and J-Symmetric Second-Order Differential Expressions

On $I = [a,b)$, $-\infty < a < b \le \infty$, consider $My = R^{-1}(-(Py')' + Qy)$ and assume that $P,Q,A : I \to \mathbb{M}_s$ (complex $s \times s$ matrices) are measurable,

$P(t)$ is invertible, $A(t) > 0$ a.e. on I, $P^{-1}, Q, A^2 \in L^1_{loc}(I)$, $P^+ = P$, $Q^+ = Q$, $R = A^+A$ where either $+$ denotes the complex conjugate transpose or the transpose. In the first case the minimal operator T_o associated to M in $H = \{y : I \to \mathbb{C}^s \mid y^*A^2y \in L^1(I)\}$ is symmetric, in the second case T_o is J-symmetric where $Jy = A^{-1}\overline{Ay}$ is a conjugation in H.

Theorem. Assume that $K > 0$ and $0 \leq c < 1$ are constants, $W, B, U, Q_j : I \to \mathbb{M}_s$, $s_j : I \to \mathbb{R}$ $(1 \leq j \leq m)$, $r, v : I \to [0,\infty)$ are measurable and $r, B \in L^1_{loc}(I)$, $s_j, Q_j, v^2P^*W \in AC_{loc}(I)$, $U > 0$, $U^2 = \mathrm{Re}\, W$, $B \geq 0$, $AB = BA$, $Q_j(t)$ are hermitian, $r \in L^1(I)$, $r|(AU^+)^{-1}| \leq Kv$, $rv|A_c^{-1}P^*WU^{-1}| \leq K(1 + \int_a^t r)$ where $A_c = (B^cA^{2(1-c)} + A^2)^{1/2}$, $|A_c^{-1}(v^2P^*W)'U^{-1}| \leq Kv$, $v^2|A_c^{-1}P^*WA^+| \leq K(1+\int_a^t r)$, $B - v^2\mathrm{Re}(P^*WQ) \leq KA^2 + \sum_{j=1}^m s_jQ_j'$, $r|s_jA_c^{-1}Q_jA_c^{-1}| \leq K(1 + \int_a^t r)$, $|s_j'A_c^{-1}Q_jA_c^{-1}| \leq K$, $|s_jA_c^{-1}Q_jP^{-1}U^{-1}| \leq Kv$. Then $d(\lambda) \leq s$ for every $\lambda \in \mathbb{C}$ where $d(\lambda) = \dim\{y \in H \mid My = \lambda y\}$. If additionally T_o is symmetric, then every real polynomial in M is limit-point.

R.C. Gilbert

Auxiliary Polynomials and Asymptotic Formulas for Solutions of a Singular Linear Ordinary Differential Equation

Consider the equation (1) $\sum_{r=0}^{n} \alpha_{n-r}(x)y^{(r)} = i^{-n}\lambda x^{-q}y$, where q is a positive integer, $\alpha_0(x) \equiv 1$, $\alpha_{n-r}(x)$ is holomorphic for x in a sector S of the complex plane, $0 < x_0 \leq |x| < \infty$, and $\alpha_{n-r}(x) \sim \sum_{k=0}^{\infty} \alpha_{n-r,h}x^{-k}$ as $x \to \infty$. Let $I_h(\mu) = \sum_{r=0}^{n} \alpha_{n-r,h}\mu^r - i^{-n}\lambda\delta_{hq}$. For certain symmetric operators, $I_0(\mu) = i^{-n}H_0(i\mu)$, where H_0 has real coefficients. Let $\beta = s - it$ be a zero of $I_0(\mu)$ of multiplicity m. Let $f_1, \ldots, f_m$ be part of a basis for (1) corresponding to β. Let $n^+(n^-)$ be the number of the f_j in L^2 for $I\lambda > 0 (<0)$. For $q = 1$ it is known that $n^+ = n^-$ if $s \neq 0$ or if $s = 0$ and m is even, but if $s = 0$ and m is odd, then $n^+ - n^- = -1(+1)$ for $H_0^{(m)}(t) > 0 (<0)$. The latter condition makes $n^+ - n^-$ switch sign over consecutive t of odd m. The following two questions will be considered for $q \geq 2$: (a) Can $n^+ - n^- \neq 0$ and retain sign for consecutive t? (b) Is $|n^+ - n^-| \geq 2$ possible?

R. M. Kauffman

Powers and Roots of Ordinary Differential Operators

We study differential expressions R of order 2N on $[1,\infty)$, such that $(Rf,f) \geq \varepsilon(f,f)$ for all f in $C_0^\infty(1,\infty)$, where $\varepsilon > 0$. We consider the problem "for every g in $L_2[1,\infty)$ find an f in $L_2[1,\infty)$ such that $Rf = g$ and $f^{(i)}(1) = 0$ for $0 \leq i \leq n-1$" as a motivation for studying the deficiency index and other problems involving boundary conditions at infinity for R. We calculate the deficiency index of R, where R is any polynomial in a positive polynomial coefficient expression L of fairly general type. We then discuss additional boundary conditions at infinity for general R which make the above problem well-posed even when the deficiency index of R is greater than N.

I. Knowles

Differential Equations and the Riemann Zeta Function

The main aim of the lecture is to present some consequences of a recently developed algorithm (c.f. [Proc. Conf. Diff. Equ. Dundee Scotland, Springer, Lecture Notes in Mathematics Vol. 964, 388-405]) that associates uniquely, a differential equation of the form

$$z'' - sb(x)\, z'(x) + s^2 c(x)\, z(x) = 0, \quad x \in [0,\infty)$$

with certain Euler products

$$R(s) = \prod_{n=1}^{\infty} \left(1 + \frac{a_n}{p_n^s}\right);$$

included here are the Euler products for $\zeta(s)/\zeta(2s)$ and $1/\zeta(s)$, where $\zeta(s)$ is the classical Riemann zeta fucntion. Some consequences of this approach include new formulae for the zeta function, the possibility of a proof for the prime number theorem via ODE asymptotics, and some connections with the automorphic wave equation and the theory of Hecke operators.

M. A. Kon

Semigroups Generated by Ordinary Differential Operators

We analyze semigroups generated by ordinary differential operators through spectral theoretic techniques. The integral kernel of the semigroup is expressed explicitly as a contour integral of the resolvent kernel, which is used to prove L^p and pointwise continuity properties of the semigroup. In particular, the semigroup is shown to be holomorphic in the right half of the complex plane, and continuous in L^p and pointwise at the origin. Smoothing properties of the semigroup operator are also considered, and the results are interpreted in terms of temporal smoothing in generalized heat equations. Results are presended in the general context of analytic functions of differential operators.

H. Kurss

Estimates of μ, the Least Limit Point of the Spectrum

Two theorems of Hinton and Lewis (1975) on sufficient conditions for $\mu = \infty$ for a real symmetric differential operator of order 2N are shown to remain valid when the pointwise constraints $p_j(x)/x^{2j} \geq -C$ are weakened, à la Brinck (1959), to average constraints $\int_J (p_j(t)/t^{2j})dt$ for all intervals J of length ≤ 1. A similar replacement is expected to be valid in $\mu = \infty$ criteria of Müller-Pfeiffer (1977) and Read (1982). The above two theorems are also strengthened to give upper and lower bounds $\mu \leq \mu_E$ or $\mu \geq \mu_E$ where, as in Eastham (1970), μ_E is the least limit point of a corresponding Euler operator.

S. J. Lee

Constructive Methods for Least-Squares Solutions

Orthogonal operator parts of linear manifolds play an important role in numerical problems. We will consider (and discuss) several applications of orthogonal operator parts:

(i) Constructive methods of operator parts

(ii) Steepest descent method for the least-squares solutions for a multi-valued operator equation.

(iii) Steepest descent method for a nondensely defined differential operator equation.

(iv) Tikhonov's method of regularization for a nondensely defined differential operator having a nonclosed range.

R. Mennicken

The Structure of Green's Function: A Simple Proof of a Theorem of M.V. Keldyš

Let E,F be B-spaces, U an open subset of $\mathbb{C}$, $T \in H(U,L(E,F))$ and $\mu \in U$. $Y \in H(U,E)$ is called a root function (RF) of T in μ if $Y(\mu) \neq 0$ and $(TY)(\mu) = 0$; let $\nu(Y)$ denote the order of the zero of TY in μ. A set of root functions $Y_1,\dots,Y_r$ of T in μ is called a canonical system (CSRF) if $\{Y_1(\mu),\dots,Y_r(\mu)\}$ is a basis of $N(T(\mu))$ and

$$\nu(Y_j) = \max\left\{\nu(Y): \begin{array}{l} Y \text{ RF of } T \text{ in } \mu, \\ Y(\mu) \notin \operatorname{span}\{Y_1(\mu),\dots,Y_{j-1}(\mu)\} \end{array}\right\}.$$

Theorem. Assume that $T^{-1} \in H(U\setminus\{\mu\}, L(F,E))$, having a pole in μ. Let $\{Y_1,\dots,Y_r\}$ be a CSRF of T in μ and set $m_j := \nu(Y_j)$. Then the following assertions hold:

1) There exist uniquely polynomials $V_j : \mathbb{C} \to F'$ of degree less than m_j such that
$$T^{-1} - \sum_{j=1}^{r} (\cdot-\mu)^{-m_j} Y_j \otimes V_j$$
is holomorphic in μ.

2) The set $\{V_1,\dots,V_r\}$ is a CSRF of T^* in μ and $\nu(V_j) = m_j$.

3) The biorthogonality relationships
$$\frac{1}{l!}\frac{d^l}{d\lambda^l} < (\cdot-\mu)^{-h} TY_i, V_j > (\mu) = \delta_{ij}\delta_{m_i-h,l}$$
hold for $h = 1,2,\dots,m_i$; $l = 0,1,\dots,m_j-1$; $i,j = 1,2,\dots,r$.

This theorem is due to Keldyš (1951,1971) Gohberg and Sigal (1970).

Gefördert durch DFG Deutsche Forschungsgemeinschaft

Recently Mennicken and Möller proved this in a simple and direct way which is presented at this meeting. An application of the theorem to boundary eigenvalue problems yields the representation of the singular parts of Green's functions in terms of eigenvectors and associated vectors of T and T*. The coefficients of the differential equations as well as those of the boundary conditions are allowed to depend holomorphically on λ; for special results cf. e.g. Langer (1939), Cole (1964) and Krall (1975).

B. Mergler

A Limit-Point-Criterion for Even Order Symmetric Differential Expressions

We consider the general 2n-th order symmetric differential expression

$$(1)\qquad \sum_{j=0}^{n}(-1)^j(p_jy^{(j)})^{(j)} + i\sum_{j=0}^{n-1}(-1)^j\ \{(q_jy^{(j)})^{(j+1)} + (q_jy^{(j+1)})^{(j)}\}$$

on $I = [1,\infty)$. For expressions of type (1) we derive a limit-point-criterion by using the theory of relatively bounded perturbations. The obtained criterion generalizes a result of Schultze (1981 , Proc. Roy. Soc. Edinb.). The method used here is a generalization of the method developed by Schultze in the above-mentioned article. The key feature is to obtain a positive lower estimate for $\|M_of\|_2^2$ where $f \in C_o^\infty(\overset{o}{I})$ and $M_oy = (-1)^n(t^{\alpha_n}y^{(n)})^{(n)} + (-1)^\ell(ct^{\alpha_\ell}y^{(\ell)})^{(\ell)}$, $n,\ell \in \mathbb{N}_o$, $n \geq 2$, $n > \ell$, $\alpha_n, \alpha_\ell \in \mathbb{R}$, $\alpha_n - 2(n-\ell) < \alpha_\ell$ and $c > 0$ on using the identity $\|M_of\|_2^2 = (M_o^2f,f)$. M_o is in the limit-point-case by a criterion of Kaufman (1977, Proc. Lond. Math. Soc.). Finally we use the following

Theorem. Let M,N ODE of type 1 with M, M+N regular (i.e. the leading coefficient is positive) and order (M) $\geq$ order (N). Furthermore there exists $\beta \in \mathbb{R}$ such that for all $f \in C_o^\infty(\overset{o}{I})$ $\|Nf\|_2^2 \leq \|Mf\|_2^2 + \beta\|f\|_2^2$. Then $\mathrm{def}(T_o(M+N)) \leq \mathrm{def}(T_o(M))$.

Thus we obtain the desired result.

D. Race

A Simplified Characterization of the Boundary Conditions Which Determine J-Selfadjoint Extensions of J-Symmetric (Differential) Operators

Various authors, including N.A. Zhikhar and I.W. Knowles, have looked at the problem of characterizing the boundary conditions which determine J-selfadjoint extensions of J-symmetric operators such as the minimal operator generated by

$$\tau y = \sum_{i=0}^{n} (-1)^{n-i} (p_i y^{(n-i)})^{(n-i)}$$

where each p_i is a complex-valued function. All the work to date has required the regularity field of this J-symmetric operator to be non-empty, a difficult condition to check in applications. We give here a more general but shorter and simpler method of proof, which yields the same results, but requires no such assumption. This answers several open questions concerning an example of J.B. McLeod (1962), namely

$$-y'' - 2i\, e^{2(1+i)x} y = \lambda y.$$

T.T. Read

Bounds for the Essential Spectrum of Some Differential Operators

It is known, both for the ordinary differential expression $\tau_1 y = -(py')' + qy$ on $[a,\infty)$ and for the partial differential expression $\tau_2 y = -\Delta y + qy$ on R^n, that if there exist $g(x)$ and $r > 0$ such that $\tau g = 0$ and $g(x) > 0$ for $|x| \geq r$, then $\inf \sigma_{ess}(T) \geq 0$, where T is the Friedrichs extension of the minimal operator. Moreover, if no such g and r exist, then $\inf \sigma_{ess}(T) \leq 0$. Thus we can obtain bounds on the essential spectrum by investigating the existence of positive solutions. We do this by using the following characterization to obtain several results on the existence or nonexistence of positive solutions.

<u>Theorem.</u> (a) $\tau_1 y = 0$ has a positive solution iff $q = u + Q'$ with $u \geq Q^2/p$.

(b) $\tau_2 y = 0$ has a positive solution iff $q = u + \operatorname{div} Q$ with $u \geq |Q|^2$.

B. Schultze

<u>Odd Order Selfadjoint Differential Expressions with Positive Real Powers as Supporting Coefficients</u>

We consider the odd order differential expression

$$My = \frac{i}{2}\sum_{\kappa=0}^{m}(-1)^{\rho_\kappa}\left\{\left(\tilde{q}_{\rho_\kappa}y^{(\rho_\kappa+1)}\right)^{(\rho_\kappa)} + \left(\tilde{q}_{\rho_\kappa}y^{(\rho_\kappa)}\right)^{(\rho_\kappa+1)}\right\}$$ on $[1,\infty)$, where

$m \in \mathbb{N}_0$, $0 \leq \rho_0 < \ldots < \rho_m = n$, $\tilde{q}_{\rho_\kappa} = a_\kappa t^{\alpha_\kappa}$ with $a_\kappa > 0$, $a_m = 1$ and $\alpha_\kappa \in \mathbb{R}$ $(\kappa = 0,\ldots,m)$ subject to the conditions if $m > 0$:

$$2 > \frac{\alpha_\kappa - \alpha_{\kappa-1}}{\rho_\kappa - \rho_{\kappa-1}} > \frac{\alpha_{\kappa+1}-\alpha_\kappa}{\rho_{\kappa+1}-\rho_\kappa} \quad (\kappa = 1,\ldots,m-1) \text{ if } m > 1 \text{ resp. } 2 > \frac{\alpha_1-\alpha_0}{\rho_1-\rho_0} \text{ if } m = 1.$$

This expression can be perturbed by

$$Ny = \frac{i}{2}\sum_{j=0}^{n}(-1)^j\left\{\left(q_j y^{(j+1)}\right)^{(j)} + \left(q_j y^{(j)}\right)^{(j+1)}\right\} + \sum_{j=0}^{n}(-1)^j\left(p_j y^{(j)}\right)^{(j)}$$

where:

$$q_j^{(k)} = o\left(t^{\frac{1}{\rho_\kappa-\rho_{\kappa-1}}\left\{(\rho_\kappa-j)\alpha_{\kappa-1}+(j-\rho_{\kappa-1})\alpha_\kappa\right\} - \frac{k}{2}\frac{\alpha_\kappa-\alpha_{\kappa-1}}{\rho_\kappa-\rho_{\kappa-1}}}\right) \quad \text{for } \kappa \in \{1,\ldots,m\}$$

such that

$$2\rho_{\kappa-1} + 2 \leq 2j+1-k \leq 2\rho_\kappa+1 \quad \text{and } k = 0,\ldots, j+1$$

$$q_j^{(k)} = o\left(t^{\alpha_0-2\rho_0+2j-k}\right) \quad \text{for } 2j+1-k = 0,\ldots,2\rho_0+1 \quad \text{and } k = 0,\ldots,j+1$$

$$p_j^{(k)} = o\left(t^{\frac{1}{\rho_\kappa-\rho_{\kappa-1}}\left\{(\rho_\kappa-j+\frac{1}{2})\alpha_{\kappa-1} + (j-\frac{1}{2}-\rho_{\kappa-1})\alpha_\kappa\right\} - \frac{k}{2}\frac{\alpha_\kappa-\alpha_{\kappa-1}}{\rho_\kappa-\rho_{\kappa-1}}}\right)$$

for $\kappa \in \{1,\ldots,m\}$

such that $2\rho_\kappa+2 \leq 2j-k \leq 2\rho_\kappa+1$ and $k = 0,\ldots,j$.

$$p_j^{(k)} = o\left(t^{\alpha_o - 2\rho_o - 1 + 2j - k}\right) \quad \text{for } 2j-k = 0,\ldots,2\rho_o+1 \quad \text{and } k = 0,\ldots,j.$$

For $M+N$ the following result is obtained:
If $\alpha_o < 2\rho_o+1$, then $M+N$ is in the limit-point case and $\sigma_e(T_o(M+N)) = \mathbb{R}$.
If $\alpha_o > 2\rho_o+1$, then $M+N$ has equal deficiency indices with
$n+1 \leq \mathrm{def}(M+N) \leq 2n+1-\rho_o$ and $\sigma_e(T_o(M+N)) = \emptyset$.

A. Zettl

Spektraltheorie abgeschlossener gewöhnlicher Differentialoperatoren

Wir untersuchen das Spektrum von nichtselbsadjungierten gewöhnlichen Differentialoperatoren. Wie kann man das wesentliche Spektrum bestimmen in der Abhängigkeit vom Verhalten der Koeffizienten?

Referees: Dr. H. Frentzen
Dr. B. Schultze

Participants

Prof. H.E. Benzinger
Department of Mathematics
273 Altgeldhall
University of Illinois
Urbana, Illinois 61 801
U S A

Prof. Dr. B.L.J. Braaksma
Department of Mathematics
University of Groningen
P.O. Box 800
9700 AV Groningen

Niederlande

Prof. R.C. B R O W N
Department of Mathematics
University of Alabama
Alabama 35 486

U S A

Prof. Dr. A. Dijksma
Department of Mathematics
University of Groningen
P.O. Box 800
9700 AV Groningen

Niederlande

Prof. Dr. M.S.P. Eastham
Mathematics Department
Chelsea College,
Manresa Road,
London, SW3 6LX

Großbritannien

Prof. Dr. W. Eberhard
Fachbereich Mathematik
Universität Duisburg
Lotharstr. 65

4100 Duisburg

Prof. Dr. W.D. Evans
Department of Pure Mathematics
University College
Cardiff, CF1 1XL

Großbritannien

Prof. W.N. Everitt
Mathematics Department
University of Dundee
Dundee, DD1 4HN

Großbritannien

Mme. J. Fleckinger
U.E.R. de Mathêmatiques,
Universitê Paul Sabatier
118, Route de Narbonne
31 062 Toulouse Cêdex

Frankreich

Dr. G. Freiling
Fachbereich Mathematik
Universität Duisburg
Lotharstr. 65

4100 Duisburg

Dr. H. Frentzen
Fachbereich Mathematik
Universität Essen
Universitätsstr. 3

4300 Essen 1

Prof. R.C. Gilbert
Mathematics Department
California State University
Fullerton, California 92 634

U S A

Prof. Dr. H. Kalf
Fachbereich Mathematik
Universität Darmstadt
Schloßgartenstr. 7

6100 Darmstadt

Prof. Dr. H.-D. Nießen
Fachbereich Mathematik
Universität Essen
Universitätsstr. 3

4300 Essen 1

Prof. R.M. Kauffman
Department of Mathematics
Western Washington University
Bellingham, WA 98 225

U S A

D. Race
Department of Mathematics
University of Birmingham
P.O. Box 363
Birmingham B15 2TT

Großbritannien

Prof. I.W. Knowles
Department of Mathematics
University of Alabama in Birmingh.
Birmingham, Alabama 35 294

U S A

Prof. T.T. Read
Department of Mathematics
Western Washington University
Bellingham
Washington 98 225

U S A

Prof. H. Kurss
Department of Mathematics
Adelphi University
Garden City
L.I., New York 11 530

U S A

Prof. Dr. A. Schneider
Abteilung Mathematik
Universität Dortmund
Postfach 500 500

4600 Dortmund 50

Prof. S.J. Lee
Department of Mathematics
Pan American University
Edinburg
Texas 78 539

U S A

Dr. B. Schultze
Fachbereich Mathematik
Universität Essen
Universitätsstr. 3

4300 Essen 1

Prof. Dr. R. Mennicken
Fachbereich Mathematik
Universität Regensburg
Universitätsstr. 31

8400 Regensburg

Dr. H.S.V. de Snoo
Department of Mathematics
University of Groningen
Groningen, P.O. Box 800

9700 AV Groningen
Niederlande

B. Mergler
Fachbereich Mathematik
Universität Essen
Universitätsstr. 3

4300 Essen 1

Prof. A. Zettl.
Department of Mathematical Sciences
Northern Illinois University
De Kalb, Illinois 60 115

U S A